

Teacher's Handbook on Constructivist Approach of Teaching in Physics at Senior Secondary Level

2015-16



Regional Institute of Education
Mysore

Acknowledgment

I sincerely acknowledge the whole hearted support provided by our honorable Principal, Prof. D.G. Rao, for the program starting from conceptualizing to developing the material as well as organising various workshops.

I also thank Prof. G Vishwanathappa, Head, Department of Extension, for the guidance and support for organising various workshops , academic faculty for their valuable contribution in the development of the handbook and accounts department staff of the institute for their timely help and support. Coordinator wishes to thank NCERT for giving opportunity to conduct the program.

Preface

Constructivist learning theories argue that tasks and discourse modes that explicitly incorporate students' former experiences can form powerful learning environments. Furthermore, research shows that students develop intrinsic motivation such as domain-specific interest, when they feel challenged and personally involved. Accordingly, constructivist teaching settings in which students participate actively, bringing in their own experiences, may be well-suited to offer this kind of experience.

In this connection "*Teacher's Handbook on Constructive Approach of Teaching in Physics at Senior Secondary Level*" has been developed as a part of a PAC programme of Regional Institute of Education Mysore.. The present handbook is a comprehensive reference document containing relevant information on constructive approach in physics at senior secondary level.

In developing and bringing out the handbook, the information has been largely drawn up from the national and international publications, and comments received from the Expert Group and Faculty Members of the Department. I hope that the handbook will be useful to the teachers in understanding the Constructivist learning and concepts in physics. It is hoped that with the feedback provided by the users of this Manual, it will be of immense value.

Suggestions/comments of the users, experts and others will be solicited for improving it further.

Santosh Kumar
Coordinator

About the Handbook

This handbook is an outcome of series of workshop held at Regional Institute of Education, Mysore. This handbook gives an orientation to constructivism in Physics and its use in classroom.

The main objectives of teachers handbook are to:

- Facilitate the organizations which conduct training programme for teachers on a regular basis
- Equip teachers to understand the nuances of the components of Physics and develop a scientific perspective.
- Acquaint with the new development and changes in the field of knowledge and constructivism as reflected in the NCF -2005 syllabi and textbooks.
- Making teachers understand not only the subject but enable to analyze the topic through interdisciplinary approach
- Empower teachers to know how learners construct knowledge and facilitate the process in classroom
- Help them to acquire different strategies, competencies and skills for transacting new upcoming areas.
- Develop the competencies among the teachers to relate the topic in context of their experiences
- Enable teachers to promote self learning/ team work in and outside the classroom for effective transaction of the curriculum
- Develop competencies to monitor the progress of each child in the classroom as well as to manage with remedial inbuilt mechanism.
- Facilitate peer group teaching and encouraging cooperative learning in the classroom.

Handbook Development Team

Muhammad Rafeek E K

Mohammad Sherif K

Savio Austine

Suresh J

Reddy Chandrasekhar

Rajashekhar.S Patil

Laxmaiah Darla

V K Manohar

Raman Namboodiri C K

Niranjana K M

Biju K

Santosh Kumar (co-ordinator)

Contents

1. MOTION IN A PLANE	1
1.1 Introduction	1
1.2 Scalars and Vectors	2
1.3 Position and Displacement Vectors	4
1.4 Types of Vectors	4
1.5 Multiplication of a vector by Real Numbers	7
1.6 Addition and Subtraction of Vectors: Graphical Method	8
1.7 Resolution of Vectors	12
1.8 Vector Addition - Analytical Method	14
1.9 Product Of Two Vectors (Extended Study)	15
1.10 Motion in a Plane	17
1.11 Motion in a Plane with constant acceleration	18
1.12 Relative Velocity in two dimension	19
1.13 Projectile Motion	19
1.14 Uniform Circular Motion	22
2. SYSTEM OF PARTICLES AND ROTATIONAL MOTION	25
2.1 Introduction	25
2.2 Rigid body	26
2.3 Centre of Mass (COM)	26
2.4 Centre of gravity	29
2.5 Motion of Centre of Mass	30
2.6 Rotational Kinematics	31
2.7 Moment of Force	33

2.8 Angular Momentum 34

2.9 Equilibrium 36

2.10 Moment of Inertia 40

2.11 Kinematics of Rotational Motion 44

2.12 Rolling Motion 48

3. WAVES 51

3.1 Introduction 51

3.2 Transverse and longitudinal waves 51

3.3 Displacement relation in a progressive wave 53

3.4 The speed of a travelling wave 54

3.5 The principle of superposition of waves 58

3.6 Reflection of waves 59

3.7 Beats 62

3.8 Doppler effect 63

3.9 Worksheets 65

4. CURRENT ELECTRICITY 73

4.1 Introduction 73

4.2 Electric Current 75

4.3 Ohm's Law 82

4.4 Resistors Combination 89

4.5 Electric Power 91

4.6 Kirchoff's laws & Measurement devices 97

5. MOVING CHARGES AND MAGNETISM 107

5.1 Introduction 107

5.2 Magnetic Force on a Charged Particle & Current carrying conductor 109

5.3 Ampere Circuital Law 114

5.4 Torque on Coil Suspended Between Two Magnets 120

5.5 Electric Motor 120

5.6 Moving coil Galvanometer 123

6. ATOMS 127

6.1 Earlier atomic theories 127

6.2 Alpha particle scattering Experiment and Rutherford's Nuclear model of Atom 128

6.3 Atomic spectra 132

6.4 The Bohr Model of Hydrogen atom 133

6.5 Line Spectra of Hydrogen Atom 135

6.6 De Broglie explanation of Bohr's Second postulate of Quantization 138

Constructivism in Physics Teaching

Introduction

Constructivism is basically a theory based on observation and scientific study about how people learn. In the classroom, the constructivist view of learning can point towards a number of different teaching practices. In the most general sense, it usually means encouraging students to use active techniques to create more knowledge and then to reflect on and talk about what they are doing and how their understanding is changing. In fact, constructivism taps into and triggers the student's innate curiosity about the world and how things work.

A first trivial version of constructivism originating in the work of Piaget holds that knowledge is actively constructed by the learner and not passively transmitted by the educator. In addition, there is the radical constructivism of von Glasersfeld (1990), in which cognition is considered adaptive in the sense that it is based on and constantly modified by a learner's experience. Beyond that, there is the social constructivist version of Vygotsky, who in an effort to challenge Piaget's ideas developed a fully cultural psychology stressing the primary role of communication and social life in meaning formation and cognition. The latter version of constructivism is accentuated by theories of sociology of scientific knowledge (SSK), which argue that all knowledge is a social construct in the frame of science and technology studies.

Constructivist Class room

Constructivist teachers encourage students to constantly assess how the activity is helping them gain understanding. By questioning themselves and their strategies, students in the constructivist classroom ideally become "expert learners." This gives them ever-broadening tools to keep learning. With a well-planned classroom environment, the students learn HOW TO LEARN.

One might look at it as a spiral. When they continuously reflect on their experiences, students find their ideas gaining in complexity and power, and they develop increasingly strong abilities to integrate new information. One of the teacher's main roles becomes to encourage this learning and reflection process.

For example: Groups of students in a science class are discussing a

problem in physics. Though the teacher knows the "answer" to the problem, she focuses on helping students restate their questions in useful ways. She prompts each student to reflect on and examine his or her current knowledge. When one of the students comes up with the relevant concept, the teacher seizes upon it, and indicates to the group that this might be a fruitful avenue for them to explore. They design and perform relevant experiments. Afterward, the students and teacher talk about what they have learned, and how their observations and experiments helped (or did not help) them to better understand the concept.

Application of Social Constructivism

Scaffolding is crucial in a constructivist environment, and more so if it's social constructivist. Students must have a solid framework to guide their learning. By playing an active role in facilitating the construction of knowledge, the instructor supports students in their accomplishment of an otherwise impossible or difficult to accomplish task. Instructors must present instruction in such a way that new information is built upon the previous so that the new can be integrated appropriately.

Because students must build upon pre-existing knowledge, instructors can use schemata to build upon and enhance a student's intuition about the incoming information. By enhancing the accretion of knowledge, learners can be guided to tune existing schemata to coincide with their experiences. As multiple mindsets and understandings interact, students are given exposure to a variety of perspectives and approaches during their construction of knowledge. New or revised schemata arise as students discover areas where these are mismatched and instructors can further aid the student's developing schema.

Theoretical Implications

Situated cognition theory, by its very nature, supports social constructivist learning because it "shifts the focus from the individual to the sociocultural setting and the activities of the people within that setting. Knowledge accrues through the lived practices of the people in a society." Increasing participation in communities of practice enable learners to be immersed and included within the context of the learning goal. Learning by participating is not only immersive and contextual, but impacts everyone in the

community.

Cognitive apprenticeships are also important. Students participate in a safe environment by which they can learn contextually without taking risks that may be necessary in the real-world environment, but giving them the confidence to be able to do so once they achieve expertise.

Case-based learning, project-based learning, and problem-based learning all provide a framework for students to learn in a social constructivist setting as well. “How can I get my students to think?” is a question asked by many dedicated instructors. One can make use of narratives to engage students in difficult to solve problems. If they are comfortable with their own ignorance of a particular situation, and the environment is supportive of that ignorance and provides resources and peers for its resolution, they will collaboratively come up with the solutions.

Assessment must be done dynamically and dialectically. Waiting until the end of the term isn't helpful because instructors may miss those points where a student may have adopted a huge misunderstanding. If applicable to the class, the instructor can ask students to self-assess, but ultimately it is in the hands of the instructor to ascertain if the student is achieving even the loosest goals set. By dialectically engaging with the students regularly, an instructor can informally assess the students and keep track of the knowledge being created.

It seems to me that education in a constructivist or social constructivist setting means more work and responsibility for everyone involved.

- It means more work for the learner: The learner must be actively involved in the learning process and must avail him or herself to the learning materials that are available.
- It means more work for the instructor: An instructor must facilitate, engage, and be in constant dialog with the students. An instructor can't rely on previously-created formulas or lesson plans to present because the curriculum is driven by the learners. An instructor must also be constantly learning.
- It means that we have a lot of questions for which we have no answers yet: Do we want students who are well-factified or well-learned? Is being well-learned elitist or can we give this to almost everyone? What does this do to standardized tests? How do we assess how we stand educationally compared to other regions or other nations?

Constructivist teaching practices

- Encouraging students to participate in demonstration represented in classroom through questioning technique.
- Setting up classroom for inquiry based learning for important concepts by asking thoughtful, open ended questions to students and encourage them to ask questions to each other.
- Put students in situations that might challenge their previous misconceptions and will create contradictions that lead to discussions.
- Framing tasks that are based on analyzing, classifying, predicting etc.
- Providing students wait time after posing questions.
- Encouraging students to elaborate initial response by further questioning.
- Try to elicit maximum from students through questioning technique and alter instructional strategy based on student's response.
- Designing curriculum and organize information around conceptual clusters of problems and questions.
- Designing group activities for some tasks according to demonstrated cognitive complexity.
- Including active techniques like experimenting, demonstration, real world problems etc. to create more knowledge.

Role of Communication and Group processing in teaching

Teaching is considered as combination of knowledge and communication skills. Two way communication between teacher and learner in teaching learning process create interest and enthusiasm of student and removes fear and inhibition. A good communication with learners helps teacher to understand their point of view and any misconceptions if they have. Communication helps teacher to understand learner's potential skills that can be used to design teaching learning strategies.

Group processing involves leadership, decision making, communication and conflict resolution. Group processing techniques help students' build team skills, allow them to reflect on their learning process and outcomes, and provide teachers with continues feedback. Students feel more positive toward the subject matter; students' increase their communication and social skills. Moreover students become more confident and self-dependent and the self-esteem rises.

MOTION IN A PLANE



<http://wonderwhizkids.com/resources/content/imagesv4/physics/concept/kinematics/image1.gif>

Most motion that we see around us is in curved, rather than straight-line, paths. Motion along a curved path is two- or three-dimensional motion, and can be described in a similar fashion to one-dimensional motion.

1.1 Introduction

When we talk about one dimensional motion the directional aspect can be taken care of by + and - signs, as in one dimension only two directions are possible. But in order to describe motion of an object in two dimensions (a plane) or three dimensions (space), we need to use vectors. Therefore, it is necessary to learn the language of vectors.

The arc made by a basketball, the orbit of a satellite, a bicycle rounding a curve, a swimmer diving into a pool, water gushing out of a pipe, and a puppy chasing its tail are some examples of motions along curved paths. In fact, most motions in nature follow curved paths rather than straight lines. Motion along a curved path on a flat surface or a plane (such as that of a ball on a pool table or a skater on an ice rink) is two-dimensional, and thus described by two-dimensional kinematics. Both two- and three-dimensional kinematics are simple extensions of the one-dimensional kinematics developed for straight line motion.

1.2 Scalars and Vectors

Learning Outcomes:

1. Differentiates between scalars and vectors by citing examples.
2. Explains multiplication of a vector by a real number.
3. Describes addition and subtraction of vectors using triangle and parallelogram laws.
4. Explains magnitude of a vector.

Activity: Group Discussion

Teacher groups the students and asks them to list out a few physical quantities. Then, they are asked to classify them as 1) Physical quantities with magnitude only and 2) Physical quantities with magnitude and direction.

Teacher consolidates the concepts of scalars and vectors.

A **Scalar** is a quantity that has only magnitude and nothing to do with spatial direction. Many physical concepts such as length, time, temperature, mass, density, charge and volume are scalars. Each has scale or size, but no associated direction. They are specified by ordinary numbers and add and subtract in the usual way.

A **Vector** is a quantity having both magnitude and direction (Magnitude just means 'size' or the 'value'.)

In one dimensional (or straight-line motion), the direction of a vector can be given simply by a plus or minus sign. In two dimensions, however, we specify the direction of a vector relative to some reference frame (i.e. coordinate system), using an arrow having length proportional to the vector's magnitude and pointing in the direction of the vector.

Vector Notation:

Graphically a vector is represented by an arrow, defining the direction, the magnitude of the vector being indicated by the length of the arrow. The tail end of the arrow is called the origin or initial point of the vector, and the head is called the terminal point. Analytically a vector is represented by a letter with an arrow over it, as \vec{A} (in print it is represented as bold **A**) and its magnitude is denoted by $|\vec{A}|$ or **A**.

[Possible misconception: Electric current is a vector quantity.

Clarification: Electric current is a scalar quantity. Though it flows in a given direction, it doesn't follow the laws of vector addition (which will be discussed later)]

Examples of Scalar quantities:

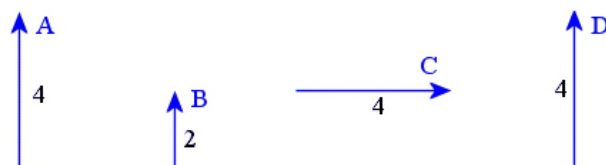
- A car moving at 35 km/h. (35km/hr is a magnitude and no direction is given).
- Mohan lives 3 km away from here. (3 km is magnitude and direction is not specified)
- It takes 3 hours to go from Delhi to Agra (3 hours is magnitude and it has no direction)

Other examples of scalar quantities include: Volume, density, temperature, mass, speed, time, length, distance, work and energy. Each of these quantities has magnitude only, and do not involve direction.

Examples of Vector Quantities:

1. I travel 30 km towards North. (magnitude is 30 km, direction is North - this is a displacement vector)
2. The train is going 80 km/h towards New Delhi (magnitude is 80 km/h, direction is 'towards Delhi' - it is a velocity vector)
3. The force on the bridge is 50 N acting downwards (the magnitude is 50 Newtons and the direction is down - it is a force vector)

Other examples of vectors include: Acceleration, momentum, angular momentum, magnetic and electric fields. Each of the examples above involves **magnitude and direction**.



The displacement vector **A** has direction 'up' and a magnitude of 4 cm. Vector **B** has the same direction as **A**, and has half the magnitude (2 cm). Vector **C** has the same magnitude as **A** (4 units), but it has different direction. Vector **D** is equivalent to vector **A**. It has the same magnitude and the same direction.

Note: It doesn't matter that **A** is in a different position to **D** - they are still considered to be **equivalent vectors** because they have the same magnitude and same direction.

We can write: $\mathbf{A}=\mathbf{D}$. We **cannot** write $\mathbf{A} = \mathbf{C}$ because even though **A**

and **C** have the same magnitude (4 cm), they have different direction. They are not equivalent.

1.3 Position and Displacement Vectors

Learning Outcomes:

1. Explains Position and Displacement Vectors by citing examples.
2. Distinguishes between path length and displacement.

Activity 1: General Discussion

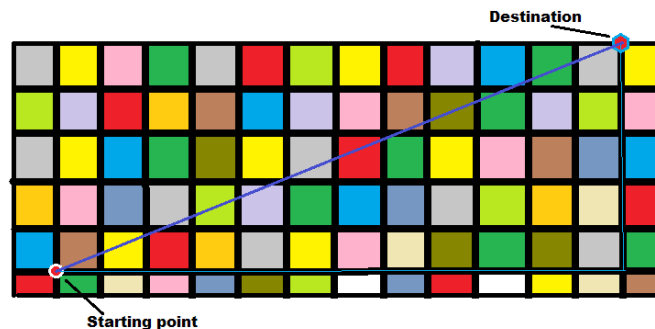
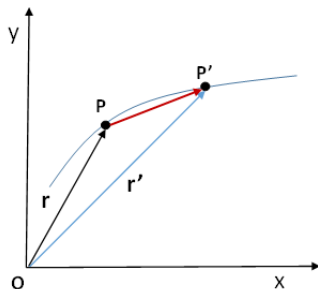
By drawing the figure in a graph board, teacher introduces the concepts of Position vector and displacement vector

Activity 2: General Discussion

Walking in a City

Suppose you want to walk from one point to another in a city with uniform square blocks and the starting and end points are as shown in the figure. The straight-line path that a helicopter might fly is also shown, but it is blocked to you and so you are forced to take a different path (as in figure). What are the path lengths traveled by the Helicopter and you?

What are the displacement of the Helicopter and you?



Teacher Consolidates the concepts of Path length (Distance) and Displacement.

1.4 Types of Vectors

Learning Outcomes:

1. Classifies different types of vectors
2. Classifies different types of vectors.

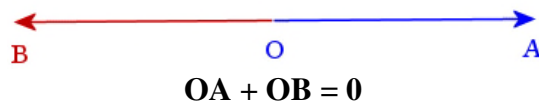
Activity: General Discussion

Using a chart, teacher explains different types of vectors like Equal vectors, Negative vector, Unit vector etc. [Chart]

Types of Vectors:

1) A **Null vector** is that vector which has zero magnitude and no specific direction. A vector which is not a null vector is a proper vector. All vectors assumed proper unless otherwise stated. It is also called **Zero vector**.

In a tug-of-war, if the teams are evenly matched at a certain instant and neither side is able to move. In this case, we can say:



The 2 force vectors **OA** and **OB**, operating in opposite directions, cancel out each other.

2) **Unit vector:** A unit vector is a vector having unit magnitude (magnitude = 1) and can take any direction. If **A** is a vector with $|A| \neq 0$, then $A/|A|$ is a unit vector having the same direction as **A**. Any vector **A** can be represented by a unit vector \hat{a} in the direction of **A** multiplied by the $|A|$. A one-dimensional unit vector is usually written \hat{i} .

3) **Equal Vector:** Two vectors are said to be equal if they have the same magnitude and same direction, wherever be their initial positions.

4) **Like Vectors:** Two vectors are said to be like vectors, if they have same direction but different magnitudes.

5) **Opposite Vectors:** The vectors of same magnitude but opposite in direction, are called opposite vectors.

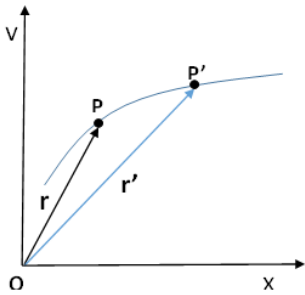
6) **Unlike Vectors:** The vectors of different magnitude acting in opposite directions are called unlike vectors.

7) **Coplanar Vectors:** Vectors lying in the same plane are called coplanar vectors and the plane in which the vectors lie is called plane of vectors (or vectorial plane).

8) **Collinear Vectors:** Vectors lying in the same line are called collinear vectors.

9) Free and Localized Vectors: So far we have seen examples of "free" vectors. We draw them without any fixed position. Another way of representing vectors is to use **directed line segments**. This means the vector is named using an **initial point** and a **terminal point**. Such a vector is called a "localized vector".

10) Position Vector: The vector which represents the relative position of an object at a particular instant is called position vector. If we assume that an object follows the curved path as shown in the figure, then to describe the position of the object moving in that plane, one need to choose a convenient point, say origin. Let P and P' be the positions of the object at time t and t' respectively. The vectors \mathbf{r} and \mathbf{r}' are called position vectors for the object at two given instants of time.



11) Displacement vector: If the object moves from P to P', the vector PP' (with tail at starting point P and tip at end point P') is called the displacement vector corresponding to the motion from point P (at time t) to point P' (at time t'). It is important that the displacement vector is the straight line joining the initial and final positions and does not depend on the actual path undertaken by the object between the two positions. The magnitude of displacement is either less or equal to the path length of an object between two points.

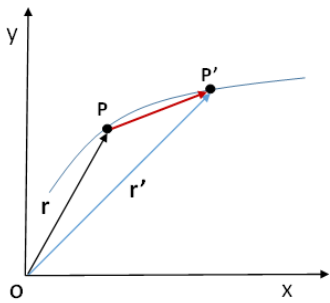
12) Velocity Vector: The average velocity (\dot{v}) of an object is the ratio of the displacement and the corresponding time interval

$$\dot{v} = \frac{\Delta r}{\Delta t} = \frac{\Delta x \hat{i} + \Delta y \hat{j}}{\Delta t} = \hat{i} \left(\frac{\Delta x}{\Delta t} \right) + \hat{j} \left(\frac{\Delta y}{\Delta t} \right)$$

$$\text{Or } \dot{v} = \dot{v}_x \hat{i} + \dot{v}_y \hat{j}$$

Since $\dot{v} = \frac{\Delta r}{\Delta t}$, the direction of the average velocity is the same as that of Δr . The velocity (instantaneous velocity is given by the limiting value of the average velocity as the time interval approaches to zero:

$V = \lim_{\Delta t \rightarrow 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt}$, the direction of velocity at any point on the path of an object is tangential to the path at that point and is in the direction of motion.



13) Acceleration Vector: The average acceleration \mathbf{a} of an object for a time interval Δt moving in x-y plane is the change in velocity divided by the time interval

$$\hat{a} = \frac{\Delta v}{\Delta t} = \frac{\Delta(\hat{v}_x \hat{i} + \hat{v}_y \hat{j})}{t} = \frac{\Delta(\hat{v}_x)}{t} \hat{i} + \frac{\Delta(\hat{v}_y)}{t} \hat{j}$$

$$\text{Or } \hat{a} = \hat{a}_x \hat{i} + \hat{a}_y \hat{j}$$

The acceleration (instantaneous acceleration) is the limiting value of the average acceleration as the time interval approaches zero;

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

1.5 Multiplication of a vector by Real Numbers

Learning Outcomes:

1. Explains the multiplication of a vector by a real number.

Activity: General Discussion

Teacher asks a student to do the following.

1. Walk 5 steps towards north from a given point. (say **A**)
2. Walk 2 times the distance as before in the same direction (say **B**)
3. Walk -2 times the distance as before in the same direction (say **C**)

The others are asked to represent the above three cases as vectors. Compare the three vectors. Teacher asks the students to complete the following worksheet.

Sn No	Vector	Magnitude	Direction	In multiple of A
1	A			
2	B			
3	C			

[Hint: **B** = 2**A** and **C** = -2**A**]

Teacher consolidates the multiplication of a vector by a real number.

Activity: General Discussion

Consider we have 3 weights tied to a beam. The first weight is $\mathbf{W}_1 = 5 \text{ N}$, the second is $\mathbf{W}_2 = 2 \text{ N}$ and the third is $\mathbf{W}_3 = 4 \text{ N}$. (Fig a)

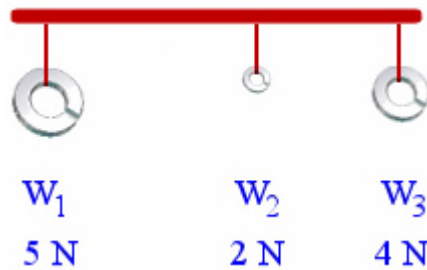


Fig a

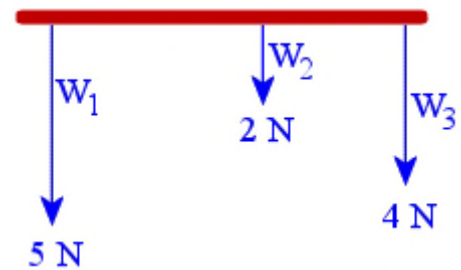


Fig b

We can represent these weights using a vector diagram (where the length of the vector represents the magnitude) as shown in Fig b. They are vectors because they all have a direction (down) and a magnitude.

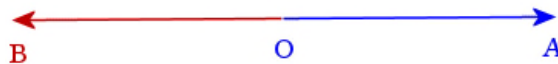
Since $5 = 2.5 \times 2$, we can write: $\mathbf{W}_1 = 2.5 \mathbf{W}_2$

Since $2 = 0.5 \times 4$, we can write: $\mathbf{W}_2 = 0.5 \mathbf{W}_3$

Since $4 = 0.8 \times 5$, we can write: $\mathbf{W}_3 = 0.8 \mathbf{W}_1$

Each of these statements is a scalar multiplication.

Vectors in Opposite Directions: Consider 2 teams playing a tug-of-war game. At the beginning of the game, they are very evenly matched and are pulling with equal force in opposite directions. We could name the vectors **OA** and **OB**. We can represent the tug of war using a vector diagram:



Where the **magnitude** of each vector is the same, but they are acting in **opposite directions**. In such a case, we indicate the opposite directions by use of a **negative sign**.

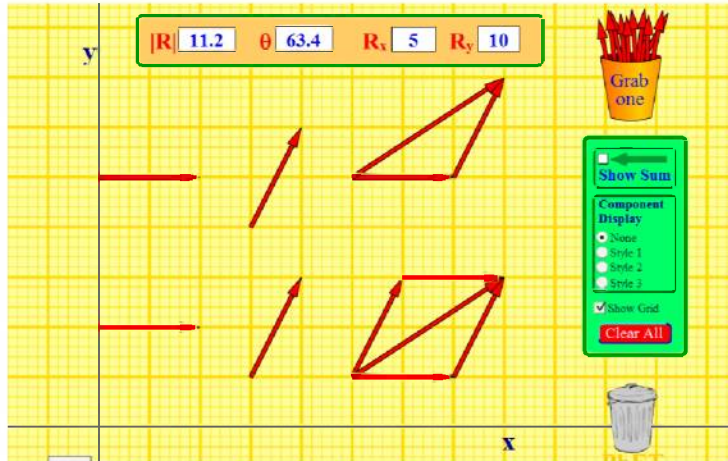
1.6 Addition and Subtraction of Vectors: Graphical Method

Learning Outcomes:

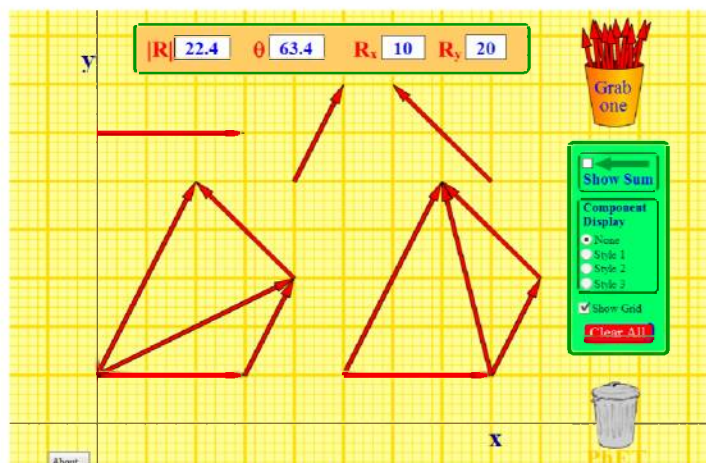
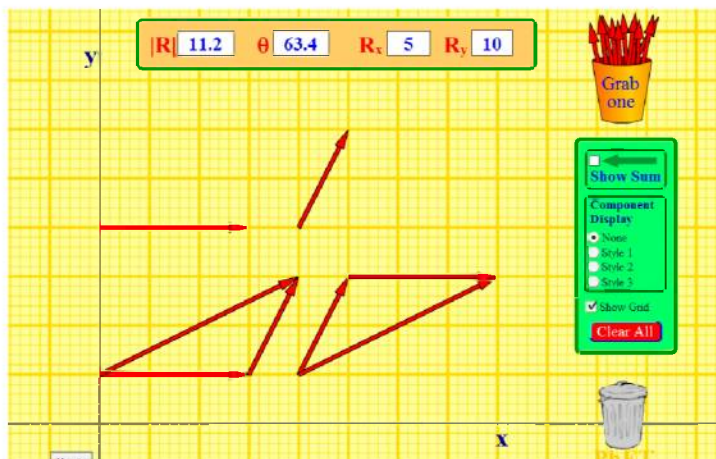
1. Explains addition and subtraction of vectors and solves numerical problems.

Activity: ICT - PhET Simulation

Using PhET simulation (<http://phet.colorado.edu/>), teacher introduces the concept of Triangle law and parallelogram law of vector addition.



Using the same applet, Commutative and Associative laws of vector addition are also explained.



NOTE: The graphical addition of vectors is limited in accuracy only by the precision with which the drawings can be made and the precision of the measuring tools. It is valid for any number of vectors.

Properties of Vector Addition:

- (i) Vector addition is commutative, i.e. $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$
- (ii) Vector addition is associative,
i.e. $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = \mathbf{B} + (\mathbf{C} + \mathbf{A}) = \mathbf{C} + (\mathbf{A} + \mathbf{B})$
- (iii) Vector addition is distributive, i.e. $m(\mathbf{A} + \mathbf{B}) = m\mathbf{A} + m\mathbf{B}$

NOTE: The addition of scalars involves only the addition of their magnitudes. But, when a vector is added with another vector we have to consider their direction also. A vector can be added with another vector provided both the vectors represent the same physical quantity. For example, the addition of a vector representing displacement of a body with another vector representing velocity of the body is meaningless.

ACTIVITY: Demonstration to verify the parallelogram law

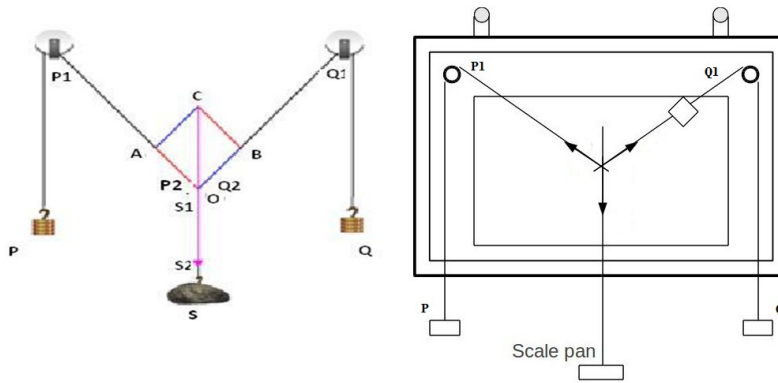
Materials Required:

Parallelogram Law of Forces apparatus (Gravesend's apparatus), Plumb line, Two hangers, with slotted weights, A body (a wooden block) whose weight is to be determined, Thin strong thread, White drawing paper sheet, Drawing pins, Mirror strip, Sharp pencil, Half meter scale, Set squares, Protractor.

Development:

1. Set up the Gravesend's apparatus and ensure its board is vertical. This can be tested using the plumb line. Test if the pulleys (let us name them - P1 and Q1) are frictionless. If you feel any friction, oil them.
2. Fix the white drawing paper sheet to the board using the drawing pins.
3. Take three pieces of strong threads and tie one end of all three together to make a knot. (Let us name this knotted end - O). This knot becomes the junction of the three threads.
4. From the other ends of the two threads, tie a weight hanger with the same slotted weights in each; we will name these weights as P and Q.
5. From the end of third thread tie the given body, which is the wooden block, which we will name as S.
6. Pass the threads with weights P and Q over the pulleys and let the third thread with the block S, stay vertical in the middle of the board.
7. The weights P, Q and the wooden block S acts as the three forces along

the three threads. At the junction O, the forces are in equilibrium.



8. Now adjust the weights P and Q (forces) such that the junction O stays in equilibrium slightly below the middle of the paper.
9. See that all the weights hang freely and that none of them touch the board or the table.
10. Mark the position of junction O on the paper using a sharp pencil.
11. Slightly disturb the weights P and Q and then leave them.
12. Once settled, note the position of junction O. Make sure that this point is very close to the earlier position.
13. Take the mirror strip and keeping it lengthwise under each thread, mark the position of the ends of the image of the thread in the mirror, covering the image by the thread. These new positions are P1, P2 for the thread with the weight P, and Q1 and Q2 for the thread with the weight Q and S1, S2 for the thread with the weight S.
14. Remove the paper from the board and with the help of the half metre scale draw lines through the points P1 and P2 to represent P, through points Q1 and Q2 to represent Q and through points S1 and S2 to represent S. These lines must meet at point O.
15. Assuming a scale of 1cm = 50 g, mark $OA = 3$ cm and $OB = 3$ cm to represent $P = 150$ g and $Q = 150$ g.
16. Complete parallelogram OACB using the set squares and join OC. This represents the resultant vector R which corresponds to the weight S.
17. Measure OC and multiply it by the scale (50 g) to get the value of the unknown weight (S).
18. For different sets of observation, change P and Q suitably.
19. We can find the weight of the wooden block (R) .
20. Take the mean of the two values to get the actual weight of the body.
21. To find the percentage error in the experiment, measure the actual weight of the body using a spring balance.

1.7 Resolution of Vectors

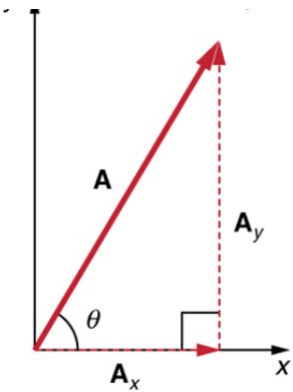
Learning Outcomes:

1. Defines unit vectors
2. Explains rectangular components of a vector

Activity: Problem Solving

Teacher asks a student to draw a vector **A** having magnitude 10cm and making an angle 30° with the X axis.

Draw a perpendicular from the tip of the vector to X and Y axes. Measure A_x and A_y from the graph.



Quantity	Measurement
A	
θ	
A_x	
A_y	
$A \sin \theta$	
$A \cos \theta$	
$\sqrt{A_x^2 + A_y^2}$	
$\sqrt{A^2 \cos^2 \theta + A^2 \sin^2 \theta}$	
$\frac{A_y}{A_x}$	
$\tan \theta$	
$\tan^{-1} \left(\frac{A_y}{A_x} \right)$	

Teacher consolidates the resolution of vectors.

How do you represent a vector in terms of rectangular components?

[Hint: $\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j}$]

How do you represent a vector in three dimension?

[Hint: $\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}$]

Statement: Resolution of a vector means the process of splitting of a vector into components.

In the example “Walking in the city” The path of an object can be represented with three vectors: one vector shows the straight-line path between the initial and final points of the motion, one vector shows the horizontal component of the motion, and one vector shows the vertical component of the motion. The horizontal and vertical components of the

motion add together to give the straight-line path **C**. When we go the opposite way, we can say, that the resultant vector **C** disintegrate to give us two perpendicular vectors – vector **A** and vector **B** or vector **C** resolves into vectors **A** and **B**.

Vectors that are perpendicular to each other are independent of each other, and their effects can be considered separately. If a vector can be taken apart so that we have only its effects on the X and Y axes, then one-dimensional equations can be used on each axis. Taking apart a vector like this is called resolving it into perpendicular components. (Figure A)

In general, such components are either:

- A) Horizontal and vertical, or
- B) Parallel and perpendicular to an inclined surface.

To resolve a vector into perpendicular components: (Figure B)

1. Place the vector on an X-Y grid.
2. Drop perpendicular lines to each axis.
3. Draw your components on each axis starting from the origin or draw the components tip-to-tail as though the vector was a resultant. If \hat{i} and \hat{j} are the unit vectors along x-axis and y-axis respectively then the vector **A** is written as $A = A_x \hat{i} + A_y \hat{j}$, where

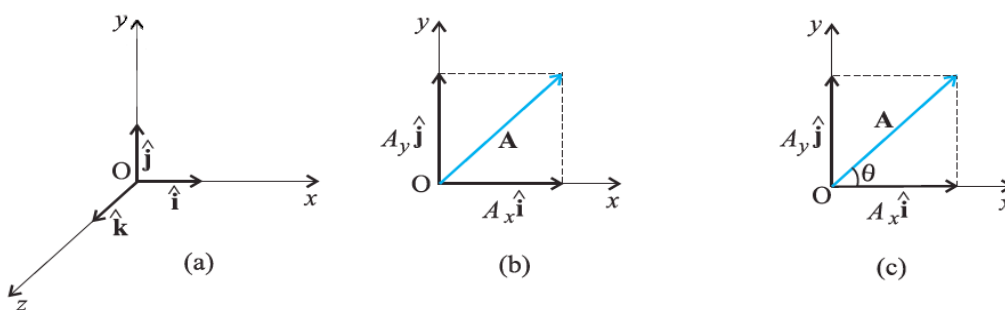
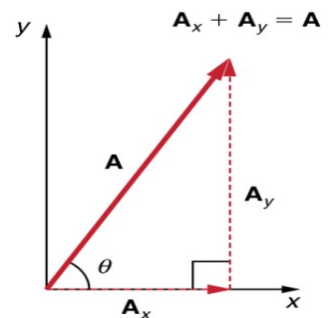
$$A_x = A \cos \theta$$

$$A_y = A \sin \theta$$

$$\text{Also, } A = \sqrt{A_x^2 + A_y^2 + 2 A_x A_y \cos \theta}$$

$$\tan(\theta) = \frac{A_y}{A_x}$$

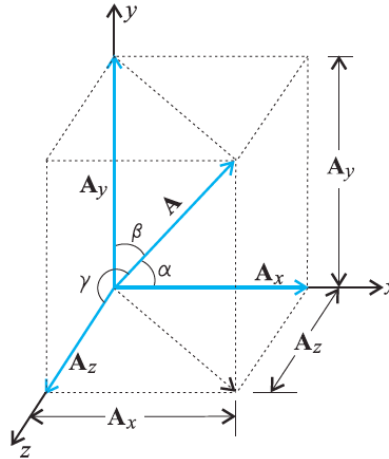
NOTE: Analytical Methods are ideal for finding vector components. (which we will see under next topic)



(a) Unit vectors \hat{i} , \hat{j} and \hat{k} lie along the x-, y-, and z-axes. (b) A vector **A** is resolved into its components A_x and A_y along x-, and y- axes. (c) A_x and A_y expressed in terms of \hat{i} and \hat{j} .

NOTE: Components of a vector are not vectors. They are scalars.

Resolution of vector in 3D:



A vector \vec{A} resolved into components along x, y and z axes

1.8 Vector Addition - Analytical Method

Learning Outcomes:

1. Explains vector addition in analytical method and solves numerical problems

Activity: Problem Solving

Teacher asks the students to write down \mathbf{A} and \mathbf{B} in terms of components.

[Hints: $\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}$ and $\mathbf{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k}$]

If $\mathbf{R} = \mathbf{A} + \mathbf{B}$, how do you represent R_x , R_y and R_z ?

[Hint:

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

$$= (A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}) + (B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k})$$

$$R_x \mathbf{i} + R_y \mathbf{j} + R_z \mathbf{k} = (A_x + B_x) \mathbf{i} + (A_y + B_y) \mathbf{j} + (A_z + B_z) \mathbf{k}$$

$$\Rightarrow R_x = A_x + B_x$$

$$\Rightarrow R_y = A_y + B_y$$

$$\Rightarrow R_z = A_z + B_z$$

1.9 Product Of Two Vectors (Extended Study)

Product of two vectors is of two types-

1. Scalar Product (Dot Product)
2. Vector Product (Cross Product)

1. Scalar Product OR Dot Product

If the product of two vectors is a scalar quantity, then the product itself is known as Scalar Product or Dot Product. The dot product of two vectors A and B having angle θ between them may be defined as the product of magnitudes of A and B and the cosine of the angle θ between them (when they are joined tail to tail).

$$\mathbf{A} \cdot \mathbf{B} = A B \cos\theta$$

Because a dot (\cdot) is used between the vectors to write their scalar product, therefore, it is also called dot product. The scalar product of vector A and vector B is equal to the magnitude, A of vector A times the projection of vector B onto the direction of A .

Why $\cos\theta$?

OK, to multiply two vectors it makes sense to multiply their lengths together but only when they point in the same direction. So we make one "point in the same direction" as the other by multiplying by $\cos\theta$.

Properties of Scalar Product

- (i) Scalar product is commutative, i.e. $A \cdot B = B \cdot A$
- (ii) Scalar product is distributive, i.e. $A \cdot (B + C) = A \cdot B + A \cdot C$
- (iii) Scalar product of two perpendicular vectors is zero.

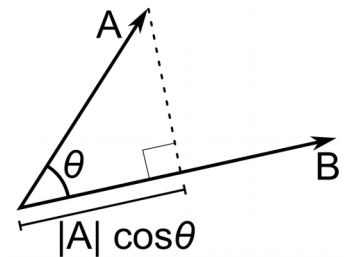
$$A \cdot B = AB \cos 90^\circ = 0$$

- (iv) Scalar product of two parallel vectors is equal to the product of their magnitudes, i.e.

$$A \cdot B = AB \cos 0^\circ = AB$$

- (v) Scalar product of a vector with itself is equal to the square of its magnitude, i.e.,

$$A \cdot A = AA \cos 0^\circ = A^2$$



(vi) Scalar product of orthogonal unit vectors

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

(vii) Scalar product in Cartesian coordinates

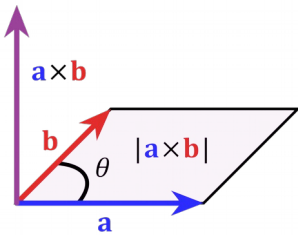
$$A \cdot B = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$A \cdot B = A_x B_x + A_y B_y + A_z B_z$$

2. Vector Product OR Cross Product

When the product of two vectors is another vector perpendicular to the plane formed by the multiplying vectors, the product is then called vector or cross product.

The cross product of two vector A and B having angle θ between them may be defined as "the product of magnitude of A and B and the sine of the angle θ , such that the product vector has a direction perpendicular to the plane containing A and B and points in the direction in which right handed screw advances when it is rotated from A to B through smaller angle between the positive direction of A and B".



$$\mathbf{A} \times \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \sin\theta \hat{a}$$

Where \hat{a} is the unit vector perpendicular to the plane containing A and B and points in the direction in which right handed screw advances when it is rotated from A to B through smaller angle between the positive direction of A and B.

Properties of Vector Product

(i) Vector product is not commutative, i.e. $A \times B \neq B \times A$

(ii) Vector product is distributive, i.e. $A \times (B+C) = A \times B + A \times C$

(iii) Vector product of two parallel vectors is zero, i.e.

$$A \times B = AB \sin 0^\circ = 0$$

(iv) Vector product of any vector with itself is zero.

$$A \times A = AA \sin 0^\circ = 0$$

(v) Vector product of orthogonal unit vectors

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 0$$

$$\hat{i} \cdot \hat{j} = \hat{k}, \hat{j} \cdot \hat{k} = \hat{i}, \hat{k} \cdot \hat{i} = \hat{j}$$

$$\hat{j} \cdot \hat{i} = -\hat{k}, \hat{k} \cdot \hat{j} = -\hat{i}, \hat{i} \cdot \hat{k} = -\hat{j}$$

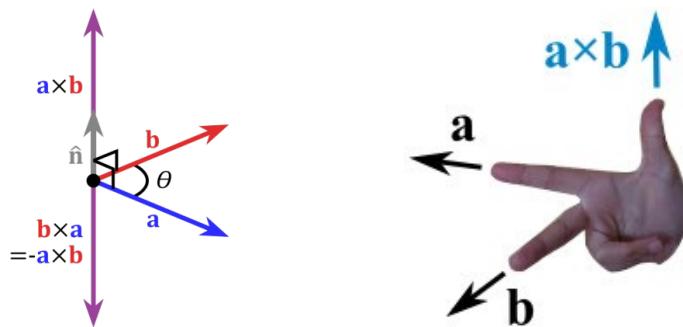
(vi) Vector product in Cartesian coordinates

$$A \times B = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$A \times B = (A_y B_z - A_z B_y) \hat{i} - (A_x B_z - A_z B_x) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$

Which Way?

The cross product could point in the completely opposite direction and still be at right angles to the two other vectors, as following



so we have the: "**Right Hand Rule**". With your right-hand, point your index finger along vector a , and point your middle finger along vector b : the cross product goes in the direction of your thumb.

1.10 Motion in a Plane

Learning Outcomes:

1. Explains position vector, velocity vector and acceleration vector

Activity: General Discussion

Teacher draws the motion of a particle on the board as shown. [figure]

Students are asked to draw the position vector of the point P.

Write the position vector \mathbf{r} in terms of rectangular components x and y .

[Hint: $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$]

Write the position vector \mathbf{r}' in terms of rectangular components. [Hint: $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$]

Draw the displacement vector ($\Delta\mathbf{r}$) from P to P'. What will be the displacement vector $\Delta\mathbf{r}$ in terms of position vector? [Hint: $\Delta\mathbf{r} = \mathbf{r}' - \mathbf{r}$]

How do you express $\Delta\mathbf{r}$ in terms of rectangular components? [Hint: $\Delta\mathbf{r} = (x'-x)\mathbf{i} + (y'-y)\mathbf{j} = \Delta x\mathbf{i} + \Delta y\mathbf{j}$]

Velocity

If the above displacement happens in Δt time, write the equation for average velocity.

$$\text{[Hint: } \dot{\mathbf{v}} = \frac{\Delta \mathbf{r}}{\Delta t} = \dot{v}_x \mathbf{i} + \dot{v}_y \mathbf{j}]$$

Then, how do you express the instantaneous velocity?

$$\text{[Hint: } \mathbf{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}]$$

Acceleration

If the velocity is changing, what will be the average acceleration in Δt

$$\text{time. [Hint: } \dot{\mathbf{a}} = \frac{\Delta \mathbf{v}}{\Delta t} = \dot{a}_x \mathbf{i} + \dot{a}_y \mathbf{j}]$$

Write the expression for instantaneous acceleration.

$$\text{[Hint: } \mathbf{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}]$$

1.11 Motion in a Plane with constant acceleration**Learning Outcomes:**

1. Formulates the equations of motion in a plane

Activity: General Discussion**1. $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$**

If velocity of a particle changes from \mathbf{v}_0 to \mathbf{v} in t seconds, what is its acceleration? [Hint: $\mathbf{a} = \frac{\mathbf{v} - \mathbf{v}_0}{t}$, $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$]

How do you write this equation in component form?

$$\text{[Hint: } \mathbf{v}_x = \mathbf{v}_{0x} + \mathbf{a}_x t, \mathbf{v}_y = \mathbf{v}_{0y} + \mathbf{a}_y t]$$

2. $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \frac{1}{2} \mathbf{a} t^2$

Let \mathbf{r}_0 and \mathbf{v}_0 be the position and velocity of a particle initially. If it changes to \mathbf{r} and \mathbf{v} in a time t , what is the average velocity?

$$\text{[Hint: } \mathbf{r} - \mathbf{r}_0 = \mathbf{v}_{\text{avg}} \times t, \Rightarrow \mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \frac{1}{2} \mathbf{a} t^2]$$

How do you write this equation in component form?

$$\text{[Hint: } \mathbf{x} = \mathbf{x}_0 + \mathbf{v}_{0x} t + \frac{1}{2} \mathbf{a}_x t^2 \text{ and } \mathbf{y} = \mathbf{y}_0 + \mathbf{v}_{0y} t + \frac{1}{2} \mathbf{a}_y t^2]$$

1.12 Relative Velocity in two dimension

Learning Outcomes:

1. Explains relative motion in two dimension and solves problems related to it.

Activity: General Discussion

If \mathbf{v}_A and \mathbf{v}_B are the velocities of A and B with respect to the ground, what will be the velocity of A related to B? [Hint: $\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$]

What will be the velocity of B related to A? [Hint: $\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A$]

Teacher consolidates the relative velocity in two dimension by solving some numerical problems.

1.13 Projectile Motion

Learning Outcomes:

1. Explains projectile motion
2. Formulates equations for time to reach maximum height, time of flight, maximum height and range of a projectile and solves problems related to them.

Activity: ICT, General discussion

The video *The Human Cannonball - Dave 'The Bullet' Smith Jr'* (<https://www.youtube.com/watch?v=uZyOzZnnwig>) is played. Where do we fix the net? What max height did he reach? How long did he was in air?

Teacher asks the students to draw the path of the projectile.

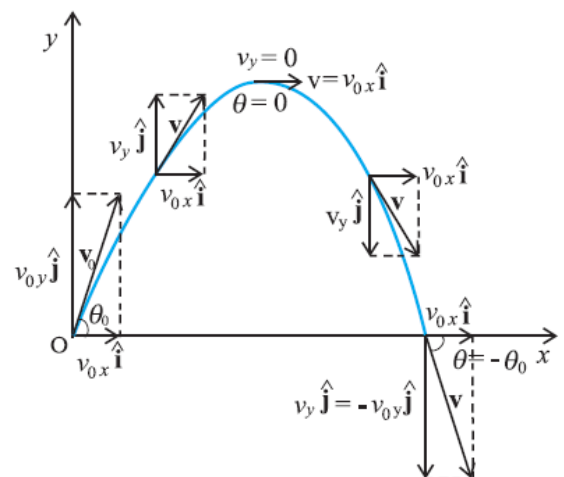
What is the acceleration acted upon the projectile along horizontal and vertical directions? [Hint: $a_x = 0$, $a_y = -g$]

If the motion starts from the origin ($x_0 = 0$, $y_0 = 0$), Write the equations of position and velocity after a time t .

[Hint: $x = v_0 t = (v_0 \cos\theta_0) t$, $y = (v_0 \sin\theta_0)t - 1/2 gt^2$

$v_x = v_{0x} = v_0 \cos\theta_0$, $v_y = (v_0 \sin\theta_0)t - gt$]

Teacher helps the students to formulate the equations for time to reach maximum height, time of flight, maximum height and range of a projectile. Projectile motion is the motion of an object thrown or projected into the air, subject to only the acceleration of gravity. Such an object is called a projectile, and its path is called its trajectory. The motion of falling objects, is a



simple one-dimensional type of projectile motion in which there is no horizontal movement. In this section, we consider two dimensional projectile motion, such as that of a football or other object (we assume air resistance to be negligible).

The key to analysing two-dimensional projectile motion is to break it into two motions, one along the horizontal axis and the other along the vertical. (This choice of axes is the most sensible, because acceleration due to gravity is vertical. So there will be no acceleration along the horizontal axis when air resistance is negligible.)

Resolving two dimensional motions into perpendicular components is possible because the components are independent.

Independence of Motion:

The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

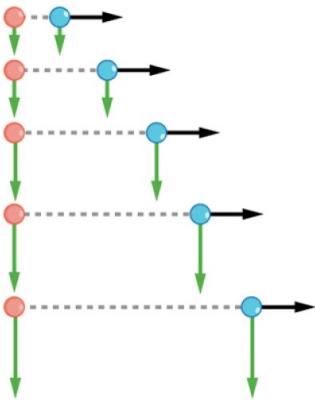
This is true in a simple scenario like that of walking in one direction first, followed by another (as we saw in “walking in a city” example. It is also true for more complicated motion involving movement in two directions at a time.

For example, let’s compare the motions of two balls. One ball is dropped from rest. At the same instant, another is thrown horizontally from the same height and follows a curved path. A stroboscope has captured the positions of the balls at fixed time intervals as they fall.

In vertical direction: It is remarkable that for each flash of the strobe, the vertical positions of the two balls are the same. This similarity implies that the vertical motion is independent of whether or not the ball is moving horizontally.

In horizontal direction: Careful examination of the ball thrown horizontally shows that it travels the same horizontal distance between flashes. This is due to the fact that there are no additional forces on the ball in the horizontal direction after it is thrown. This result means that the horizontal velocity is constant, and affected neither by vertical motion nor by gravity (which is vertical). The two-dimensional curved path of the horizontally thrown ball is composed of two independent one-dimensional motions (horizontal and vertical).

Comparing free fall and projectile motion using multi-flash photography. The picture on left shows the motions of two identical balls - one falls from rest, the other has an initial horizontal velocity. Each subsequent position is an equal time interval. In the figure on right hand side, the arrows represent horizontal and vertical velocities at each



position. The ball on the right has an initial horizontal velocity, while the ball on the left has no horizontal velocity.

Defining a Coordinate System

It is important to set up a coordinate system when analyzing projectile motion. One part of defining the coordinate system is to define an origin for the x and y positions. Often, it is convenient to choose the initial position of the object as the origin such that $x_0 = 0$ and $y_0 = 0$. It is also important to define the positive and negative directions in the x and y directions. Typically, we define the positive vertical direction as upwards, and the positive horizontal direction is usually the direction of the object's motion. When this is the case, the vertical acceleration, g takes a negative value (since it is directed downwards towards the Earth). However, it is occasionally useful to define the coordinates differently.

For example, if you are analyzing the motion of a ball thrown downwards from the top of a cliff, it may make sense to define the positive direction downwards since the motion of the ball is solely in the downwards direction. If this is the case, g takes a positive value. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls. This shows that the vertical and horizontal motions are independent.

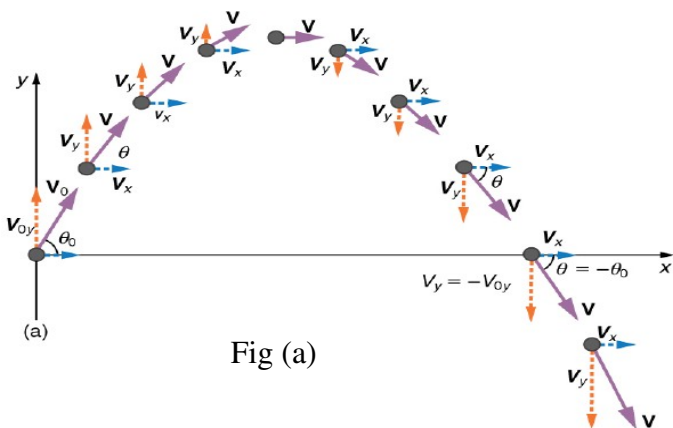


Fig (a)

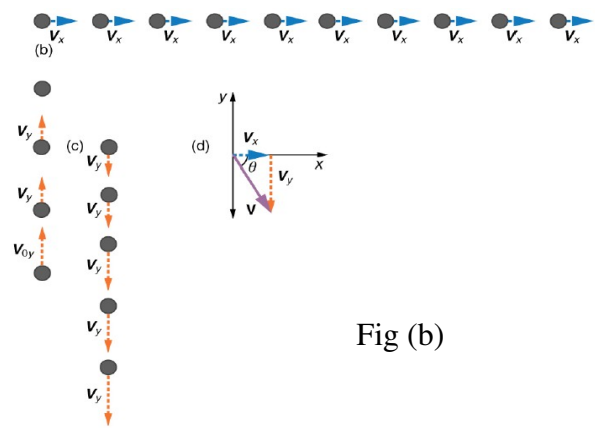


Fig (b)

Fig (a) and (b) Dividing the two-dimensional projectile motion into two independent one-dimensional motions along the vertical and horizontal axes.

NOTE: Free fall can be perceived as a special case of projectile motion with horizontal component of velocity = 0.

Points to remember:

1. The horizontal motion is simple, because $a_x = 0$ and v_x is thus constant.
2. The velocity in the vertical direction begins to decrease as the object rises; as the object falls towards the Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity.
3. At its highest point, the vertical velocity is zero.
4. The x - and y -motions are recombined to give the total velocity at any given point on the trajectory.

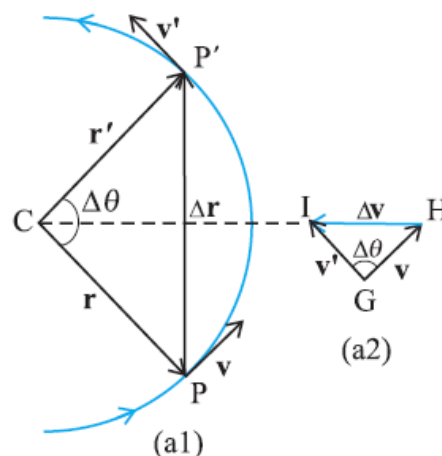
1.14 Uniform Circular Motion**Learning Outcomes:**

1. Explains uniform circular motion
2. Formulates equation for centripetal acceleration

Activity: General Discussion

Teacher asks the students to find out some examples of circular motion from daily life. What is the difference between uniform motion and uniform circular motion?

Draw the path of a particle moving in uniform circular motion. At the point P, draw the position vector and velocity vector. If the particle reaches at P' after a time Δt , draw the position vector and velocity vector. Draw the displacement vector.



Teacher helps the students to formulate the expression for centripetal acceleration using the similar triangles.

$$[\text{Hints: } \frac{\Delta v}{v} = \frac{\Delta r}{r} \Rightarrow \Delta v = \frac{v \Delta r}{r} \Rightarrow a_c = \frac{v^2}{r}]$$

Centripetal force

The word centripetal (not to be confused with the word centrifugal) means center seeking.

To understand the importance of a centripetal force, it is important to have a sturdy understanding of the Newton's first law of motion - the law of inertia which states that, "Objects in motion tend to stay in motion with the same speed and the same direction unless acted upon by an unbalanced force."

Moving objects will tend to naturally travel in straight lines; an unbalanced force is only required to cause it to turn. Thus, the presence of an unbalanced force is required for objects to move in circular motion. This force is called centripetal force. The acceleration towards the centre is called centripetal acceleration.

A larger force is needed if

- (i) the speed v of the ball is increased,
- (ii) the radius r of the circle is decreased,
- (iii) the mass m of the ball is increased.

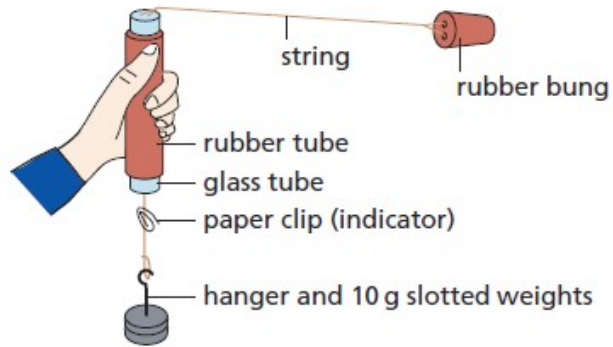
The rate of change of direction, and so the acceleration a , is increased by (i) and (ii).

Should the force be greater than the string can bear, the string breaks and the ball flies off with steady speed in a straight line along the tangent, i.e. in the direction of travel when the string broke (as Newton's first law of motion predicts). It is not thrown outwards.

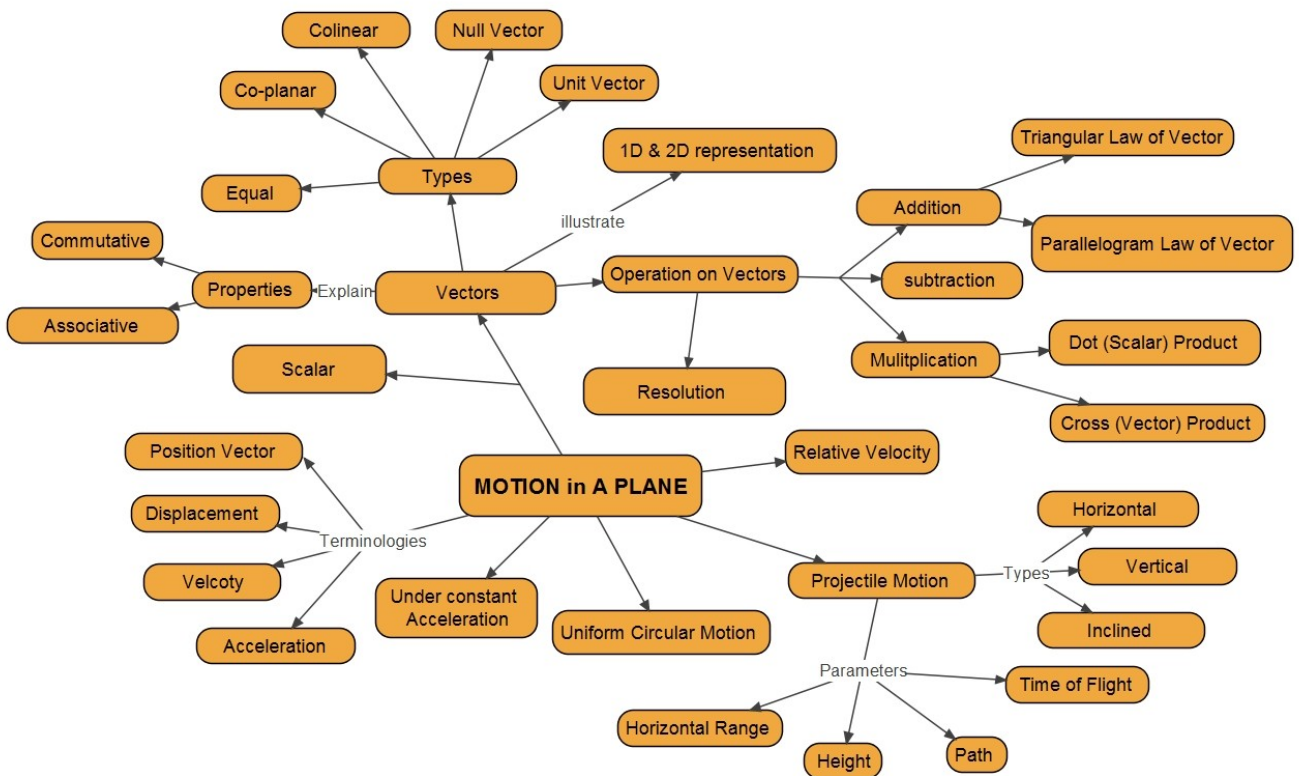
Whenever a body moves in a circle (or circular arc) there must be a centripetal force acting on it. Inthrowing the hammer it is the pull of the athlete's arms acting on the hammer towards the centre of the whirling path. When a car rounds a bend along a road, a frictional force is exerted inwards by the road on the car's tyres.

ACTIVITY: Factors deciding centripetal force

1. Use the apparatus as shown in figure to investigate the various factors that affect circular motion. This simple apparatus can be made at home or in any school physics lab.
2. Pass a thread through a hollow pipe
3. Make sure the rubber bung (or any small mass) is tied securely to the string and that the area around you is clear of other students.



4. The paper clip acts as an indicator to aid keeping the radius of the circular motion constant.
5. Spin the rubber bung at a constant speed while adding more weights to the holder. We can see that the radius of the orbit decreases.
6. If the rubber bung is spun faster, more weights must be added to the holder to keep the radius constant.
7. Are these findings in agreement with the formula for the centripetal force, $F = mv^2/r$?



SYSTEM OF PARTICLES AND ROTATIONAL MOTION



2.1 Introduction

In the previous chapters we discussed kinematics, dynamics and circular motion where both the size and shape of the body were neglected and the body is treated as point mass. In general an object consists of many number of particles, and motion of each particle is also to be considered in rotational motion. Rolling motion is the combination of linear motion and rotational motion.

As a physicist we love to look at something complicated and find in it something simple, and familiar. In this chapter we learn about mechanics of rotational body. Our attention is focused on the concept of rigid body, its motion with the interpretation of centre of mass and centre of gravity. we also introduce the concept of moment of inertia of a rigid body, derive equation for rotational kinetic energy and show how the angular momentum of a rigid body is conserved in the absence of external torque. At the end of this chapter we can understand that rolling motion is not a new task, it is just the combination of linear and rotational motion. By understanding rotational dynamics we will be able to derive moment of inertia of different types rigid bodies.

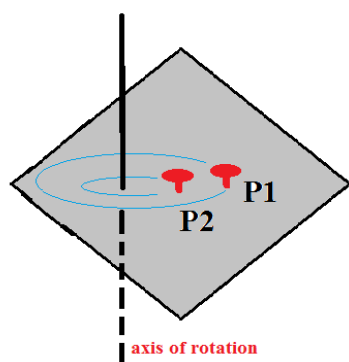
2.2 Rigid body

Learning Outcomes:

Describes concept of Rigid Body

Activity: Group Discussion

Ask the students to take a piece of card board of irregular shape and pierce a needle through it at any point, preferably away from geometric centre of the body, and let the needle be perpendicular to the plane of the card board. Place two drawing pins on the board and ask the students to observe the path of the pins as card board is rotated. Let the students identify the path of pins is circle. Now have them mark some other points on the card board and identify the path of each point during rotation. Ask the students where the centre of the circle in each case.



After the activity teacher summarize the following points-

1. A rigid body is a body in which all particles of the body remain fixed relative to one another. Its shape and size remains unchanged on application of external force
2. When a rigid body rotates, every point of it describes a circular path, whose centre lies on the axis of rotation (for example, when an insect is stacked to a point on a rotating fan, then the path followed by insect is circular)
3. Different particles of a rotating body may have different linear velocities
4. Angular displacement and hence angular velocity is the same for all the particles of a rotating rigid body

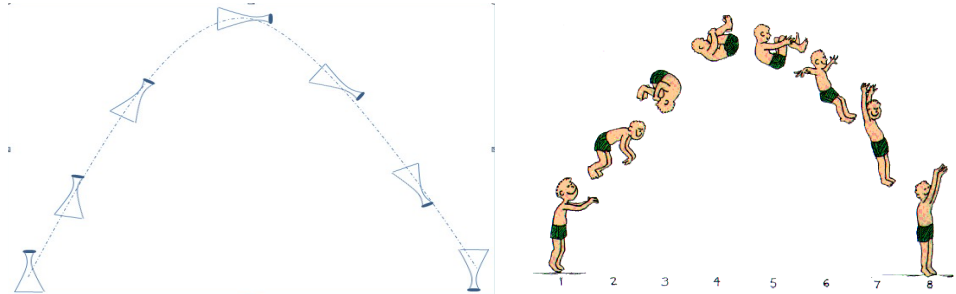
2.3 Centre of Mass (COM)

Learning Outcomes:

- Describes concept of centre of Mass.

Activity: Group Discussion

Teacher discuss the motion of the below mentioned pictures and summarize the important points -



- It's a point in an object (System of particles), which moves as if the whole mass of the system is concentrated at that point and all the external forces applied act at that point.
- If we have a two particle system the COM would there at exactly half of the distance between them when their masses are same. (Its applicable for 'n' particles system).

Think: Where would be COM for one particle system?

➤ General Representation of formula to find the centre of mass

- In 1-D: $X_{\text{COM}} = \frac{\sum m(i)x(i)}{\sum m(i)}$
- In 2-D: $X_{\text{COM}} = \frac{\sum m(i)x(i)}{\sum m(i)}$, $Y_{\text{COM}} = \frac{\sum m(i)y(i)}{\sum m(i)}$
- In 3-D: $X_{\text{COM}} = \frac{\sum m(i)x(i)}{\sum m(I)}$, $Y_{\text{COM}} = \frac{\sum m(i)y(i)}{\sum m(I)}$, $Z_{\text{COM}} = \frac{\sum m(i)z(i)}{\sum m(i)}$

* Here X,Y,Z represent the axis of Co-Ordinate System. x, y, z represent the distance to the position of the particle from the origin.

➤ The COM of the body in space can be found using

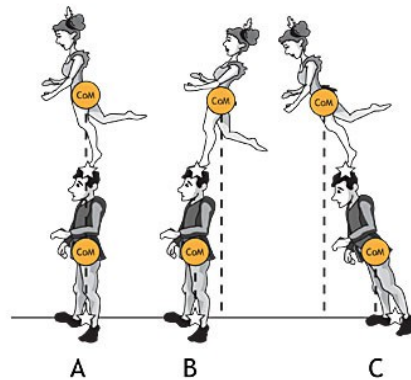
$$r_{\text{COM}} = \frac{\sum m(i)r(i)}{\sum m(i)} \quad \text{where 'r' represents } r = x_i i^{\wedge} + y_j j^{\wedge} + z_k k^{\wedge}$$

* Here 'r' represents vector of 'r'.

Extended Activity:

Standing on your tip toes is hard enough, but imagine trying to do it while staying balanced on top of someone else's head. Or, just as difficult, imagine staying upright with someone standing on your head. To keep balanced, the Nanjing acrobats must be aware of their centers of mass, and

the various forces called torque—that might cause them to rotate and fall out of balance. In this unit students will also learn how these two concepts make it possible for the acrobats to achieve their amazing acts of balance.



In Rigid Bodies:

- In symmetric bodies the COM lies within the body where as in asymmetric bodies the centre of mass may lie in plane of the body (eg: Banana)

Here in above examples we dealt with particle system which were finite, but when we consider rigid bodies its becomes difficult to find summation of all the particles, in fact its more tedious to find the number of particles in a rigid body. So to solve these kind of problems we can use Differential Equation as the tool and resolve the problem.

$$X_{COM} = \frac{1}{M} \int x \cdot dm, Y_{COM} = \frac{1}{M} \int y \cdot dm, Z_{COM} = \frac{1}{M} \int z \cdot dm.$$

Where M = total mass of the object, dm = mass of the particles taken into consideration, and x, y, z are distance from the origin.

Again for objects having continuous, uniform distribution of particles have uniform density (ρ). $\rho = M/V$ Mass per unit Volume...

Then $dm = (\rho \cdot V) dv$

where dv is volume occupied by a mass element dm , and V is total volume of the rigid body. Then the equations changes to

$$X_{COM} = \frac{1}{V} \int x \cdot dv, Y_{COM} = \frac{1}{V} \int y \cdot dv, Z_{COM} = \frac{1}{V} \int z \cdot dv.$$

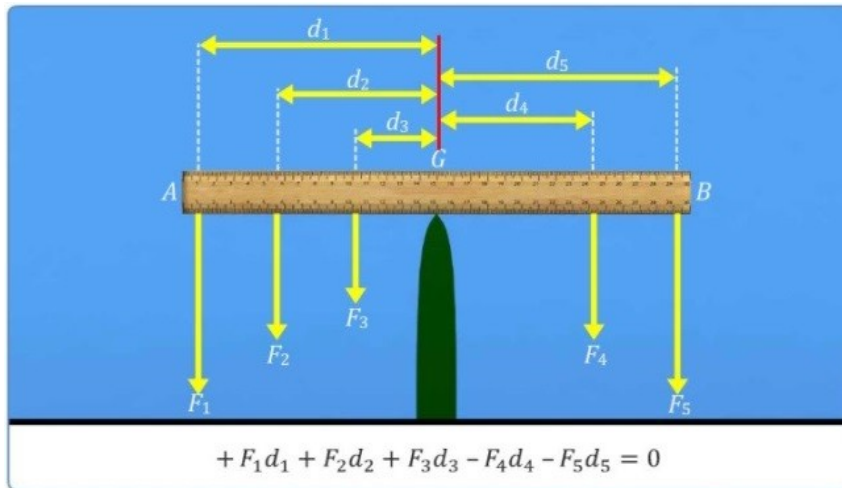
2.4 Centre of gravity

Learning Outcomes:

- Describes concept of centre of gravity

Activity 1: Group Discussion

Tools required: Yardstick/meter stick/cane/PVC pipe or any stick of a similar general size, clay or other weight, measuring tape.



(Source:

<http://www.learnnext.com/nextgurukul/wiki/concept/ICSE/IX/Physics/Principle-of-Moments.htm>,

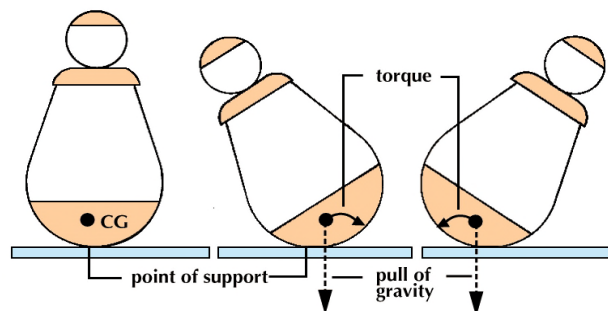
http://etc.usf.edu/clipart/36300/36326/centergrav_36326.htm)

Hold the stick/pipe in such a way that the ends will be resting on the index fingers. Slowly slide the fingers together until they meet. The point where the fingers will meet will be exactly the centre of gravity of the stick. When we first support the stick with two fingers, one finger (the one that is closer to the centre of gravity) will generally be holding a little more of the weight than the other. When we try to move your fingers closer together, the one that is carrying less weight will slide more easily. This finger will continue to slide more easily until it gets closer to the centre of gravity than the other finger, at which point the situation will reverse and the other finger will begin to slide faster. Your left and right fingers simply alternate moving until they meet at the centre of gravity, where both fingers support equal weight. Now increase the weight of the stick by attaching some clay/wax to some point. Again repeat the same

procedure to find the centre of gravity. Now is it the same point as the earlier one? If not, to which side the centre of gravity got shifted? Attach some more clay/wax to some other points and repeat the same. Always it will be the stick's centre of gravity where we can balance the stick on just one finger.

Activity 2:

Observe the movement of toy in the following figure. When the centre of gravity and point of support lies on same vertical line it is observed stationary. When you displace the point of support of the toy as shown in the figure, the centre of gravity and point of support are not lying on same vertical line, thereby it causes turning effect



It is a point where the entire weight of the body is supposed to act. The force acting on the centre of gravity is a vector sum of force on each particle of the system. The direction of this net force is directed always towards the centre of the earth.

2.5 Motion of Centre of Mass

Learning Outcomes:

- Generalize the concept of the motion of Centre of mass.

Activity: Group Discussion

1. As we know that rate of change of displacement is velocity.
So, the rate of change of displacement of centre of mass gives velocity of centre of mass

$$\frac{dR(cm)}{dt} = V(cm)$$

2. Hence, Velocity of centre of mass is given by,
3. $V_{cm} = (m_1v_1 + m_2v_2 + \dots + m_nv_n) / (m_1 + m_2 + \dots + m_n)$, (1)
4. As we know that rate of change of velocity is acceleration.

So, the rate of change of velocity of centre of mass gives acceleration of centre of mass

$$\frac{dV(cm)}{dt} = a(cm)$$

5. Hence, the acceleration of centre of mass is

$$\mathbf{a}_{cm} = (m_1\mathbf{a}_1 + m_2\mathbf{a}_2 + \dots + m_n\mathbf{a}_n)/(m_1+m_2+\dots+m_n) \quad (2)$$

6. From equation (1),

$$(m_1+m_2+\dots+m_n) \mathbf{V}_{cm} = (m_1\mathbf{v}_1+m_2\mathbf{v}_2 + \dots + m_n \mathbf{v}_n)$$

Therefore momentum of centre of mass is the vector sum of momenta of all individual particles of the system

$$M\mathbf{V}_{cm} = (m_1\mathbf{v}_1+m_2\mathbf{v}_2 + \dots + m_n \mathbf{v}_n)$$

From equation (2)

$$(m_1+m_2+\dots+m_n) \mathbf{a}_{cm} = (m_1\mathbf{a}_1+m_2\mathbf{a}_2 + \dots + m_n \mathbf{a}_n)$$

7. therefore, force on centre of mass is the vector sum of the forces acting on individual particles of the system i.e.,

$$8. M\mathbf{a}_{cm} = (m_1\mathbf{a}_1+m_2\mathbf{a}_2 + \dots + m_n \mathbf{a}_n) \text{ or } \mathbf{F}_{cm} = \mathbf{F}_1 + \mathbf{F}_2 + \dots + \mathbf{F}_n$$

9. It gives clear picture of linear motion of a rolling body and it helps to calculate the translatory kinetic energy. $KE_{translatory} = (1/2)MV_{cm}^2$

Test your-self.....

Problem: A girl of mass 'm' is standing on a platform of mass 'M' kept on smooth frictionless surface. If the girl starts to move on platform with a speed 'V' relative to the platform with what velocity relative to the surface does the platform recoil?

HINT: let V_g, V_p are the velocities of girl and plank respectively, From data, $V_{gp} = V = V_g - V_p$

$$\Rightarrow V_g = V + V_p \quad (1)$$

As there is no force on the system, momentum of system is constant.

$$\text{i.e., } mV_g + MV_p = 0 \quad (2)$$

from (1) and (2)

$$V_p = \frac{mV}{m+M}$$

2.6 Rotational Kinematics

Learning Outcomes:

- Describes concept of Rotational motion.
- Describe the terminology related to the rotational motion.

Activity: Group Discussion

Q) Why do tornadoes spin at all? And why do tornadoes spin so rapidly?

A) The answer is that air masses that produce tornadoes are themselves rotating, and when the radii of the air masses decrease, their rate of rotation increases.



Q) Have you observed the motion of an Ice skater ?

The skater starts her rotation with outstretched limbs and increases her spin by pulling them in toward her body. The same physics describes the exhilarating spin of a skater and the wrenching force of a tornado.

Clearly, force, energy, and power are associated with rotational motion.

look at angular acceleration—the rotational analogs of linear acceleration.

Angular Velocity

Uniform circular motion is a motion in a circle at constant speed and, hence, constant angular velocity. Recall that angular velocity ω was defined as the time rate of change of angle θ :

$$\omega = \frac{\Delta\theta}{\Delta t} ,$$

where θ is the angle of rotation

* The relationship between angular velocity ω and linear velocity v is given by

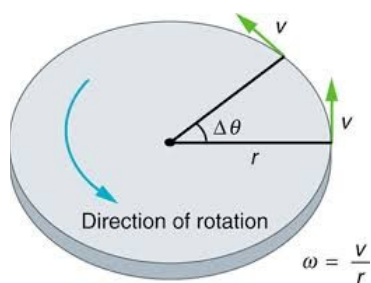
$$v = r\omega \text{ or } \omega = \frac{v}{r} ,$$

where r is the radius of curvature. According to the sign convention, the counter clockwise direction is considered as positive direction and clockwise direction as negative.

Angular Acceleration

Angular acceleration a is defined as the rate of change of angular velocity.

In equation form, angular acceleration is expressed as follows:



$$a = \frac{\Delta\omega}{\Delta t}$$

where $\Delta\omega$ is the change in angular velocity and Δt is the change in time. The units of angular acceleration are (rad/s)/s or rad/s^2 . If ω increases, then a is positive. If ω decreases, then a is negative.

It would be useful to know how linear and angular acceleration are related. In circular motion, linear acceleration is *tangent* to the circle at the point of interest, as seen in Thus, linear acceleration is called **tangential acceleration**(a_t).

Relation between linear velocity and angular velocity

From the above fig.7.16, the linear displacement in time Δt is PP'

Therefore, we know $PP' = r (\Delta\theta)$

On differentiating both sides w.r.t 'time'

$$\frac{PP'}{\Delta t} = r \frac{\Delta\theta}{\Delta t}$$

$$V = r \omega$$

On differentiating above equation w.r.t 'time', $a_t = r\alpha$ (where, a_t = tangential acceleration).

2.7 Moment of Force

Learning Outcomes:

- Describes concept of moment of force

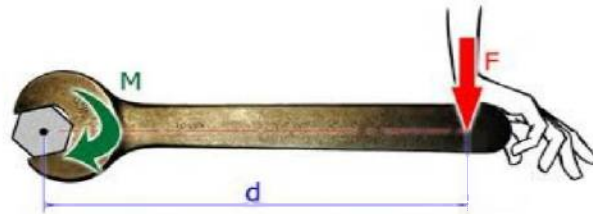
Activity: Group Discussion

The handle on a door is at the outside edge so that it opens and closes easily. A much larger force would be needed if the handle were near the hinge. Similarly it is easier to loosen a nut with a long spanner than with a short one.

The **turning effect of a force** is called the **moment of the force**. It depends on both the size of the force and how far it is applied from the pivot or **fulcrum**. It is measured by multiplying the force by the perpendicular distance of the line of action of the force from the fulcrum. The unit is the **newton metre** (N m).

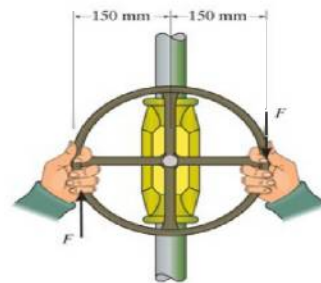
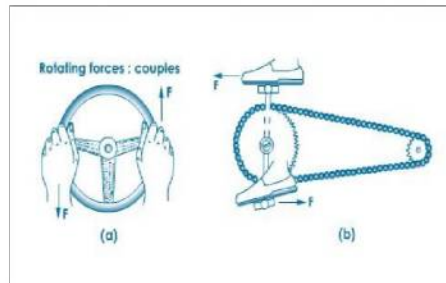
moment of a force = force \times perpendicular distance of the line of action of the force from fulcrum

$$\text{Moment of force} = \mathbf{r} \times \mathbf{F}$$



Couple:

Two equal, parallel and opposite forces acting on rigid body known as couple.

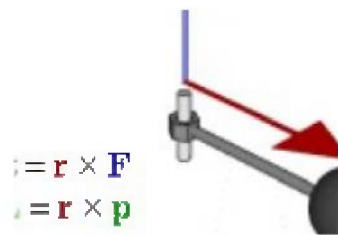


Moment of couple or Torque:

Turning effect given by the couple of forces is known as moment of couple or torque and is measured by,

$$\mathbf{T} = \mathbf{r} \times \mathbf{F}$$

Direction of torque can be known by using cross product rule of vectors or Maxwell's screw rule.



2.8 Angular Momentum

Learning Outcomes:

- Describes angular momentum
- Describe conservation of angular momentum.

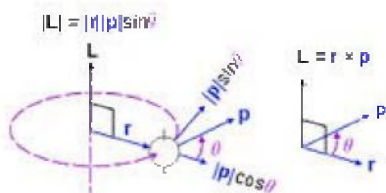
Activity: Group Discussion

Like moment of force, angular momentum is also vector product. It could also be referred as moment of (linear) momentum. ie., angular momentum is rotational analogue for linear momentum.

Mathematically angular momentum is represented as,

$$\begin{aligned} \mathbf{L} &= \mathbf{r} \times \mathbf{p} \\ \mathbf{L} &= r p \sin\theta \end{aligned} \tag{1}$$

Where, \mathbf{r} = position vector
 \mathbf{P} = linear momentum



Note: from the figure we can understand the direction of angular momentum

Differentiating equation (1) on both sides w.r.t 't'

$$\frac{dL}{dt} = \frac{d(r \times p)}{dt}$$

On simplification, $\frac{dL}{dt} = \tau$

Thus, we can conclude that, the rate of change of angular momentum of a particle with respect to time is equal to the torque acting on it.

$$\frac{dp}{dt} = F_{\text{ext}} = 0$$

Therefore, $\frac{dL}{dt} = \tau_{\text{ext}} = 0$

Hence, $\frac{dL}{dt} = 0$

Ie, $L = \text{constant}$

Conservation of Angular Momentum:

An object that moves in a circle is not in uniform motion. For example, the moon's nearly circular motion about the earth is an accelerated motion. Although the speed of the moon in its orbit is constant, the moon is always changing the direction of its motion. For this reason, linear momentum is not conserved for the motion of the moon because the requirement for motion of unchanging direction in a straight line is violated. However, there is a conserved quantity associated with revolving motion. You have undoubtedly seen examples of it as you have watched ice skaters in their spins, divers or gymnasts in their tucks, or a falling cat twisting to land on its feet. Think again of the moon revolving about an axis through the center of the earth. If we multiply the mass of the moon by its constant speed and then again by the constant radius of the circular orbit, we create a quantity that is unchanging during the motion of the moon. We call this quantity **angular momentum**, *angular momentum ! mass "speed" distance to axis of revolution*. In those cases where the mass is not concentrated in a small volume, we must think of extended objects subdivided into many small mass "particles" such that the angular momentum of the whole object is the sum of angular momenta of its parts. Just as net forces change linear momentum, some kinds of forces can change angular momentum. Forces that impart an additional "twist" (technically called a **torque**) to the revolving object can change its orbital

speed and, hence, its angular momentum. Brakes exert torques on rotating wheels to reduce angular momentum. However, there is a class of forces that does not change the angular momentum. These are forces that act along the line that connects the moving object to the axis of revolution. In the case of the moon and earth, this is the line from the center of the moon to the center of the earth. Forces along this special line, such as the force of gravity attracting the moon to the earth, impart (nor subtract) no additional “twist” to the system.

We say that with this in mind **Law of Conservation of Angular Momentum**: In the absence of any net torque, the angular momentum of an object revolving about an axis is conserved, i.e., the product of the mass, the speed, and the distance from the object to the axis of revolution is a constant in time. An ice skater makes use of this principle. Using her feet and with her hands outstretched, she creates enough torque on her body to begin to spin slowly about a vertical axis through the center of her body. Then she quickly draws her hands inward to her chest. The force she uses to draw her hands inward exerts no torque on her (it is along the line connecting the masses of her hands to the axis of rotation). Under these conditions her angular momentum is conserved. But since she has decreased the distance of her hands (revolving masses) from the axis of revolution, her speed of revolving motion must increase accordingly so that the angular momentum is conserved. If one thing decreases (distance), the other (speed) must increase. Thus, she begins to spin very rapidly. It is the same conservation law that operates when divers and cats execute their maneuvers.

2.9 Equilibrium

Learning Outcomes:

- Describes equilibrium.
- Describe the conditions of equilibrium.

Activity: Group Discussion

Q) What is Equilibrium? What are the conditions for Equilibrium?

A) What might desks, bridges, buildings, trees, and mountains have in common—at least in the eyes of a physicist? The answer is that they are ordinarily motionless relative to the Earth. Furthermore, their acceleration is zero because they remain motionless. That means they also have something in common with a car moving at a constant velocity, because anything with a constant velocity also has an acceleration of zero. Now,

the important part—Newton’s second law states that $\text{net } F = ma$, and so the net external force is zero for all stationary objects and for all objects moving at constant velocity. There are forces acting, but they are balanced. That is, they are in *equilibrium*.

The First Condition for Equilibrium:

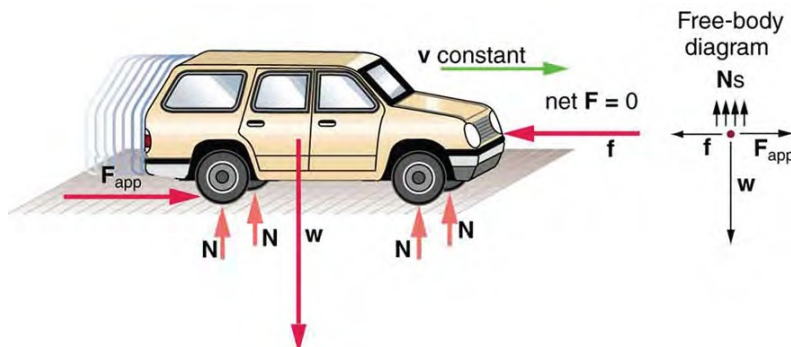
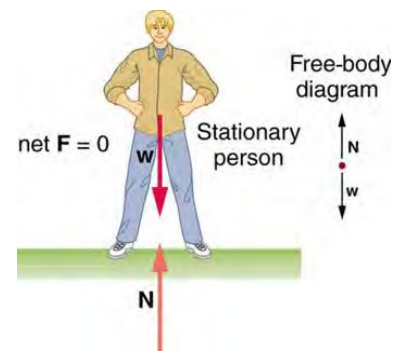
The first condition necessary to achieve equilibrium is the one already mentioned: the net external force on the system must be zero. Expressed as an equation, this is simply

$$\text{net } \mathbf{F} = 0$$

Note that if $\text{net } F$ is zero, then the net external force in *any* direction is zero. For example, the net external forces along the typical x - and y -axes are zero. This is written as

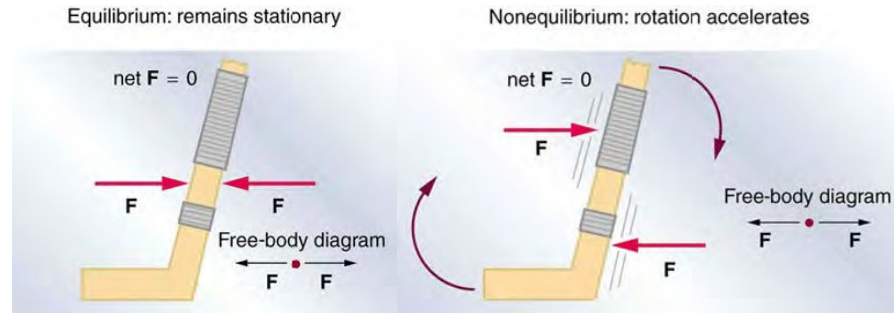
$$\text{net } F_x = 0 \text{ and } F_y = 0$$

Figures illustrate situations where $\text{net } F = 0$ for both static equilibrium (motionless), and dynamic equilibrium (constant velocity). This motionless person is in static equilibrium. The forces acting on him add up to zero. Both forces are vertical in this case.



This car is in dynamic equilibrium because it is moving at constant velocity. There are horizontal and vertical forces, but the net external force in any direction is zero. The applied force F_{app} between the tires and the road is balanced by air friction, and the weight of the car is supported by the normal forces, here shown to be equal for all four tires. However, it is not sufficient for the net external force of a system to be zero for a system to be in equilibrium. Consider the two situations illustrated in above pictures, where forces are applied to an ice hockey stick lying flat on ice. The net external force is zero in both situations shown in the figure; but in one case, equilibrium is achieved, whereas in the other, it is not. The ice hockey stick remains motionless whereas with the same forces applied in different places, the stick experiences accelerated rotation. Therefore, we

know that the point at which a force is applied is another factor in determining whether or not equilibrium is achieved.



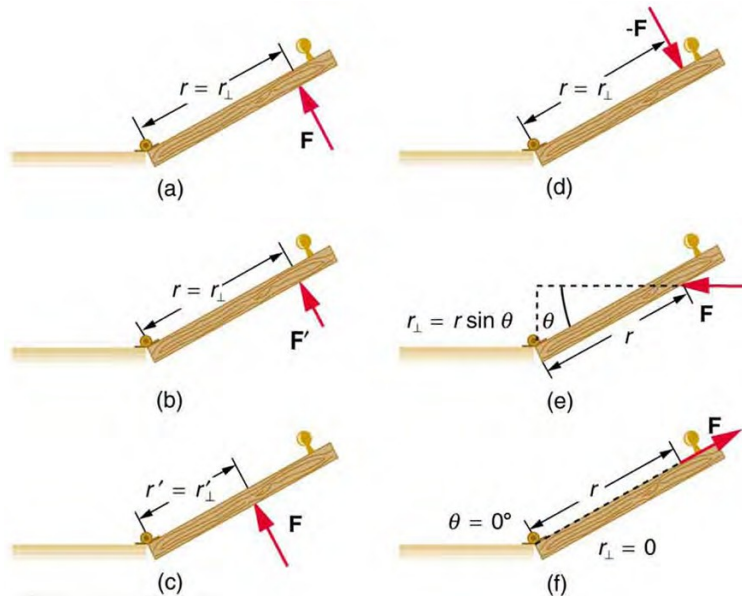
An ice hockey stick lying flat on ice with two equal and opposite horizontal forces applied to it. Friction is negligible, and the gravitational force is balanced by the support of the ice (a normal force). Thus, $\text{net } F = 0$. Equilibrium is achieved, which is static equilibrium in this case. The same forces are applied at other points and the stick rotates—in fact, it experiences an accelerated rotation. Here $\text{net } F = 0$ but the system is *not* at equilibrium. Hence, the $\text{net } F = 0$ is a necessary—but not sufficient—condition for achieving equilibrium.

The Second Condition for Equilibrium:

The second condition necessary to achieve equilibrium involves avoiding accelerated rotation (maintaining a constant angular velocity). A rotating body or system can be in equilibrium if its rate of rotation is constant and remains unchanged by the forces acting on it. To understand what factors affect rotation, let us think about what happens when you open an ordinary door by rotating it on its hinges.

Several familiar factors determine how effective you are in opening the door. First of all, the larger the force, the more effective it is in opening the door—obviously, the harder you push, the more rapidly the door opens. Also, the point at which you push is crucial. If you apply your force too close to the hinges, the door will open slowly, if at all. Most people have been embarrassed by making this mistake and bumping up against a door when it did not open as quickly as expected. Finally, the direction in which you push is also important. The most effective direction is perpendicular to the door—we push in this direction almost instinctively.

Torque is the turning or twisting effectiveness of a force, illustrated here for door rotation on its hinges (as viewed from overhead). Torque has both magnitude and direction.



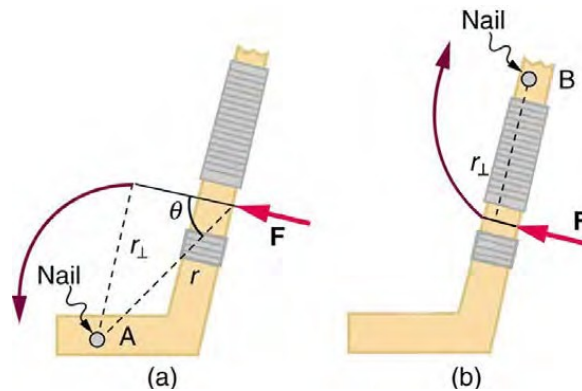
(a) Counterclockwise torque is produced by this force, which means that the door will rotate in a counterclockwise due to \mathbf{F} . Note that r_{\perp} is the perpendicular distance of the pivot from the line of action of the force. (b) A smaller counterclockwise torque is produced by a smaller force \mathbf{F}' acting at the same distance from the hinges (the pivot point). (c) The same force as in (a) produces a smaller counterclockwise torque when applied at a smaller distance from the hinges. (d) The same force as in (a), but acting in the opposite direction, produces a clockwise torque. (e) A smaller counterclockwise torque is produced by the same magnitude force acting at the same point but in a different direction. Here, θ is less than 90° . (f) Torque is zero here since the force just pulls on the hinges, producing no rotation. In this case, $\theta = 0^\circ$, The magnitude, direction, and point of application of the force are incorporated into the definition of the physical quantity called torque. **Torque** is the rotational equivalent of a force. It is a measure of the effectiveness of a force in changing or accelerating a rotation (changing the angular velocity over a period of time). In equation form, the magnitude of torque is defined to be,

$$\tau = rF \sin \theta$$

where τ (the Greek letter tau) is the symbol for torque, r is the distance from the pivot point to the point where the force is applied, F is the magnitude of the force, and θ is the angle between the force and the vector directed from the point of application to the pivot point, as seen in An alternative expression for torque is given in terms of the **perpendicular lever arm** r_{\perp} as shown in Figure which is defined as

so that,

$$r_{\perp} = r \sin \theta$$

$$\tau = r_{\perp} F$$


A force applied to an object can produce a torque, which depends on the location of the pivot point. (a) The three factors r , F , and θ for pivot point A on a body are shown here— r is the distance from the chosen pivot point to the point where the force F is applied, and θ is the angle between F and the vector directed from the point of application to the pivot point. If the object can rotate around point A, it will rotate counterclockwise. This means that torque is counterclockwise relative to pivot A. (b) In this case, point B is the pivot point. The torque from the applied force will cause a clockwise rotation around point B, and so it is a clockwise torque relative to B. The perpendicular lever arm r_{\perp} is the shortest distance from the pivot point to the line along which F acts; it is shown as a dashed line in Figures. Note that the line segment that defines the distance r_{\perp} is perpendicular to F , as its name implies. It is sometimes easier to find or visualize r_{\perp} than to find both r and θ . In such cases, it may be more convenient to use $\tau = r_{\perp} F$ rather than $\tau = rF \sin \theta$ for torque, but both are equally valid.

The **SI unit of torque** is newtons times meters, usually written as $\text{N} \cdot \text{m}$

2.10 Moment of Inertia

Learning Outcomes:

- Describes moment of inertia
- Describe the moment of inertia of rigid bodies.

Activity: Group Discussion

What is Inertia rotational motion(moment of inertia)?

For a point mass the **Moment of Inertia** is the mass times the square of perpendicular distance to the rotation reference axis and can be expressed as.

$$I = mr^2$$

For a rigid body, $I = m_1r_1^2 + m_2r_2^2 + \dots$ OR $I = \sum m_i r_i^2$

Where m_1, m_2 are masses of particles 1 and 2 and r_1, r_2 are the respective distances from the axis of rotation.

Moment of inertia tells you how difficult it is to rotate an object. It is equivalent to the mass in linear problems. Moment of inertia is proportional to an object's mass and to its distance from the rotational axis squared. The distance from the rotational axis dominates over the object's mass due to the square power. Thus the more mass an object has at its 'edges' the more moment of inertia it has. Thus if you grab a big long pole and hold it at the center, it is fairly easy to rotate. However, if you hold the same pole at the end and try to rotate it -not so easy! Finally, moment of inertias can be added as long as the rotating parts are rotating around the same axis. For example, the moment of inertia of a pole with a mass at its end is the sum of the individual moment of inertias of each object involved.

Examples

Flywheels have large moments of inertia to smooth out mechanical motion. This example is in a Russian museum.

The moment of inertia, otherwise known as the angular mass or rotational inertia, of a rigid body determines the torque needed for a desired angular acceleration about a rotational axis. It depends on the body's mass distribution and the axis chosen, with larger moments requiring more torque to change the body's rotation. It is an extensive (additive) property: the moment of inertia of a composite system is the sum of the moments of inertia of its component subsystems (all taken about the same axis). One of its definitions is the second moment of mass with respect to distance from an axis r , integrating over the entire mass.



Relation Between Angular Momentum, Angular Velocity and Moment of Inertia

Moment of inertia I is defined as the ratio of the ANGULAR MOMENTUM ' L ' of a system to its ANGULAR VELOCITY ω around a principal axis, that is

$$I = \frac{\omega}{L}$$

Calculation Of Moment Of Inertia

For bodies constrained to rotate in a plane, it is sufficient to consider their moment of inertia about an axis perpendicular to the plane. For bodies free to rotate in three dimensions, their moments can be described by a symmetric 3×3 matrix; each body has a set of mutually perpendicular principle axes for which this matrix is diagonal and torques around the axes act independently of each other.

About an axis

The moment of inertia about an axis of a body is calculated by summing mr^2 for every particle in the body, where r is the perpendicular distance to the specified axis. To see how moment of inertia arises in the study of the movement of an extended body, it is convenient to consider a rigid assembly of point masses. (This equation can be used for axes that are not principal axes provided that it is understood that this does not fully describe the moment of inertia).

Consider the kinetic energy of an assembly of N masses m_i that lie at the distances r_i from the pivot point P , which is the nearest point on the axis of rotation. It is the sum of the kinetic energy of the individual masses. This shows that the moment of inertia of the body is the sum of each of the mr^2 terms, that is,

$$E_K = \sum_{i=1}^N \left(\frac{1}{2} m_i \mathbf{v}_i \cdot \mathbf{v}_i \right) = \sum_{i=1}^N \left(\frac{1}{2} m_i (\omega r_i)^2 \right) = \frac{1}{2} \omega^2 \sum_{i=1}^N m_i r_i^2.$$

$$I_P = \sum_{i=1}^N m_i r_i^2.$$

Thus, moment of inertia is a physical property that combines the mass and distribution of the particles around the rotation axis. Notice that rotation about different axes of the same body yield different moments of inertia.

Example 1: Find the MI of system of two particles about an axis XX^1

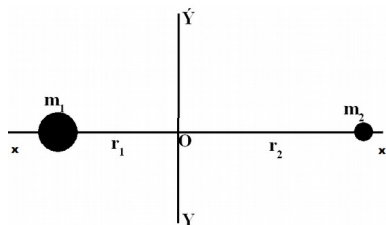
$$\text{M.I of } m_1 \text{ about } X = m_1 r_1^2$$

$$\text{M.I of } m_2 \text{ about } X^1 = m_2 r_2^2$$

$$\text{Therefore MI of system about } XX^1 = m_1 r_1^2 + m_2 r_2^2$$

Example 2: Three particles of masses m each are arranged at the corners of a triangle ABC respectively. Find the MI of system about the axis YY^1

$$\text{MI Of } m \text{ at } A = m \times 0 = 0$$



$$\text{MI of } M \text{ at } B = ml \frac{r}{2} J = \frac{mr^2}{4}$$

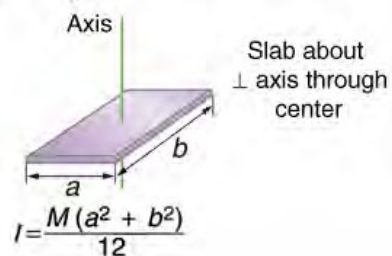
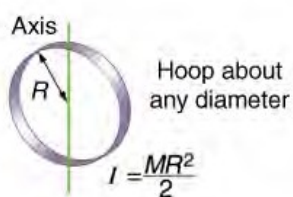
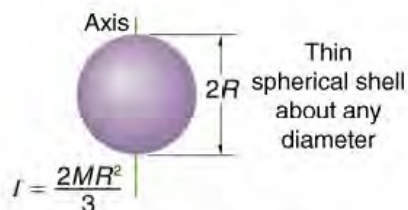
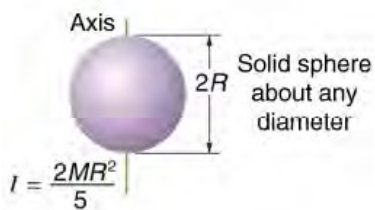
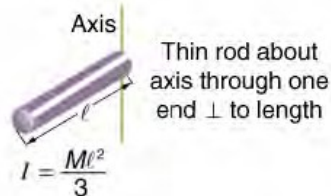
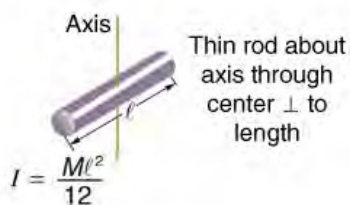
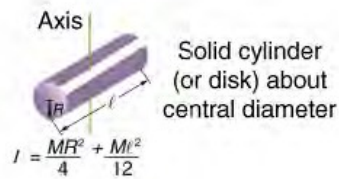
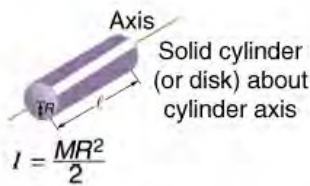
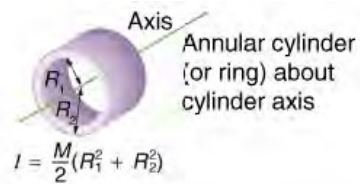
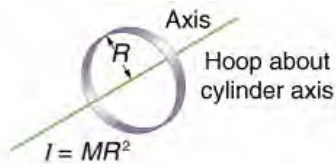
$$\text{MI of } m \text{ at } C : m r^2$$

Total MI of System of three particles About an axis XY is,

$$I = 0 + \frac{mr^2}{4} + mr^2$$

$$I = \frac{5mr^2}{4}$$

Moment of Inertia for Various Objects



2.11 Kinematics of Rotational Motion

Learning Outcomes:

- Describes Kinematics of Rotational Motion
- Describes Dynamics of Rotational Motion
- Describe the conservation of angular momentum.

Activity: Group Discussion

Just by using our intuition, we can begin to see how rotational quantities like θ , ω , and α are related to one another. For example, if a motorcycle wheel has a large angular acceleration for a fairly long time, it ends up spinning rapidly and rotates through many revolutions. In more technical terms, if the wheel's angular acceleration α is large for a long period of time t , then the final angular velocity ω and angle of rotation θ are large.

The wheel's rotational motion is exactly analogous to the fact that the motorcycle's large translational acceleration produces a large final velocity, and the distance traveled will also be large. Kinematics is the description of motion. The **kinematics of rotational motion** describes the relationships among rotation angle, angular velocity, angular acceleration, and time. Let us start by finding an equation relating ω , α , and t . To determine this equation, we recall a familiar kinematic equation for translational, or straight-line, motion:

$$v = v_0 + at \text{ (constant } a\text{)}$$

Note that in rotational motion $a = at$, and we shall use the symbol a for tangential or linear acceleration from now on. As in linear kinematics, we assume a is constant, which means that angular acceleration α is also a constant, because $a = r\alpha$. Now, let us substitute $v = r\omega$ and $a = r\alpha$ into the linear equation above:

$$r\omega = r\omega_0 + r\alpha t.$$

The radius r cancels in the equation, yielding

$$\omega = \omega_0 + at \text{ (constant } a\text{)},$$

where ω_0 is the initial angular velocity. This last equation is a *kinematic relationship* among ω , α , and t —that is, it describes their relationship without reference to forces or masses that may affect rotation. It is also precisely analogous in form to its translational counterpart. Starting with the four kinematic equations we developed in **One-Dimensional Kinematics**, we can derive the following four rotational kinematic equations (presented together with their translational counterparts):

ROTATIONAL	TRANSLATIONAL	RELATIONSHIP
θ	x	$\Theta = X/R$
ω	v	$\omega = v/r$
α	a	$\alpha = at/r$

ROTATIONAL	TRANSLATIONAL	
$\theta = \omega t$	$x = vt$	
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$	(constant α , a)
$\theta = \omega_0 t + 1/2 \alpha t^2$	$x = v_0 t + 1/2 at^2$	(constant α , a)
$\omega^2 = \omega_0^2 + 2\alpha\theta$	$v^2 = v_0^2 + 2ax$	(constant α , a)

In these equations, the subscript 0 denotes initial values (θ_0 , x_0 , and t_0 are initial values), and the average angular velocity $\bar{\omega}$ - and average velocity \bar{v} - are defined as follows:

$$\bar{\omega} = \omega_0/2 + \omega/2 \quad \text{and} \quad \bar{v} = v_0/2 + v/2$$

The equations given above in Table can be used to solve any rotational or translational kinematics problem in which a and α are constant.

Dynamics of Rotational Motion: Rotational Inertia

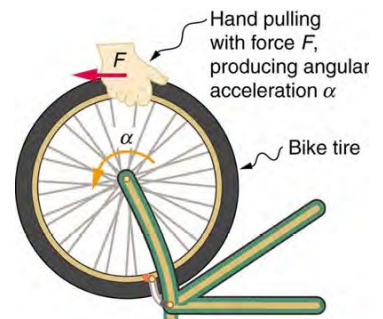
If you have ever spun a bike wheel or pushed a merry-go-round, you know that force is needed to change angular velocity as seen in figure.

Force is required to spin the bike wheel. The greater the force, the greater the angular acceleration produced. The more massive the wheel, the smaller the angular acceleration. If you push on a spoke closer to the axle, the angular acceleration will be smaller.

In fact, your intuition is reliable in predicting many of the factors that are involved. For example, we know that a door opens slowly if we push too close to its hinges. Furthermore, we know that the more massive the door, the more slowly it opens. The first example implies that the farther the force is applied from the pivot, the greater the angular acceleration; another implication is that angular acceleration is inversely proportional to mass.

These relationships should seem very similar to the familiar relationships among force, mass, and acceleration embodied in Newton's second law of motion. There are, in fact, precise rotational analogs to both force and mass.

To develop the precise relationship among force, mass, radius, and angular acceleration, consider what happens if we exert a force F on a point mass m that is at a distance r from a pivot point, as shown in Figure. Because the force is perpendicular to r , an acceleration $a = F/m$ is obtained in the direction of F . We can rearrange this equation such that $F = ma$ and



then look for ways to relate this expression to expressions for rotational quantities. We note that $a = r\alpha$, and we substitute this expression into $F = ma$, yielding $F = mr\alpha$.

Recall that **torque** is the turning effectiveness of a force. In this case, because \mathbf{F} is perpendicular to r , torque is simply $\tau = Fr$. So, if we multiply both sides of the equation above by r , we get torque on the left-hand side. That is,

$$rF = mr^2\alpha$$

or

$$\tau = mr^2\alpha$$

This last equation is the rotational analog of Newton's second law ($F = ma$), where torque is analogous to force, angular acceleration is analogous to translational acceleration, and mr^2 is analogous to mass (or inertia). The quantity mr^2 is called the **rotational inertia** or **moment of inertia** of a point mass m a distance r from the center of rotation.

Angular Momentum in case of Rotation about a Fixed Axis

It is interesting to watch the performance of a flying acrobats, gymnasts, ice-skaters and half-pipe snowboarders. They tuck in their arms and scrunch up their bodies while spinning in the air. By keeping their arms and legs tucked close to their centers of mass, they are able to rotate faster. Why is it so?

1. Why do the fliers scrunch up in the air while flipping and twisting?
2. What happens to the rate at which they spin when they change shape in the air?
3. How the direction of an aircraft can be changed?
4. What makes a somersault diver to make multiple turns in air and reach water with less splash of water?

The general expression for the total angular momentum of the system is The rigid body is made up of several particles and we focus on one such particle first. For a typical particle, the angular momentum, $\mathbf{l} = \mathbf{r} \times \mathbf{p}$

But, $\mathbf{r} = \mathbf{OP} = \mathbf{OC} + \mathbf{CP}$ and $\mathbf{p} = m\mathbf{v}$

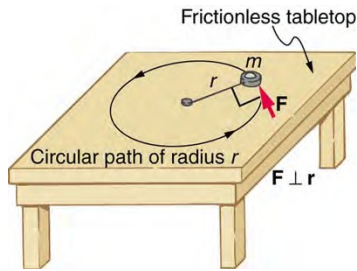
Then, $\mathbf{l} = (\mathbf{OC} \times m\mathbf{v}) + (\mathbf{CP} \times m\mathbf{v})$

But, $(\mathbf{CP} \times m\mathbf{v}) = (r \cdot mv) = (r \cdot m\omega r) = (mr^2\omega)$

Let $l_z = (mr^2\omega)$

Then $\mathbf{l} = l_z \hat{\mathbf{z}} + \mathbf{OC} \times m\mathbf{v}$

In general, for a particle, the angular momentum \mathbf{l} is not along the axis of rotation. i.e. for a particle, \mathbf{l} and $\boldsymbol{\omega}$ are not necessarily parallel. But, for a particle, the linear momentum \mathbf{p} and \mathbf{v} are always parallel to each other.



Considering all such particles, the total angular momentum of the rigid body,

Where,

Consider a special case: rotation of a rigid body about an axis which is symmetric. For every particle which has a velocity v_i , there is another particle of velocity $-v_i$ located diametrically opposite on the circle. Hence $L_i=0$. Again, and

But, and for rotation about a fixed axis, if the moment of inertia does not change with time, then,

Conservation of Angular momentum

When the net external torque is zero ($\tau=0$) then $I\alpha$ is zero, or $I\omega$ is constant. Hence for systems with zero net external torque on increasing I ω decreases and on decreasing I ω increases.

Activity 1

Sit in a swivel (rotatable) chair with your arms folded and feet not resting on, i.e., away from the ground .and ask your friend to rotate the chair rapidly. As the chair is rotating with a constant angular velocity stretch the arms horizontally and see what happens. Fold back your arms and see what happens now. Why is it happening?

Activity 2

Hang a bicycle wheel with the help of a rope. Rotate it fast and then keep the rotating wheel in a vertical plane. Surprisingly, we can see that the wheel remains in the vertical plane without falling down. This is due to the conservation of angular momentum.

Activity 3

Sit in a swivel chair without touching the ground and holding a rotating wheel. Let the wheel rotate in such a manner so that its angular momentum is upward. At this stage we and the chair are at rest. Now if the wheel is turned over, its angular momentum is now downward. But since there is nothing to exert a significant torque on the system, the angular momentum must have the original value upward. Hence, the man and the stool begin to rotate with angular momentum twice that of the wheel.

Questions:

On doubling the moment of inertia of an isolated system its angular speed

1. Reduces by half
2. Increases by two

3. Remains the same
4. Reduces by four times

The property of this extremely important quantity I as a measure of rotational inertia of the body has been put to a great practical use. The machines, such as steam engine and the automobile engine, etc., that produce rotational motion have a disc with a large moment of inertia, called a flywheel. Because of its large moment of inertia, the flywheel resists the sudden increase or decrease of the speed of the vehicle. It allows a gradual change in the speed and prevents jerky motions, thereby ensuring a smooth ride for the passengers on the vehicle.

2.12 Rolling Motion

Learning Outcomes:

- Describes rolling motion.
- Describes Kinetic energy of rolling motion.

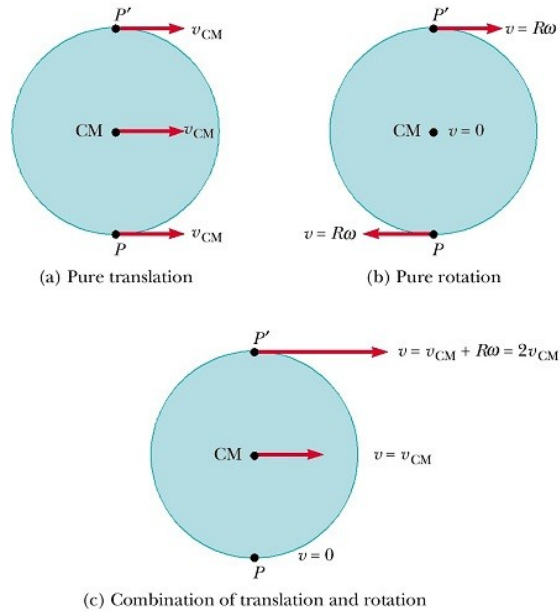
Activity: Group Discussion

One of the most common motions observed in daily life is the rolling motion. All wheels used in transportation have rolling motion. For specificness we shall begin with the case of a disc, but the result will apply to any rolling body rolling on a level surface. We shall assume that the disc rolls without slipping. This means that at any instant of time the bottom of the disc which is in contact with the surface is at rest on the surface. We have remarked earlier that rolling motion is a combination of rotation and translation. We know that the translational motion of a system of particles is the motion of its centre of mass. The rolling motion (without slipping) of a disc on a level surface. Note at any instant, the point of contact P_0 of the disc with the surface is at rest; the centre of mass of the disc moves with velocity, v_{cm} . The disc rotates with angular velocity ω about its axis which passes through C ; $v_{cm} = R\omega$, where R is the radius of the disc.

Kinetic Energy of Rolling Motion

Our next task will be to obtain an expression for the kinetic energy of a rolling body. The kinetic energy of a rolling body can be separated into kinetic energy of translation and kinetic energy of rotation. This is a special case of a general result for a system of particles, according to which the kinetic energy of a system of particles (K) can be separated into the kinetic energy of motion of the centre of mass (translation) ($MV^2/2$)

and kinetic energy of rotational motion about the centre of mass of the system of particles (K'). Thus,



$$k = k^1 + \frac{mv_{cm}^2}{2}$$

We assume this is general result and apply it to the case of rolling motion. In our notation, the kinetic energy of the centre of mass, i.e., the kinetic energy of translation, of the rolling body is

$$\frac{mv_{cm}^2}{2}$$

where m is the mass of the body and v_{cm} is the centre of the mass velocity. Since the motion of the rolling body about the centre of mass is rotation, K' represents the kinetic energy of rotation of the body;

$$k^1 = \frac{I\omega^2}{2}$$

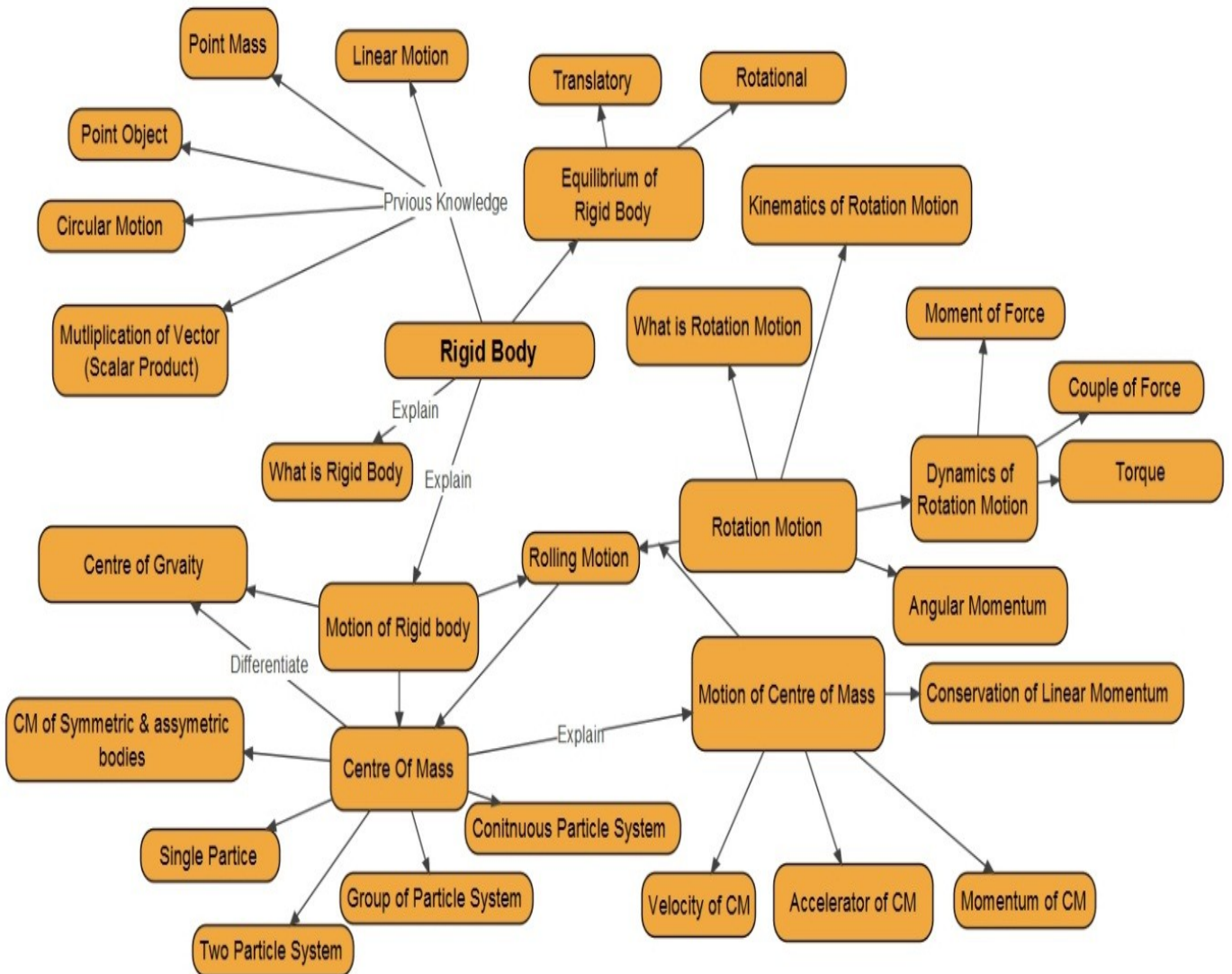
where I is the moment of inertia about the appropriate axis, which is the symmetry axis of the rolling body. The kinetic energy of a rolling body, therefore, is given by

$$K = \frac{1}{2} I \omega^2 + \frac{1}{2} m v_{cm}^2$$

$I = mk^2$ where k = the corresponding radius of gyration of the body and $v_{cm} = R \omega$, we get

$$K = \frac{1}{2} m v_{cm}^2 \left[1 + \frac{k^2}{R^2} \right]$$

The above equation applies to any rolling body: a disc, a cylinder, a ring or a sphere.



WAVES



3.1 Introduction

Learning Outcomes:

1. Explains different types of waves by citing examples
2. Identifies the properties required the medium for the propagation of mechanical waves.

Activity: General Discussion, ICT

Teacher brushes up the idea of waves in which a disturbance travels from one point to another point. Teacher draws three columns in the board and asks the students to name the waves they are familiar with and writes such a way that the mechanical waves in the first column, e.m. waves in the second column and matter waves in the third column.

- What is the difference between the waves in the first and second column (Hint- mechanical wave require medium and e.m. waves does not require)
- What is the name of the waves in the first column and second column. (Teacher writes the heading of the column)

- What type of waves are the third column (Hint – These wave associated with subatomic particle atom and even molecules. Teacher writes the heading of third column)

Teacher consolidates three types of waves with examples and their properties.

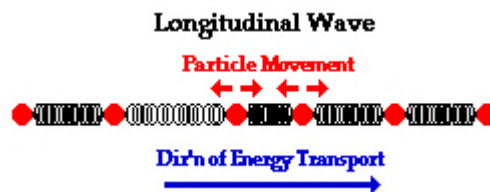
3.2 Transverse and longitudinal waves

Learning Outcomes:

1. Explains transverse and longitudinal waves

Activity: General Discussion, ICT

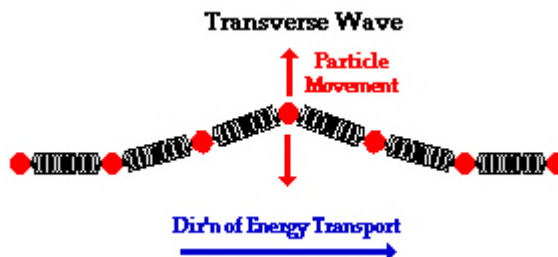
Teacher shows an ICT of longitudinal waves or demonstrates the production of longitudinal waves in a coiled spring. A knot is tied at any point on the string.



How wave propagates in this? (Hint – In the form of compressions and rarefactions)

What is the direction of motion of the knot w.r.t the direction of motion of the wave (Hint- Parallel to the propagation of wave).

Teacher shows an ICT of transverse waves or demonstrates the production of transverse waves in a coiled spring. A knot is tied on the spring



How wave propagates in this. (Hint- In the form of crest and troughs)

What is the direction of motion of the particles of the knot w.r.t the propagation of waves (Mutually perpendicular) Whether both waves can propagate through any medium (Hint- No transverse wave can propagate through solids and strings. But longitudinal waves can propagate through solids and fluids)

What is the reason behind this (Hint – Transverse waves propagates through a medium due to rigidity modulus and longitudinal waves propagates due to bulk modulus)

Teacher consolidates the difference between longitudinal and transverse waves, the properties of the medium required for the propagation of transverse and longitudinal waves.

3.3 Displacement relation in a progressive wave

(Amplitude and Phase; Wavelength and Angular Wave Number; Period, Angular Frequency and Frequency)

Learning Outcomes:

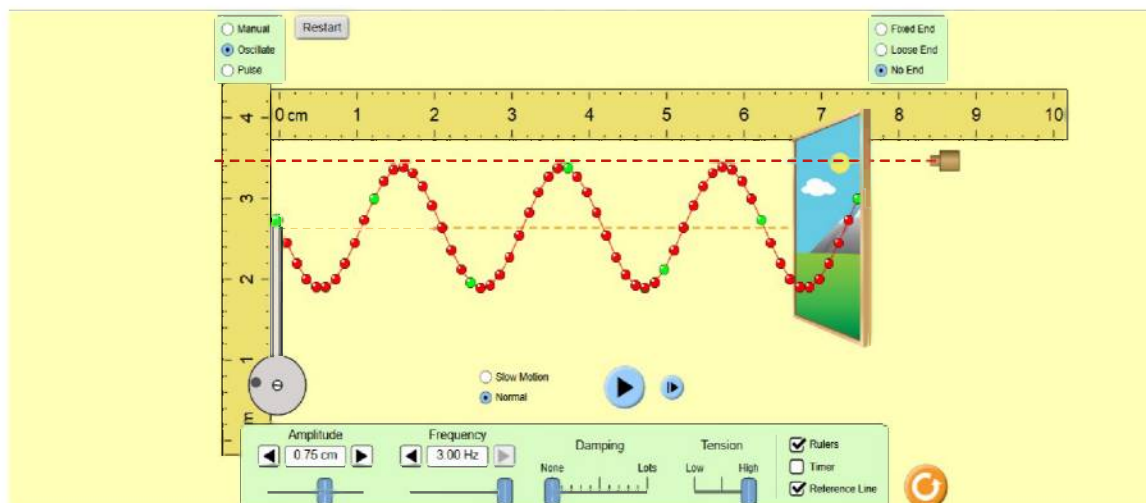
1. Explains displacement relation of a progressive wave.
2. Defines different parameters related to displacement relation in this equation.
3. Explains amplitude, phase, wave length, frequency, angular wave number.
4. Formulates relation connecting frequency and angular frequency, wavelength and angular wave number and solves numerical problems.

Activity: General discussion, PhET Simulation

Teacher shows the PhET Simulation

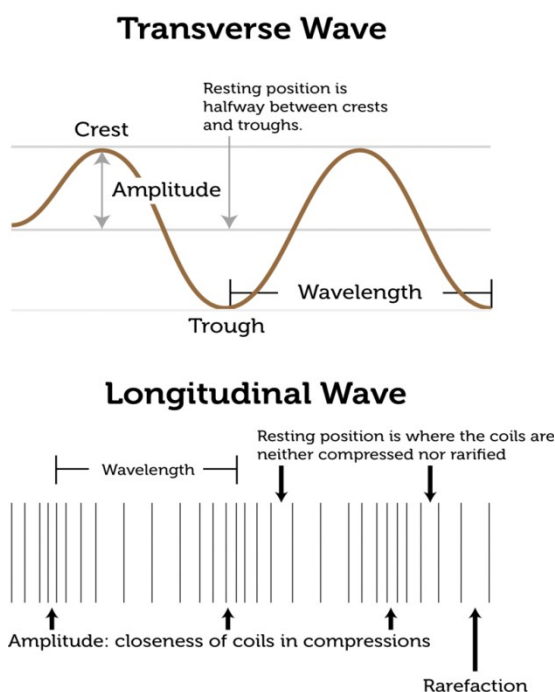
(http://phet.colorado.edu/sims/html/wave-on-a-string/latest/wave-on-a-string_en.html) and familiarise various parameters related to a wave.

(Amplitude, Period, Frequency, Angular Frequency, Wavelength, Angular Wave number, Phase)



Teacher asks the students to observe the position of a particular particle in the wave. How can we express the vertical displacement y which is varying with x and t . Using the previous knowledge of Oscillatory motion, teacher writes the equation $y(x,t) = a \sin(kx - \omega t + \phi)$ and explains different parameters in the equation.

Teacher gives the following worksheet and the students are asked to draw wave forms



Teacher consolidates the displacement relation of a progressive wave, the terms angular frequency, frequency, period, angular wave number and helps the students to solve numerical problems.

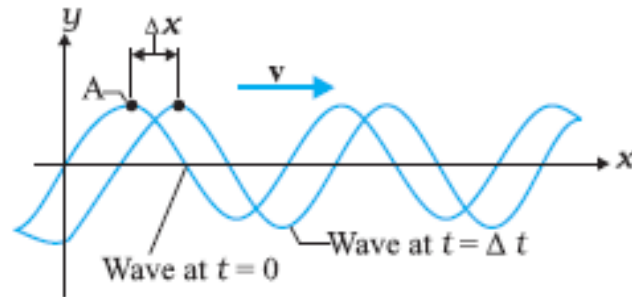
3.4 The speed of a travelling wave

Learning Outcomes:

1. Formulates the equation for speed of a travelling wave
2. Formulates the equation for speed transverse wave in a stretched string and speed of longitudinal waves
3. solves numerical problems related to speed of transverse wave in a stretched string and speed of longitudinal waves
4. Formulates the equation of speed of sound in a medium and solves problem related to it
5. Identifies and interprets the factors affecting the speed of sound in a medium under isothermal and adiabatic condition

Activity: General discussion

Teacher draws the displacement for various elements of a string at two different instants of time as shown.



What is the equation of the wave at A moving along +ve X direction.

[Hint: $y(x, t) = A \sin(kx - \omega t)$]

What is the equation of the wave at B moving along +ve X direction.

[Hint: $y(x + \Delta x, t + \Delta t) = A \sin[k(x + \Delta x) - \omega(t + \Delta t)]$]

Observe the shape of the wave at A and B. What do you find?

[Hint – In both points, the shapes are same]

In order to maintain the shape, what should be the nature of the argument in the above equation. [Hint – the argument should be constant.

i.e. $kx - \omega t = \text{constant}$

How can we find out the velocity from this equation?

Teacher helps the students to derive the equation

[Hint

i.e. $(kx - \omega t) = k(x + \Delta x) - \omega(t + \Delta t)$]

$k\Delta x = \omega \Delta t$

OR $v = \omega \lambda$

Teacher consolidates the relation among velocity, frequency and wavelength and helps the students to solve the problems

Speed of a Transverse Wave on Stretched String

Teacher ties two strings of equal length at both ends, one has slightly different thickness. Wave pulses are produced simultaneously at one end of the two strings. The students are asked to observe carefully.

Which wave has more speed?

[Hint: Wave in the string with less linear mass density, $v \propto \frac{1}{\mu}$]

Teacher takes two strings of same linear mass density and ties one of them tightly and other one loosely. Waves are produced as before.

Now, Which wave has more speed?

[Hint: Wave in the string with more tightly tied, $v \propto T$]

How do you arrive at an expression for speed of wave in a stretched string?

[Using dimensional analysis

$$[\mu] = [ML^{-1}]$$

$$[T] = [F] = [ML^{-1}T^{-2}]$$

Using the dimensional formulae of μ and T , how do you obtain the dimension of v ?

$$v = \sqrt{\frac{T}{\mu}} \quad]$$

Teacher consolidates that the speed of wave is determined by inertia and elasticity of median.

Speed of a Longitudinal Wave Speed of Sound

Activity: General discussion, problem solving

Teacher starts the discussion by asking what are the factors depend on the speed of longitudinal waves in a fluid

[Hints: Bulk modulus(B) and density(ρ) of medium]

Teacher helps the students to formulate the equation for speed of Longitudinal wave using the method of dimensional analysis.

$$[v = \sqrt{\frac{B}{\rho}} \quad]$$

Then, what will be the equation for speed of sound in solids?

[Hint: Replace B by Y , young's modulus

$$v = \sqrt{\frac{Y}{\rho}} \quad]$$

Teacher consolidates equations of speed of longitudinal waves in a fluid and solid.

Newton's formula for velocity of sound in air, Laplace correction

Activity: General discussion

Teacher checks the previous knowledge of speed of longitudinal waves.

What is the speed of longitudinal wave in a fluid?

$$[\text{Hint: } v = \sqrt{\frac{B}{\rho}} \quad]$$

Why then the speed of sound waves are higher in liquids and solids than that of gas.

[Hint: Bulk modulus of solids and liquids are higher than that of gases.]

What is the relation between P and V in the case of ideal gases

[Hint: $PV = Nk_B T$]

In isothermal process How can we write this equation.

[Hint $PV = \text{a const}$]

Teacher explains that Newton considered the propagation of sound waves in ideal gas as a result of isothermal changes

Then what will be B in this case?

$$[\text{Hint: } PV = a \text{ const} \Rightarrow P\Delta v + V\Delta P = 0 \Rightarrow P = - \frac{-\Delta P}{\frac{\Delta V}{V}}]$$

$\Rightarrow P = B$

Now, How will you modify this equation.

$$[\text{Hint } v = \sqrt{\frac{P}{\rho}} \Rightarrow \text{Newton's Formula}]$$

How can we estimate the speed of sound in air at STP if the mass of one mole of air $29 \times 10^{-3} \text{kg}$

[Hint: Mass of 1 mol of gas = $29 \times 10^{-3} \text{kg}$

Volume of 1 mol of gas = 22.4 litre = $22.4 \times 10^{-3} \text{m}^3$

i.e Pressure at STP = $1.01 \times 10^5 \text{N/m}^2$

Density of 1 mol of gas = 1.29kg/m^3

$$\therefore v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{1.01 \times 10^5}{1.29}} = 280 \text{ms}^{-1}]$$

Do you know the value of speed of sound in air?

[Hint: 331ms^{-1}]

The value obtained above is about 15% smaller compared to the experimental value of 331ms^{-1} . What is it so?

Teacher explains that Newton erroneously considered the pressure variation in air during the propagation of sound is isothermal. Laplace corrected it by considering the pressure variation as adiabatic. What is the equation for adiabatic process in ideal gas

[Hint: $PV^\gamma = a \text{ constant}$]

Teacher helps the students to arrive the Laplace equation.

$$[\text{Hint: } \Delta(PV^\gamma) = 0 \Rightarrow PV^{\gamma-1}\Delta V + V^\gamma\Delta P = 0 \Rightarrow \gamma P = - \frac{\Delta P}{\frac{\Delta V}{V}} \Rightarrow B =$$

γP]

Now, what will be the equation.

$$[\text{Hint } v = \sqrt{\frac{\gamma P}{\rho}} \Rightarrow \text{Laplace's Formula}]$$

Teacher consolidates Newton's Formula and Laplace's correction

3.5 The principle of superposition of waves

Learning Outcomes:

1. Explains the super position principle of waves and interference
2. Solves problems related to superposition and interference

Activity: General discussion, demonstration, ICT

Teacher produces wave pulses in opposite direction along a rope with the help of students such that the crest of one pulse overlap with the crest on another one or showing an ICT in which wave trains are in using in opposite direction and explains the superposition principle and help the students to state the principle.

If $y_1(x, t)$ and $y_2(x, t)$ are the displacement of any element of string, then what will be the net displacement of that element.

[Hint: $y(x, t) = y_1(x, t) + y_2(x, t)$]

Teacher shows a video in which two waves of same wavelength, amplitude and velocity moving along +ve X direction, with const. phase difference ϕ .

Compare the amplitude, and wavelength of the two waves

[Hint: Equal]

How can we write the equation for first wave

[Hint : $y_1(x, t) = A \sin(kx - \omega t)$]

How can we write the equation for the second wave

[Hint : $y_2(x, t) = A \sin[kx - \omega t + \phi]$

Apply super position principle and write the equation for the resultant wave. [Hint: $y(x, t) = y_1(x, t) + y_2(x, t)$

$y(x, t) = 2A \cos \sin[kx - \omega t +]$

What is the amplitude of the resultant wave

[Hint: $2A \cos$

What is the phase of resultant wave

[Hint : $kx - \omega t + .$

Assume that the two waves are in same phase i.e $\phi = 0$, How can we modify the above equation.

[Hint : $y(x, t) = 2A \sin(kx - \omega t)$

Now what is the amplitude of resulting wave

[Hint : $2A$]

If $\phi = \pi$ How will modify the above equation.

[Hint : $y(x, t) = 0$]

Teacher consolidates constructive and destructive interference and helps the students to identify the requirements for constructive and destructive interference

3.6 Reflection of waves

Standing Waves and Normal Modes

Learning Outcomes:

1. Formulates of equation of reflected waves from rigid boundary and open boundary and identifies the phase difference between incident and reflected wave.
2. Formulates of equation of standing waves in a stretched string and identifies nodes, antinodes, and harmonies.
3. Performs expt. of sonometer
4. Solves simple problems related to standing waves in a stretched string
5. Formulates the equation of the frequency of standing waves.
6. Draws standing waves in an open and closed pipe
7. Solves problems related to standing waves in open and closed pipes

Reflection of waves and standing waves

Activity: group discussion, ICT

Teacher shows an video showing reflection of transverse waves on a stretched string from closed or open end. Teacher can also demonstrate the reflection as follows.

[Material required : A rope and a weight less ring a rod and a clamp]

Tie one end of the rope in the ring and insert the ring through a fixed rod

Experiment: insert the rod through the ring and tie one end of the rope with the ring. Clamp the ring and the rod. Produce pulses on the other end by jerking rope up and down.

What type of wave are produced

[Hint: transverse]

What is the phase difference between the incident and reflected wave at the boundary.

[Hint: opposite phase or π phase difference]

Now teacher removes the camping of the ring from the rod and again jerking the other end

What is the phase difference between the incident and reflected wave at the boundary

[Hint: No phase difference as zero]

How can we represent mathematically the incident wave

[Hint: $y(x,t) = A \sin(kx - \omega t)$]

How can we represent the reflected wave from closed boundary

[Hint: $y(x,t) = A \sin(kx + \omega t + \pi) = A \sin'(kx + \omega t)$]

How can we represent the reflected wave from the open boundary
 [Hint: $y(x,t) = A \sin(kx + \omega t)$]

Teacher consolidates the reflection of waves on string from open boundary and closed boundary and solves problems based on interference.

Standing Waves on Stretched String

Activity : Demonstration, ICT, group discussion

Teacher demonstrates the standing waves produced in a string using electrical tuning fork. The teacher can also show an ICT showing standing waves on a string

What is the boundary condition in this

[Hint: Closed boundary]

What is the equation for a wave travelling right

[Hint: $y_1(x,t) = A \sin(kx - \omega t)$]

What is the equation for a wave travelling left after reflection

[Hint : $y_2(x,t) = A \sin(kx + \omega t)$]

What should be the equation for resultant wave

[Hint: $y(x,t) = y_1(x,t) + y_2(x,t) = A \sin(kx - \omega t) + A \sin(kx + \omega t)$

$y(x,t) = (2A \sin kx) \cos \omega t$]

Which is the amplitude term in this equation

[Hint : $2A \sin kx$]

For which values of kx , $\sin kx$ become 0

[Hint: $kx = n\pi$ when $n = 0, 1, 2, 3$ etc...

i. e. $x = n\pi$

OR $x = \frac{n\pi}{k}$ where $n = 0, 1, 2$ etc]

What is the name of these points.

[Hint: nodes]

For which values of kx $\sin kx$ become 1

[Hint: $kx = \frac{(2n+1)\pi}{2}$ where $n = 0, 1, 2, 3$

$x = \frac{(2n+1)\pi}{2k}$

$x = \frac{(2n+1)\lambda}{4}$ where $n = 0, 1, 2, 3$]

What is the name of these points ?

[Hint: antinodes]

Consider the case of a stretched string of length L fixed at both ends.

Teacher shows a string vibrating in first mode of vibrating in an expt.

At the fixed ends node or antinode is forming

In order to get node at $x = L$ What is the relation between L and λ

[Hint: $L = \frac{n\lambda}{2}$ When $n = 1, 2, 3$]

What wave lengths are possible for a standing wave in a stretched string of length L

[Hint: $\lambda = \frac{2L}{n}$ $n = 1, 2, 3$]

What will be the frequencies corresponding to this wave lengths

[Hint: $v = n \lambda$ $n = 1, 2, 3$]

What will be name of these frequencies

[Hint : natural frequencies or modes of oscillation]

What is the relation between nodes antinodes and harmonies

What is the relation of second and third harmonies with fundamental or first harmonic

[Hint : They are integral multiples of first harmonic]

Teacher consolidates the discussion by giving numerical problems related to standing waves on a stretched string.

Vibrations of air column in pipes closed at one end and both ends are open

Activity : General discussion ICT

Teacher shows an ICT of standing waves produced in an air column in the fundamental mode

Node or anti node is produced at $x = L$

[Hint: anti node]

In order to produce antinode at the open end what should be the relation between L and λ

[Hint: $L = (2n + 1) \frac{\lambda}{4}$ $n = 0, 1, 2$ etc]

What is the value of λ from above equation

[Hint: $\lambda = \frac{4L}{2n + 1}$]

What is the frequency corresponding to this wave length

[Hint: $V = f \lambda$]

For $n = 0$, what is the name of the mode

[Hint: $V_1 = \frac{V}{4L}$, fundamental mode]

For $n = 1$, what is the frequency

[Hint: $V_2 = 3V_1 = 3V$]

What is the relation between fundamental frequency and higher frequencies

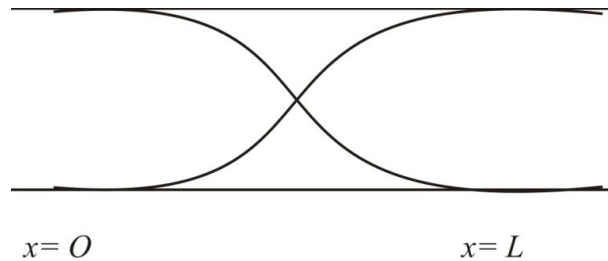
[Hint: Higher frequencies are odd multiples of fundamental frequency]

What is the relation among modes, nodes and antinodes

Teacher consolidates the discussion by helps students to draw different modes of vibration in closed pipe and solves numerical problems related to oscillation of air column in one end closed pipes.

Standing Waves in Both end Open Pipe

Teacher draws the fundamental mode of vibrations of air column in a pipe open at both ends as shows.



In this figure at what points antinodes are formed.

[Hint: at $x = 0$ and $x = L$]

To satisfy this what should be the relation between λ and L

[$L = n\lambda/2$ where $n = 0, 1, 2, 3$]

What is the frequency equation

[Hint: $v = n\lambda f$ where $n = 1, 2, 3$ etc]

Put $n = 1$, what is v_1

[Hint: $v_1 = v/2$]

Put $n = 2$, what is v_2

[Hint: $v_2 = 2v_1$]

What is the relation between higher modes and fundamental modes

Teacher consolidates the discussion by helping the students to draw different modes of vibration and helps to some numerical problems related to the vibration of air column in a pipe open at both ends

3.7 Beats

Learning Outcomes:

1. Explains beats and solves numerical problems related to it.

Activity:

General discussion, ICT showing the production of beats, Problem solving

Teacher starts the class by showing an ICT in which beat can be heard. Teacher refresh the idea of interference of waves. Consider two waves of slightly different frequencies v_1 , and v_2

What is the equation for the displacement of the particle of a medium when a wave of frequency travelling through it

[Hint: $x_1 = A \cos \omega_1 t$]

What is the equation for the displacement of the particle at the same point of the medium when a wave of frequency ω_2 travelling through it

[Hint: $x_2 = A \cos \omega_2 t$]

According to super position principle what is the resultant displacement

[Hint: $x = x_1 + x_2 = A \cos \omega_1 t + A \cos \omega_2 t = 2 A \cos t + t$]

In this equation which term represent amplitude term

[Hint: the first term cost is the amplitude term.]

How can we write this amplitude term in terms of frequency

[Hint: $\cos 2\pi t = \cos \pi(v_1 - v_2)t$]

What is the condition for this term to become maximum

[Hint: $\cos \pi(v_1 - v_2)t = \pm 1$

OR $\pi(v_1 - v_2)t = n\pi$ where $n = 0, 1, 2, 3$ etc.

i.e $t =$

What is the condition at which the value of $\cos \pi(v_1 - v_2)t$ become zero

[Hint: $\cos \pi(v_1 - v_2)t = 0$

$\pi(v_1 - v_2)t =$ when $n = 1, 3, 5$

$t =$

What is the relation between intensity and amplitude

[Hint: When amplitude is maximum, intensity of sound also maximum]

What is the time interval between two minimum intensities

[Hint

Similarly how many maximum heard in one second

[Hint: $2(v_1 - v_2)$]

What is the difference between one maxima and next minima

[Hint: $2(v_1 - v_2) - (v_1 - v_2) = (v_1 - v_2)$]

Teacher consolidates that one maximum and one minimum constitute beat and help the students to arrive a definition for beat. Teacher help the students to solve numerical problems related to beat frequency and practical applications of beats.

3.8 Doppler effect

Learning Outcomes:

1. Defines Doppler effect
2. Formulates the equation of apparent frequency
3. Solves problems related to Doppler effect.

Activity: Demonstration of Doppler effect using ICT, General discussion, problem solving

Teacher demonstrates the Doppler effect in which the source is moving towards the observer.

Teacher draws the figure in the board. Suppose the source emits a compression pulse a time $t=0$.

What is the time(t_1) required for the pulse to reach near the observer

[Hint:]

The source emits the next pulse in a time T seconds. what is the distance travelled by the source during this time

[Hint: $L - v_s T$]

What is the time(t_2) required for this pulse to reach near the observer

[Hint:

What is the time interval between these pulses

[Hint:

What is the apparent frequency heard by the observer

[Hint:

What change in pitch feels the observer

[Hint: pitch increases]

If the source is moving away from the observer what is the equation for apparent frequency

[Hint:

What change in pitch feels the observer

[Hint: pitch decreases]

Suppose the source is stationary and the observer is moving towards the source as shown. What is the distance between two compression pulse

[Hint:

What is the velocity of the wave with respect to the observer

[Hint: $v + v_o$]

What is the time interval between the two compressions

[Hint:

What is the apparent frequency heard by the observer

[Hint:

What change in frequency the observer feel

[Hint: frequency increases]

What will be the equation if the observer is moving away from the source

[Hint:

What change in frequency the observer feel

[Hint: frequency decreases]

Suppose both the source and observer are moving as shown

What is the time at which the first compression reach at the observer

[Hint:

What is the distance travelled by source and observer during the period T

[Hint:

What is the time at which the second compression reach at the observer

[Hint:

What is the apparent period of sound heard by the observer

[Hint:

What is the apparent frequency heard by the observer

[Hint:

When the source moves towards the observer and the observer moves away from the source what will be the equation for apparent frequency

[Hint:

3.9 Worksheets

1. Travelling wave moving towards positive X direction

[$y = a \sin(kx - \omega t)$]

Complete the tables (here all are filled , give blank for students) in order

to find 'y' at different positions 'x' ($0, \frac{\lambda}{4}, \frac{\lambda}{2}, 3\frac{\lambda}{4}, \lambda$) by keeping 't'

constant and draw wave pattern for different time interval ($0,$

$\frac{T}{4}, \frac{T}{2}, 3\frac{T}{4}, T$)

t	0				
ωt	0				
x	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$3\frac{\lambda}{4}$	λ
kx	0	$\frac{\pi}{2}$	π	$3\frac{\pi}{2}$	2π
kx- ωt	0	$\frac{\pi}{4}$	π	$3\frac{\pi}{2}$	2π
a sin (kx- ωt)	0	a	0	-a	0

t	$\frac{T}{4}$				
ωt	$\frac{\pi}{2}$				
x	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$3\frac{\lambda}{4}$	λ
kx	0	$\frac{\pi}{2}$	π	$3\frac{\pi}{2}$	2π

$kx - \omega t$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$3\frac{\pi}{2}$
$a \sin(kx - \omega t)$	$-a$	0	a	0	$-a$

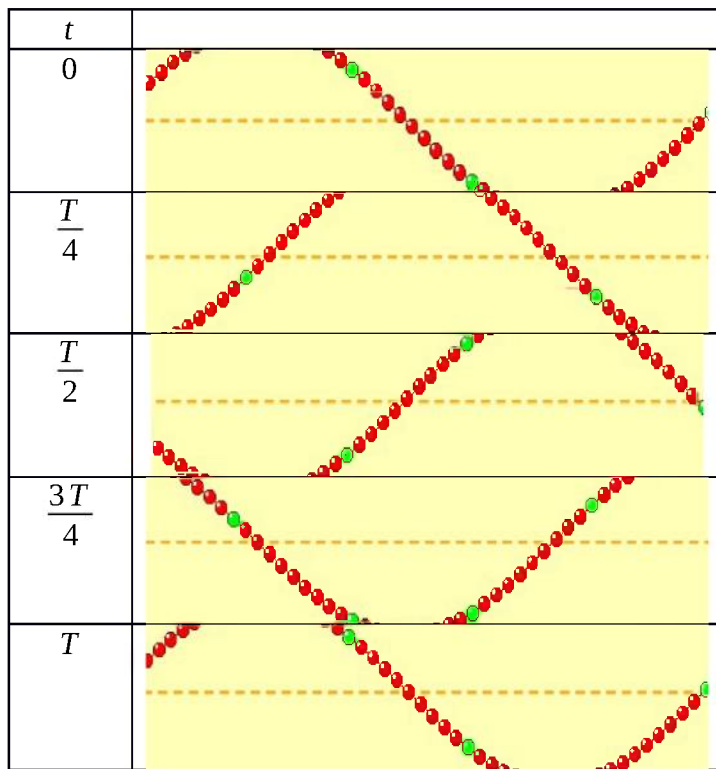
t	$\frac{T}{2}$				
ωt	π				
x	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$3\frac{\lambda}{4}$	λ
kx	0	$\frac{\pi}{2}$	π	$3\frac{\pi}{2}$	2π
$kx - \omega t$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π
$a \sin(kx - \omega t)$	0	$-a$	0	a	0

t	$3\frac{T}{4}$				
ωt	$3\frac{\pi}{2}$				
x	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$3\frac{\lambda}{4}$	λ
kx	0	$\frac{\pi}{2}$	π	$3\frac{\pi}{2}$	2π
$kx - \omega t$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π
$a \sin(kx - \omega t)$	0	$-a$	0	a	0

t	T				
ωt	2π				
x	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$3\frac{\lambda}{4}$	λ
kx	0	$\frac{\pi}{2}$	π	$3\frac{\pi}{2}$	2π
$kx - \omega t$	-2π	$-3\frac{\pi}{4}$	$-\pi$	$-\frac{\pi}{2}$	0
$a \sin(kx - \omega t)$	0	a	0	$-a$	0

2. Travelling wave moving towards Negative direction [$y=a \sin(kx+\omega t)$]

Complete the tables (here all are filled , give blank for students) in order to find 'y ' at different positions 'x' ($0, \frac{\lambda}{4}, \frac{\lambda}{2}, 3\frac{\lambda}{4}, \lambda$) by keeping ' t' constant and draw wave pattern for different time interval ($0, \frac{T}{4}, \frac{T}{2}, 3\frac{T}{4}, T$)



t	0				
ωt	0				
x	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$3\frac{\lambda}{4}$	λ
kx	0	$\frac{\pi}{2}$	π	$3\frac{\pi}{2}$	2π
$Kx+\omega t$	0	$\frac{\pi}{4}$	π	$3\frac{\pi}{2}$	2π
$a \sin (kx+\omega t)$	0	a	0	-a	0

t	$\frac{T}{4}$				
ωt	$\frac{\pi}{2}$				
x	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$3\frac{\lambda}{4}$	λ
kx	0	$\frac{\pi}{2}$	π	$3\frac{\pi}{2}$	2π
kx+ ωt	$\frac{\pi}{2}$	π	$3\frac{\pi}{2}$	2π	$5\frac{\pi}{2}$
a sin (kx+ ωt)	a	0	-a	0	a

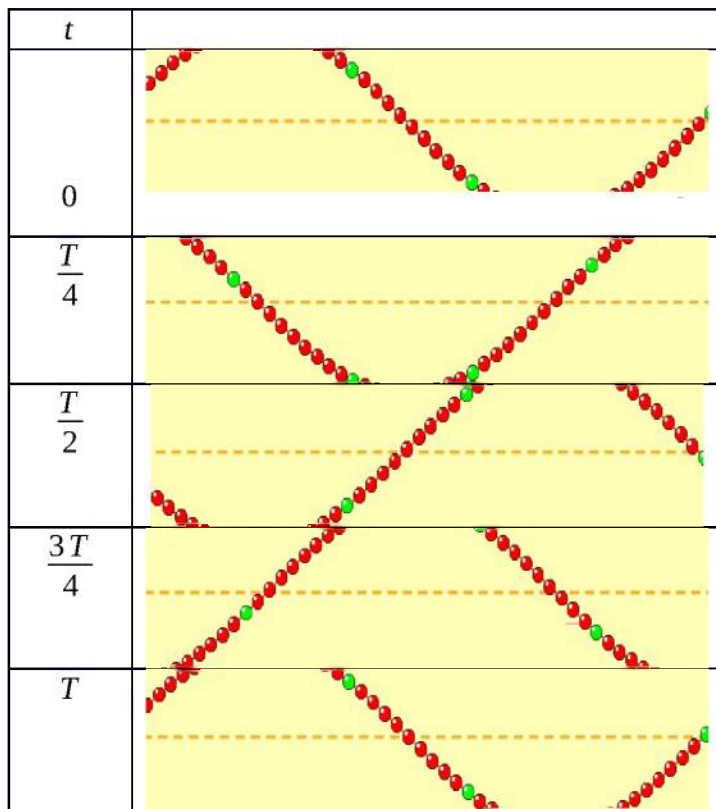
t	$\frac{T}{2}$				
ωt	π				
x	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$3\frac{\lambda}{4}$	λ
kx	0	$\frac{\pi}{2}$	π	$3\frac{\pi}{2}$	2π
Kx+ ωt	π	$3\frac{\pi}{2}$	2π	$5\frac{\pi}{2}$	3π
a sin (kx+ ωt)	0	-a	0	a	0

t	$3\frac{T}{4}$				
ωt	$3\frac{\pi}{2}$				
x	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$3\frac{\lambda}{4}$	λ
kx	0	$\frac{\pi}{2}$	π	$3\frac{\pi}{2}$	2π
Kx+ ωt	$3\frac{\pi}{2}$	2π	$5\frac{\pi}{2}$	3π	$7\frac{\pi}{2}$
a sin (kx+ ωt)	-a	0	a	0	-a

t	T				
ωt	2π				
x	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$3\frac{\lambda}{4}$	λ
kx	0	$\frac{\pi}{2}$	π	$3\frac{\pi}{2}$	2π
$Kx + \omega t$	2π	$5\frac{\pi}{2}$	3π	$7\frac{\pi}{2}$	4π
$a \sin(kx + \omega t)$	0	a	0	-a	0

3. Standing wave [$y=2a \sin kx \cos \omega t$]

Complete the tables (here all are filled , give blank for students) in order to find 'y' at different positions 'x' ($0, \frac{\lambda}{4}, \frac{\lambda}{2}, 3\frac{\lambda}{4}, \lambda$) by keeping 't' constant and draw wave pattern for different time interval ($0, \frac{T}{4}, \frac{T}{2}, 3\frac{T}{4}, T$)



t	0				
ωt	0				
$\cos\omega t$	1				
x	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$3\frac{\lambda}{4}$	λ
kx	0	$\frac{\pi}{2}$	π	$3\frac{\pi}{2}$	2π
$2a \sin kx$	0	$2a$	0	$-2a$	0
$2a \sin kx \cos\omega t$	0	$2a$	0	$2a$	0

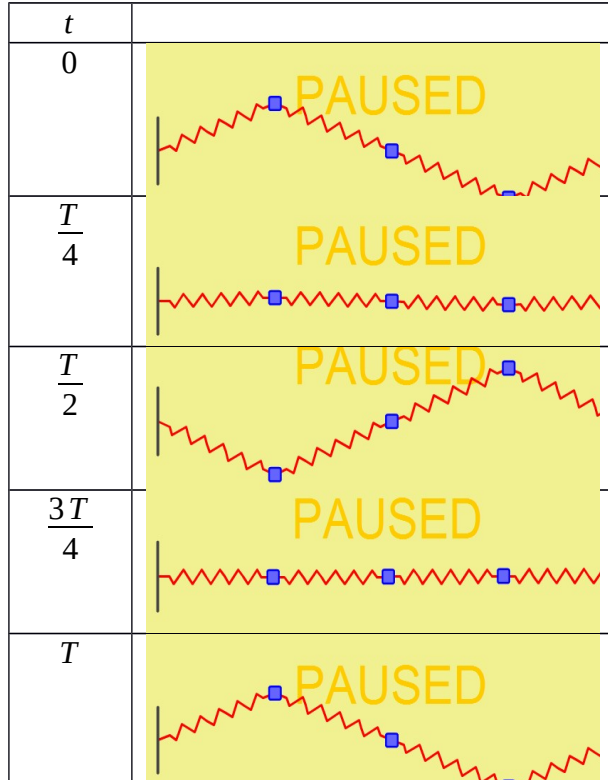
t	$\frac{T}{4}$				
ωt	$\frac{\pi}{2}$				
$\cos\omega t$	0				
x	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$3\frac{\lambda}{4}$	λ
kx	0	$\frac{\pi}{2}$	π	$3\frac{\pi}{2}$	2π
$2a \sin kx$	0	$2a$	0	$-2a$	0
$2a \sin kx \cos\omega t$	0	0	0	0	0

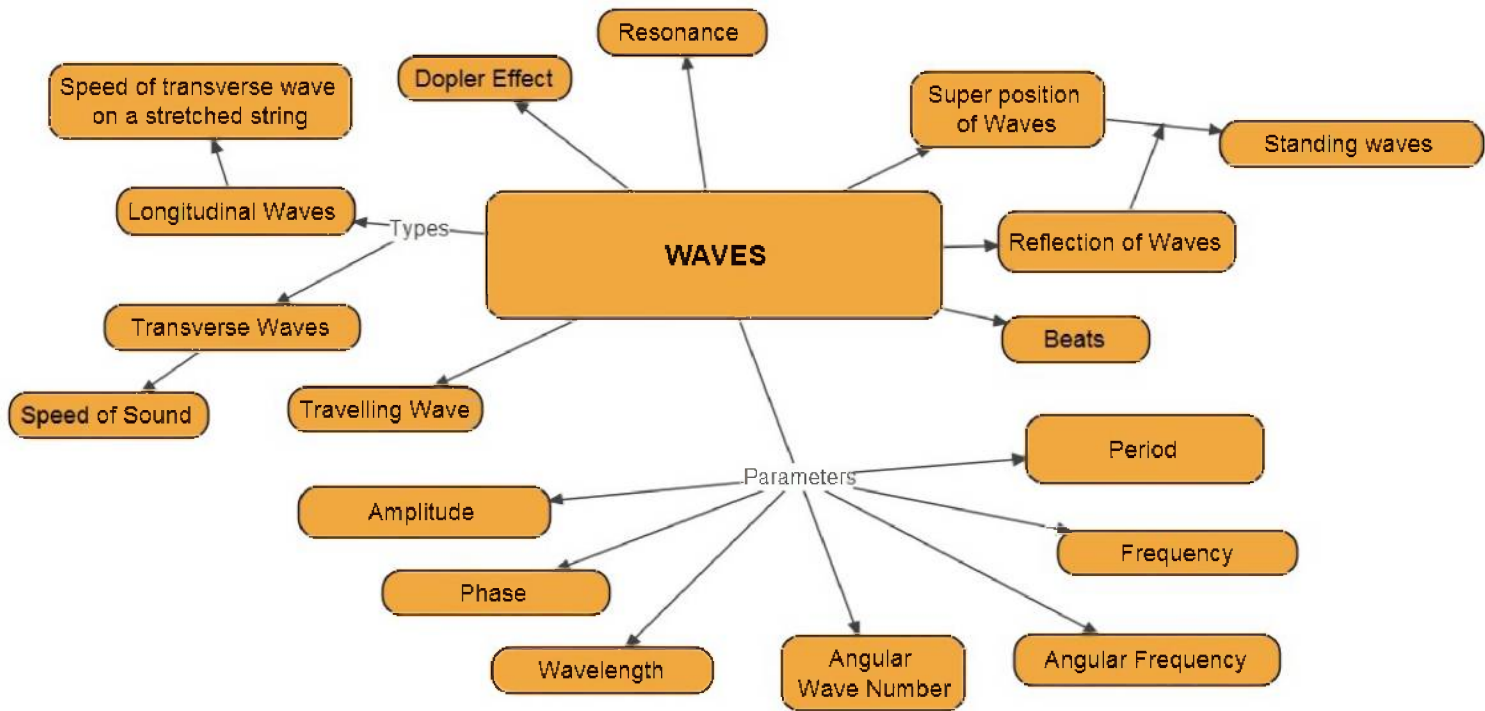
t	$\frac{T}{2}$				
ωt	π				
$\cos\omega t$	-1				
x	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$3\frac{\lambda}{4}$	λ
kx	0	$\frac{\pi}{2}$	π	$3\frac{\pi}{2}$	2π
$2a \sin kx$	0	$2a$	0	$-2a$	0
$2a \sin kx \cos\omega t$	0	$-2a$	0	$2a$	0

t	$3\frac{T}{4}$				
ωt	$3\frac{\pi}{2}$				
$\cos\omega t$	0				

x	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$3\frac{\lambda}{4}$	λ
kx	0	$\frac{\pi}{2}$	π	$3\frac{\pi}{2}$	2π
$2a \sin kx$	0	$2a$	0	$-2a$	0
$2a \sin kx \cos \omega t$	0	0	0	0	0

t	T				
ωt	2π				
$\cos \omega t$	1				
x	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$3\frac{\lambda}{4}$	λ
kx	0	$\frac{\pi}{2}$	π	$3\frac{\pi}{2}$	2π
$2a \sin kx$	0	$2a$	0	$-2a$	0
$2a \sin kx \cos \omega t$	0	$2a$	0	$-2a$	0





CURRENT ELECTRICITY



4.1 Introduction

Natural observable examples of electrical current are lightning, static electricity, and the solar wind, the source of the polar auroras.

Man-made occurrences of electric current include the flow of conduction electrons in metal wires such as the overhead power lines that deliver electrical energy across long distances and the smaller wires within electrical and electronic equipment.

Eddy currents are electric currents that occur in conductors exposed to changing magnetic fields. Similarly, electric currents occur, particularly in the surface, of conductors exposed to electromagnetic waves. When oscillating electric currents flow at the correct voltages within radio antennas, radio waves are generated. In electronics, other forms of electric current include the flow of electrons through resistors or through the vacuum in a vacuum tube, the flow of ions inside a battery or a neuron, and the flow of holes within a semiconductor.

History of Current Electricity

Long before any knowledge of electricity existed people were aware of shocks from electric fish. Ancient Egyptian texts dating from 2750 BC

referred to these fish as the "Thunderer of the Nile", and described them as the "protectors" of all other fish. Patients suffering from ailments such as gout or headache were directed to touch electric fish in the hope that the powerful jolt might cure them. In 15th century, Arabs coined the word for lightning (raad) applied to the electric ray.



Thales of Miletus made a series of observations on static electricity around 600 BC, from which he believed that friction rendered amber magnetic, in contrast to minerals such as magnetite, which needed no rubbing. Thales was incorrect in believing the attraction was due to a magnetic effect, but later science would prove a link between magnetism and electricity. According to a controversial theory, the Parthians may have had knowledge of electroplating, based on the 1936 discovery of the Baghdad Battery, which resembles a galvanic cell, though it is uncertain whether the artifact was electrical in nature.

Electricity would remain little more than an intellectual curiosity for millennia until 1600, when the English scientist William Gilbert made a careful study of electricity and magnetism, distinguishing the lodestone effect from static electricity produced by rubbing amber. He coined the new Latin word *electricus* ("of amber" or "like amber", from the Greek word for "amber") to refer to the property of attracting small objects after being rubbed. This association gave rise to the English words "electric" and "electricity", which made their first appearance in print in Thomas Browne's *Pseudodoxia Epidemica* of 1646.

Further work was conducted by Otto von Guericke, Robert Boyle, Stephen Gray and C. F. du Fay. In the 18th century, Benjamin Franklin conducted extensive research in electricity, selling his possessions to fund his work. In June 1752 he is reputed to have attached a metal key to the bottom of a dampened kite string and flown the kite in a storm-threatened sky. A succession of sparks jumping from the key to the back of his hand showed that lightning was indeed electrical in nature. He also explained the apparently paradoxical behavior of the Leyden jar as a device for

storing large amounts of electrical charge in terms of electricity consisting of both positive and negative charges.

In 1791, Luigi Galvani published his discovery of bioelectricity, demonstrating that electricity was the medium by which nerve cells passed signals to the muscles. Alessandro Volta's battery, or voltaic pile, of 1800, made from alternating layers of zinc and copper, provided scientists with a more reliable source of electrical energy than the electrostatic machines previously used. The recognition of electromagnetism, the unity of electric and magnetic phenomena, is due to Hans Christian Oersted and André-Marie Ampere in 1819-1820; Michael Faraday invented the electric motor in 1821, and Georg Ohm mathematically analyzed the electrical circuit in 1827. Electricity and magnetism (and light) were definitively linked by James Clerk Maxwell, in particular in his "On Physical Lines of Force" in 1861 and 1862.

4.2 Electric Current

Learning Outcomes:

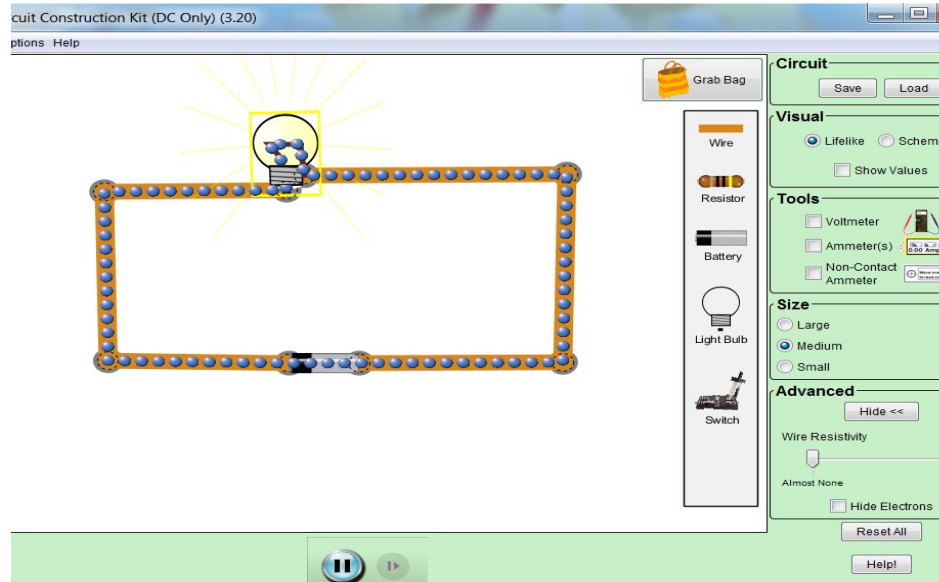
1. Explain the absence of current in conductors in the absence of electric field.
2. Describe the mechanism of electrical conduction in presence of electric field.
3. Demonstrate the motion of electrons using mechanical model.
4. Define drift velocity, relaxation time & electric current.
5. Differentiate between conventional current and electronic current.
6. Derive expression for current using current density **for a given finite area.**
7. **Describe conduction of current in plasma.**

Activity: Group Discussion

Open PhET simulation - <https://phet.colorado.edu/en/simulation/circuit-construction-kit-dc>

Connect a bulb with a wire as shown. The bulb doesn't glow even though there are free electrons in the wire and they are in motion.

The number of electrons entering the bulb is equal to the number of electrons leaving the bulb. (In this simulation the spheres represent electrons and their motion represents the direction of motion of electrons and not the actual motion of electrons.) Hence, the net current flowing is zero, or we can say there is no observable current.



Analogy

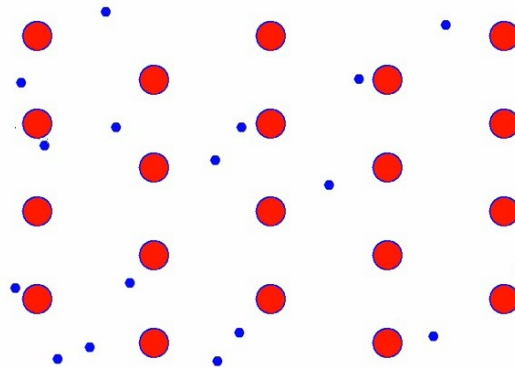
The flow of water through a garden hose represents the directed flow of positive charge (the protons in the water molecules) at a rate of perhaps several million coulombs per second. There is no net transport of charge, however, because there is a parallel flow of negative charge (the electrons in the water molecules) of exactly the same amount moving in exactly the same direction.

Now, insert a battery into the circuit. Then the bulb glows. In this situation the charges starts to flow through the bulb.

How does the Bulb Glow when Battery is Inserted?

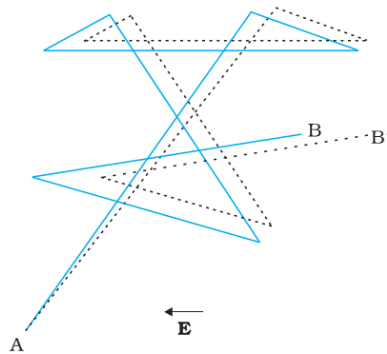
Actually, electrons in a conductor do not move forward as straight as it may seem from the simulation. Rather, they are moving randomly in all directions, and hence the net velocity in any direction becomes zero. Their movement can be understood from this simple video -

https://www.youtube.com/watch?v=dyX5I_io7bg



In the above snapshot of the video, blue dots represent electrons which are continuously and randomly striking red dots representing lattice which are constituted of relatively heavy positive ions.

When an external electric field is applied, all the electrons are simultaneously drifted opposite in direction to applied field. In the diagram, path AB is travelled in absence of electric field, and path AB' is travelled in presence of electric field, which is provided by the battery in the simulation.



Activity: 'Motion of Electrons in a Conductor'



This demonstration is an analog of electrical conduction in a resistive material. The marbles (analog of the charge carriers) move under the influence of gravity (analog of the electric field) down the inclined plane. After moving a short distance the marble collides (i.e. scatters) with one of the nails (an impurity in the metal) losing some of its energy. On average the marble moves down the plane with some average drift velocity. The current of marbles can only be sustained if the lecturer acts as a battery and continually lifts the marbles up to the top of the board thus replacing the energy which has been dissipated in the collisions. It can be mentioned that the nails presumably are heated by the collisions, the analog of Joule heating. Adjusting the tilt of the board is the analog of varying the electric field. Small tilt angles give slow drift velocity and vice-versa.

Estimating Drift Velocity

Electrical signals are known to move very rapidly. Telephone conversations carried by currents in wires cover large distances without noticeable delays. Lights come on as soon as a switch is flicked. Most electrical signals carried by currents travel at speeds on the order of 10^8 m/s, a significant fraction of the speed of light. Interestingly, the individual charges that make up the current move much more slowly on average, typically drifting at speeds on the order of 10^{-4} m/s.

How do we reconcile these two speeds, and what does it tell us about standard conductors?

The high speed of electrical signals results from the fact that the force between charges acts rapidly at a distance. Thus, when a free charge is forced into a wire, the incoming charge pushes other charges ahead of it, which in turn push on charges farther down the line. Hence, when charged particles are forced into this volume of a conductor, an equal number are quickly forced to leave. The repulsion between like charges makes it difficult to increase the number of charges in a volume. Thus, as one charge enters, another leaves almost immediately, carrying the signal rapidly forward. Good conductors have large numbers of free charges in them. In metals, the free charges are free electrons. The distance (l) that an individual electron can move between collisions with atoms or other electrons is quite small, and can be obtained by using 2nd equation of motion as,

$$l = \frac{1}{2} a t^2 = \frac{1}{2} \frac{eE}{m} t^2$$

Between any two collisions, the electrons get accelerated until they strike positive ions in the lattice and come to rest, and further repeat the process.

The time passed between successive collisions is regarded as the relaxation time (τ).

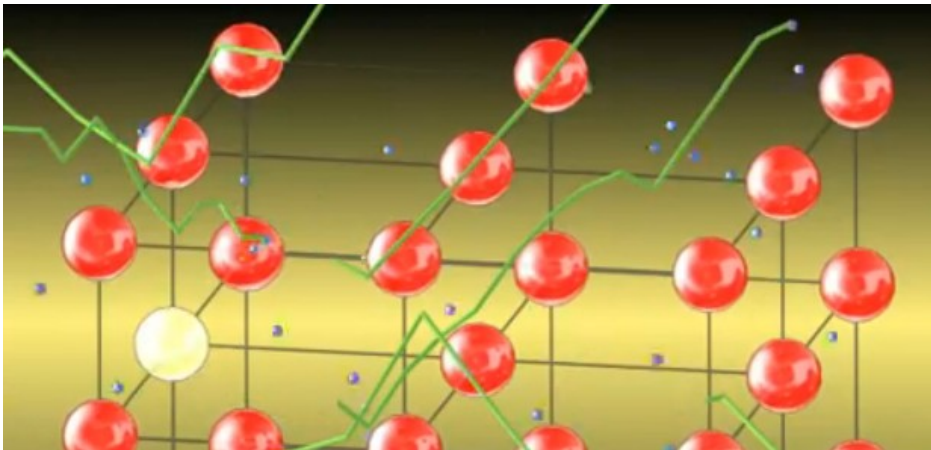
And though the electrons are accelerated between any two collisions in random directions, but this motion produces a constant velocity in the direction of flow of current, which is called drift velocity (v_d), which can be obtained as –

$$v_d = \frac{l}{t} = \frac{1}{t} \frac{eE}{m} t^2 = \frac{1}{2} \frac{eE}{m} t$$

The electron paths thus appear nearly random, like the motion of atoms in a gas. But there is an electric field in the conductor that causes the electrons to drift in a direction opposite to applied field. Drift velocity is quite small, since there are so many free charges. If we have an estimate

of the density of free electrons in a conductor, we can calculate the drift velocity for a given current. The larger the density, the lower the velocity required for a given current. The collisions normally transfer energy to the conductor, requiring a constant supply of energy to maintain a steady current. It can easily demonstrated using this animation –

<https://www.youtube.com/watch?v=KprFTxjQAoE>



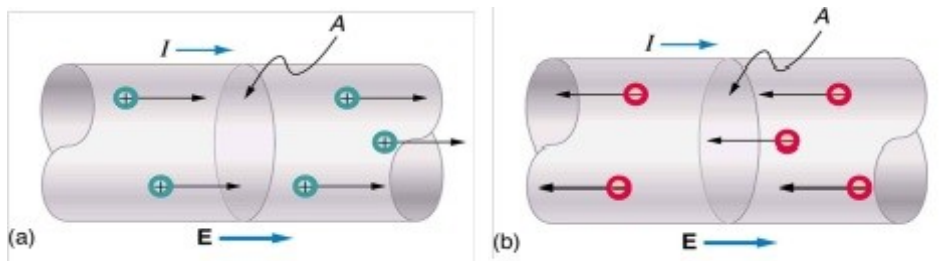
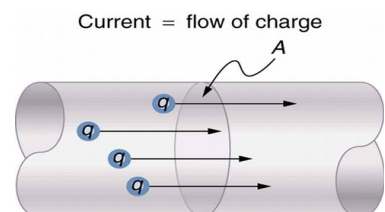
Definition of Electric Current

The rate of flow of charge is known as electric current. If ΔQ is the charge flowing in Δt time, then current I is equal to $\frac{\Delta Q}{\Delta t}$.

Conventional current is defined to move in the direction of the electric field which is the direction of movement of positive charges and is opposite to the movement of negative charges.

The flow of electrons is sometimes referred to as electronic flow. A very clear demonstration of current flow in a battery resistor circuit is given in the link below -

<https://phet.colorado.edu/en/simulation/legacy/battery-resistor-circuit>



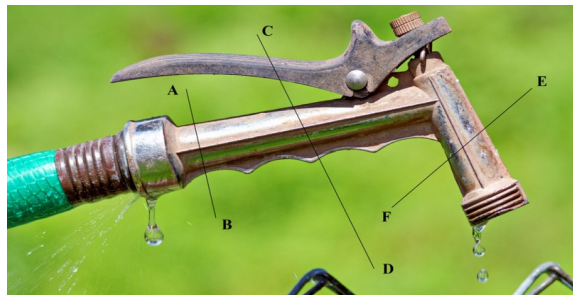
Problem

If 0.3 mA current flows during the operation of a given calculator, current being carried by electrons, then how many electrons per second pass through it?

After calculation, we observe that the number of electrons passing per second is quite high, i.e. 1.88×10^{15} electrons. There are so many charged particles moving, even in small currents, that individual charges are not noticed, just as individual water molecules are not noticed in water flow.

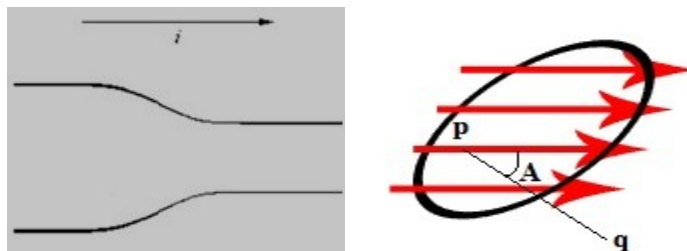
Conservation of Charge & Current Uniformity

Under steady-state conditions, the current is the same for planes AB , CD , and EF and indeed for all planes that pass completely through the conductor, no matter what their location or orientation. This follows from the fact that charge is conserved. Under the steady-state conditions assumed here, an electron must pass through plane AB for every electron that passes through plane EF . In the same way, if we have a steady flow of water through a garden hose, a drop of water must leave the nozzle for every drop that enters the hose at the other end. The amount of water in the hose is a conserved quantity.



Current Density

Consider a conductor of non-uniform cross-section as shown in the first figure below.



Let I be the current flowing from left to right. Current is same in both wider and narrower cross sections but current per unit area is less in the

wider section and more in the narrower section. This current per unit area is represented as current density (j). Again consider the next diagram. If the area element ΔS is considered at some angle, say angle A to the direction of current propagation, then

$$j = \frac{\Delta i}{\Delta S \cos A}$$

$$\text{Or, } \Delta i = \vec{j} \cdot \vec{\Delta S}$$

For a finite area,

$$i = \int \vec{j} \cdot \vec{dS}$$

Conduction in Gases and Plasmas

In air and other ordinary gases below the breakdown field, the dominant source of electrical conduction is via relatively few mobile ions produced by radioactive gases, ultraviolet light, or cosmic rays. Since the electrical conductivity is low, gases are dielectrics or insulators. However, once the applied electric field approaches the breakdown value, free electrons become sufficiently accelerated by the electric field to create additional free electrons by colliding, and ionizing, neutral gas atoms or molecules in a process called avalanche breakdown. The breakdown process forms a plasma that contains enough mobile electrons and positive ions to make it an electrical conductor. In the process, it forms a light emitting conductive path, such as a spark, arc or lightning. Due to their lower mass, the electrons in a plasma accelerate more quickly in response to an electric field than the heavier positive ions, and hence carry the bulk of the current.

Nerve Conduction – Electrocardiograms

Electric currents in the vastly complex system of billions of nerves in our body allow us to sense the world, control parts of our body, and think. Nerve conduction is a general term for electrical signals carried by nerve cells. It is one aspect of bioelectricity, or electrical effects in and created by biological systems.

An electrocardiogram (ECG) is a record of the voltages created by the wave of depolarization and subsequent repolarization in the heart. The ECG measures components of depolarization and repolarization of the heart muscle and can yield significant information on the functioning and malfunctioning of the heart.

Simulation On Neuron

Stimulate a neuron and monitor what happens. Pause, rewind, and move forward in time in order to observe the ions as they move across the neuron membrane. http://cnx.org/content/m42352/1.3/neuron_en.jar

4.3 Ohm's Law

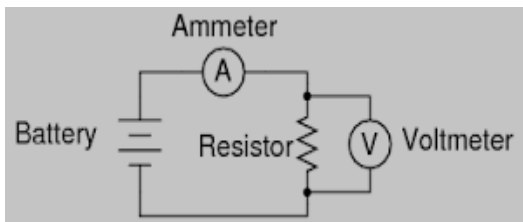
Learning Outcomes:

1. Define & verify Ohm's law.
2. Deduce the resistivity law experimentally.
3. Determine the temperature coefficient of resistance for a given material.
4. Describe thermistors and superconductors along with their applications.
5. Apply colour coding technique and determine the resistance of a given resistor.

Activity: Group Discussion

Materials required for experiment: connecting wires, ammeter, voltmeter, 1m resistance wire, rheostat and a key.

Procedure: Connect the resistance wire, ammeter, rheostat and the key as shown in diagram. Connect the voltmeter across the resistance wire. Change the value of current using rheostat and measure the voltmeter reading as shown in the table.



Current(mA)	Voltage(V)	V/I
4.0	8.0	2
4.5	9.0	2
5.0	10.0	2
5.5	11.0	2
6.0	12.0	2

Result : It is found that V is proportional to I.

This is the underlying rule in Ohm's law which can be defined as follows:

Statement of Ohm's Law: *The electric current I flowing through a substance is proportional to the voltage V across its ends, i.e., $V=IR$, where R is called the resistance of the substance, provided the temperature and other physical conditions remain constant throughout the experiment.*

Alternatively it can be expressed in terms of charge density, current density and resistivity as,

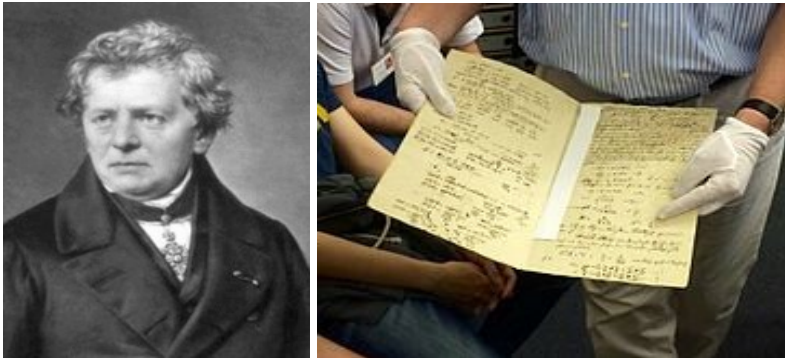
$$\sigma = j\rho$$

An interactive simulation of Ohm's law produced by PhET is available at - <https://phet.colorado.edu/en/simulation/ohms-law>

Story of Ohm's Law

In January 1781, before Georg Ohm's work, Henry Cavendish experimented with Leyden jars and glass tubes of varying diameter and

length filled with salt solution. He measured the current by noting how strong a shock he felt as he completed the circuit with his body. Cavendish wrote that the "velocity" (current) varied directly as the "degree of electrification" (voltage). He did not communicate his results to other scientists at the time, and his results were unknown until Maxwell published them in 1879.



Ohm did his work on resistance in the years 1825 and 1826, and published his results in 1827 as the book *Die galvanische Kette, mathematisch bearbeitet* ("The galvanic circuit investigated mathematically"). He drew considerable inspiration from Fourier's work on heat conduction in the theoretical explanation of his work. For experiments, he initially used voltaic piles, but later used a thermocouple as this provided a more stable voltage source in terms of internal resistance and constant voltage. He used a galvanometer to measure current, and knew that the voltage between the thermocouple terminals was proportional to the junction temperature. He then added test wires of varying length, diameter, and material to complete the circuit. He found that his data could be modelled through the equation

$$x = \frac{a}{b + l},$$

where x was the reading from the galvanometer, l was the length of the test conductor, a depended only on the thermocouple junction temperature, and b was a constant of the set-up.

When Ohm first published his work, critics called his work a "web of naked fancies" and the German Minister of Education proclaimed that "a professor who preached such heresies was unworthy to teach science." The prevailing scientific philosophy in Germany at the time asserted that

experiments need not be performed to develop an understanding of nature because nature is so well ordered, and that scientific truths may be deduced through reasoning alone. Also, Ohm's brother Martin, a mathematician, was battling the German educational system. These factors hindered the acceptance of Ohm's work, and his work did not become widely accepted until the 1840s. Fortunately, Ohm received recognition for his contributions to science well before he died. In the 1850s, Ohm's law was known as such and was widely considered proved, and alternatives, such as "Barlow's law", were discredited, in terms of real applications to telegraph system design, as discussed by Samuel F. B. Morse in 1855.

Resitivity of a Conductor

Procedure - Take three resistance wires (eg. Nichrome) of length 1m, 2m and 3m respectively. Connect a 2V battery and an ammeter across all and measure the resistance of the wires by using Ohm's law. It can be seen that the resistance is directly proportional to the length. Now repeat the same experiment by choosing 1m length resistance wires having different cross-sectional areas. It can be seen that the resistance is inversely proportional to the cross-sectional area.

Result –

Hence it can be deduced that, $R \propto \frac{l}{A}$

The constant of proportional to be introduced in above relation is termed as resistivity (ρ), which has a constant value for a given material. Hence, the relation

$$R = \rho \frac{l}{A}$$

An interactive simulation of law of resistivity produced by PhET is available at – <https://phet.colorado.edu/en/simulation/resistance-in-a-wire>

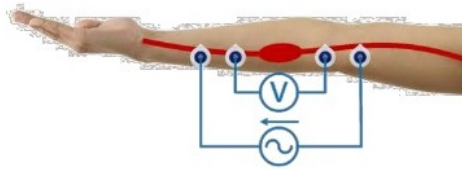
Applications of Resistivity Law

Impedance phlebography, or impedance plethysmography (IPG), is a non-invasive medical test that measures small changes in electrical resistance of the chest, calf or other regions of the body. For the chest, the technique was developed by NASA to measure the split second impedance changes within the chest, as the heart beats, to calculate both cardiac output and lung water content. The key to this technique is the fact that resistance can

be related to the volume of the calf between the inner electrodes. The volume is the product of the length L and the cross-sectional area of the calf. Solving for A shows that,

$$R = \rho \frac{l}{A} = \rho \frac{l}{V_{\text{calf}}/l}$$

Thus, resistance is inversely proportional to volume, a fact that is exploited in diagnosing deep venous thrombosis.



Temperature Coefficient of Resistance

Resistance of any material varies with temperature. For temperature range that is not too great, this variation can be represented approximately as a linear relation -

$$R_T = R_0 [1 + \alpha (T - T_0)]$$

where, R_T and R_0 are the values of the resistance at temperature T and T_0 respectively. T_0 is often taken to be either room temperature or 0°C , α is the temperature coefficient of resistivity. Pure metals have a small, positive value of α , which means that their resistance increases with increasing temperature. From temperature measurements of R , we can find α . To do this we will plot resistance values versus T , and approximate the results with a straight line. The intercept of this line with the resistance axis is R_0 , and the slope divided by R_0 is the values of α .

Procedure

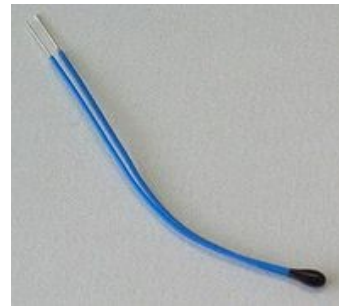
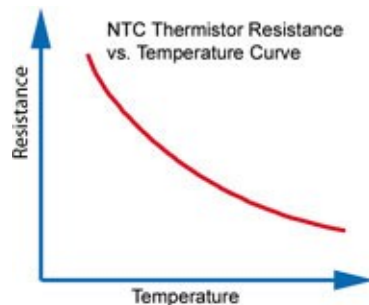
Take a metallic container and fill it with water. Take a strong wooden cylinder (2 inches diameter & 3-5 inches height) and wind copper wire around it (around 50 turns) and connect both ends of wire to a multimeter, set to measure resistance. Immerse the cylinder into water, such that copper wires are not touching the container on their way out. Insert a thermometer, it should be immersed in water. Stir the water thoroughly and start heating it via a water heater. Record the initial values of temperature and resistance. When the resistance starts to change, measure around 10 readings, and then close the power supply to the heater. Again, start taking the readings as the temperature falls. Take the mean of readings taken during heating and cooling of the cylinder with copper coil. Record the readings as shown in the table.

Temperature ($^{\circ}\text{C}$)	Resistor-1 (ohms)	Resistor-2 (ohms)	Resistor-3 (ohms)

Now, a graph must be plotted between resistance and temperature. From the graph, slope divided by R_0 gives the temperature coefficient of resistance.

Thermistor

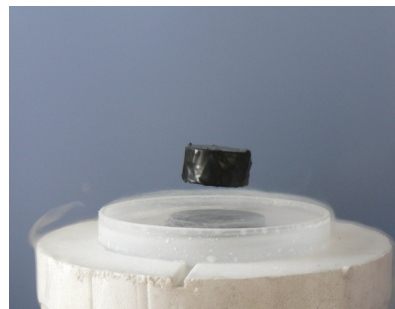
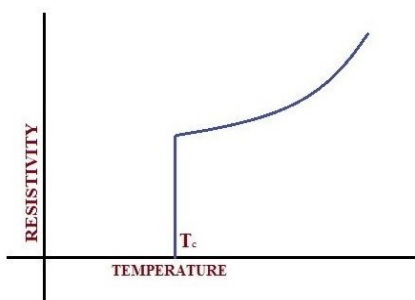
One of the most interesting devices in the study of temperature dependence of resistance is thermistor. Thermistors are thermally sensitive resistors whose prime function is to exhibit a large, predictable and precise change in electrical resistance when subjected to a corresponding change in body temperature. Negative Temperature Coefficient (NTC) thermistors exhibit a decrease in electrical resistance when subjected to an increase in body temperature and Positive Temperature Coefficient (PTC) thermistors exhibit an increase in electrical resistance when subjected to an increase in body temperature.



Because of their very predictable characteristics and their excellent long term stability, thermistors are generally accepted to be the most advantageous sensor for many applications including temperature measurement and control. Since the negative temperature coefficient of silver sulphide was first observed by Michael Faraday in 1833, there has been a continual improvement in thermistor technology. A thermistor's change in electrical resistance due to a corresponding temperature change is evident whether the thermistor's body temperature is changed as a result of conduction or radiation from the surrounding environment or due to "self heating" brought about by power dissipation within the device. When a thermistor is used in a circuit where the power dissipated within the device is not sufficient to cause "self heating", the thermistor's body temperature will follow that of the environment.

Superconductors

Superconductivity was first noticed when liquid mercury was cooled to liquid Helium temperatures (4.2K) while its resistivity was being plotted against temperature. Dutch physicist Heike Kamerlingh Onnes performed this experiment in 1911. The resistance was observed to decrease linearly, and finally approached to zero, at a particular temperature, now referred to as critical temperature.



Since that time, other elements and combinations of elements have been shown to possess a superconducting state at various temperatures. The current flowing in the resistor doesn't produce any heat since the resistance becomes zero. It excludes or repels any magnetic field that comes near it, like a magnetic mirror. This is called the Meissner effect where a magnet will float, or levitate, above a ceramic of superconducting material.

The following demonstration can be seen to get a glimpse of superconductivity - <https://www.youtube.com/watch?v=nWTSzBWEsms>

Future of superconductivity:

1. Today, about 9% the power generated at the electrical power stations is wasted as heat in the wires which carry it from the station to the end user. If these wires were superconductors, there would be a tremendous savings.
2. MRI (magnetic resonance imaging) machines use powerful magnets to create the fields necessary to help physicians see into the body without having to perform surgery. Using superconductors would allow stronger magnets to be built, providing clearer pictures of various types of cells and tissues.
3. SQUIDS are magnetically sensitive sensors, sensitive enough to be able to detect electrical activity within the brain. They would be used to help

diagnose and track changes in brain activity to determine if medicines or treatments are helping.

3. Future trains will use superconductors to provide a method to levitate the train above the tracks, reducing friction with the wheels, allowing the train to travel faster with less energy.

4. If computers used superconductors, then they could be made smaller, with smaller wires which would not heat up due to resistance, which would operate faster because the computer chips are closer to each other.

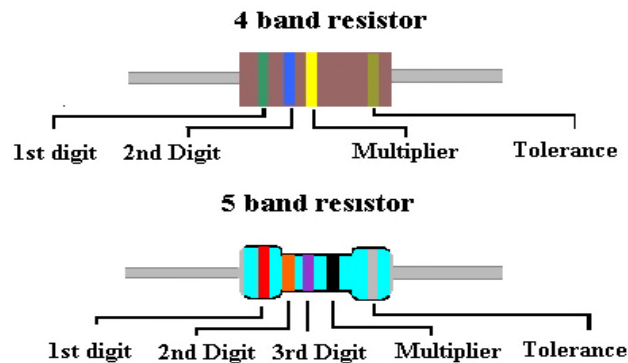
Suggested Projects

Students can carry out guided projects on following topics –

1. Resistance thermometer: Iron coil in liquid nitrogen or flame varies current.
2. Effect of temperature on current in carbon or tungsten filaments.

Colour Coding of Resistors

It can be either of the following two types :



Worksheet problem : Choose 5 different resistors from laboratory, and complete the table :

Resistor No	Resistors value by colour code	Measured resistors value by multi-meter	Difference
1			
2			
3			
4			

Whether the measured values of resistances by multimeter matches with values obtained from the colour code of resistors? Find the percentage error of the multimeter measurement.

4.4 Resistors Combination

Learning Outcomes:

1. Deduce experimentally the law of resistor combination in series and parallel combination of resistors.

Activity: Group Discussion

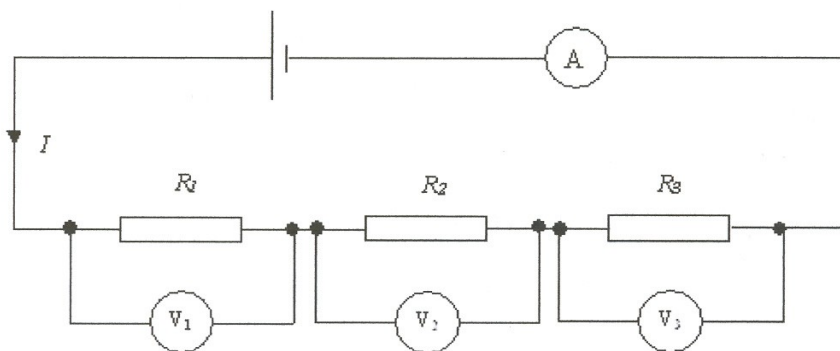
Analogy: Consider a situation where you and your friends have gone for a movie. After the movie ends, the public rushes towards the exit gates present in the theatre. Now you have two choices. Either, all your friends can try to exit together through the same gate, or can get distributed around and try to take exit through different gates. In which of the two cases will it be easier to take exit?

Obviously, in the second case. The same happens in the case of resistors too. The resistor combination offering minimum resistance is the one where the current gets distributed.

Series Resistance Combination

Materials Required : Standard resistors - 3, battery 12 V, ammeter, voltmeter or DMM, connecting wires.

Experiment : Connect the different components as shown in the diagram and find out potential difference across each resistor(V_1, V_2, V_3) and potential difference across the circuit (V) using voltmeter or Digital Multimeter (DMM) . Also find out the current through each resistor (I_1, I_2, I_3) and current through the circuit(I) . Write down the values of potential difference and current in tabular form as given below-



From above table find out the relation between total voltage to the voltage across each resistance and total current in the circuit to the current through each resistor.

Result :
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Discussion: In a parallel circuit, there are multiple loops. So, the current splits up among the loops with the individual loop currents adding to the total current. The current going IN to a junction will always be equal to the current going OUT of a junction. Sum of potential drop across each resistor is equal to the total potential applied.

4.5 Electric Power

Learning Outcomes:

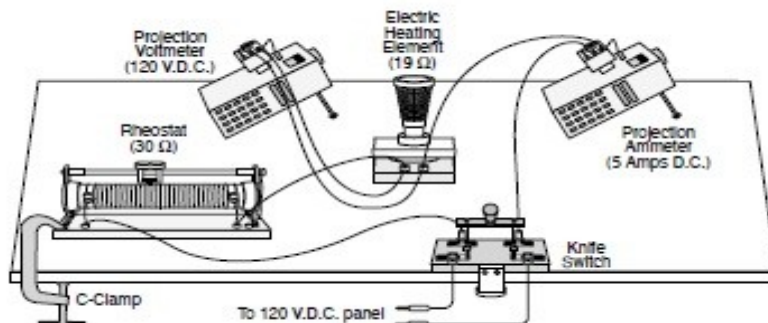
1. Determine the expression for power of an electrical circuit, i.e Watt's law.
2. Compare the power consumed in series and parallel combination of resistors.

Describe the electrical hazards and the influence of shock on human body.

Activity: Group Discussion

Materials Required : Knife switch, Projection Voltmeter (120 V.D.C.), Projection Ammeter (5 Amps D.C.), Rheostat (30 ohms), Electric Heating Element (19 ohms), Variable resistor, glow coil, volt and amp meter.

Procedure : The electric heating element (glow coil) in this demo has a resistance of about 19 ohms. When the knife-switch is closed, a current flows through the coil, causing the coil to heat up. Watt's law states that the rate of energy transfer P equals the current times the voltage. Measure the current and voltage using ammeter and voltmeter respectively. The following circuit picture can be used for reference.

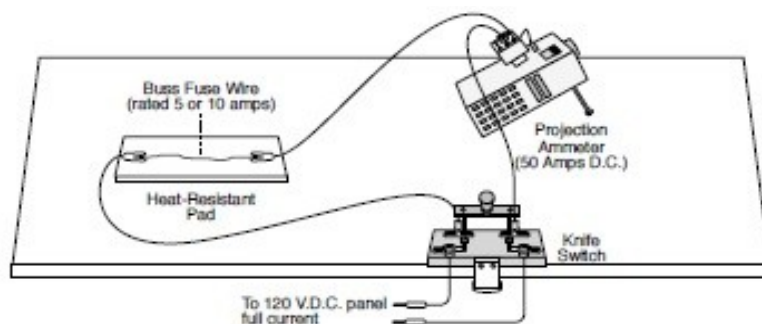


Result : The following relationship is established, i.e $P = VI$

Thermal Effect of Current

Materials Required : Switch, Heat-Resistant Pad, Buss Fuse Wire (rated 5 or 10 amps), Ammeter (50 Amps D.C.)

Procedure : The fuse wire in this demo is rated to stand 5 amps (or 10 amps). The D.C. power panel is adjusted to deliver much more current. When the knife switch is closed, the current rises sharply, then the fuse wire melts with a flash of light and a puff of smoke. As soon as the fuse melts, power must be turned off. The following circuit picture can be used for reference.



Problem:

Make a list of the power ratings on a range of appliances in your home. Explain why something like a toaster has a higher rating than a digital clock. Estimate the energy consumed by these appliances in an average day (by estimating their time of use). Some appliances might only state the operating current. If the household voltage is 120 V, then use $P=VI$. Check out the total power of your classroom in school. Suppose that the building was closed all weekend and that these lights were left on from 2 p.m. Friday until 7 a.m. Monday. What would it cost estimating the rate of electricity supply is Rs.4 per unit? How about for an entire year of weekends?

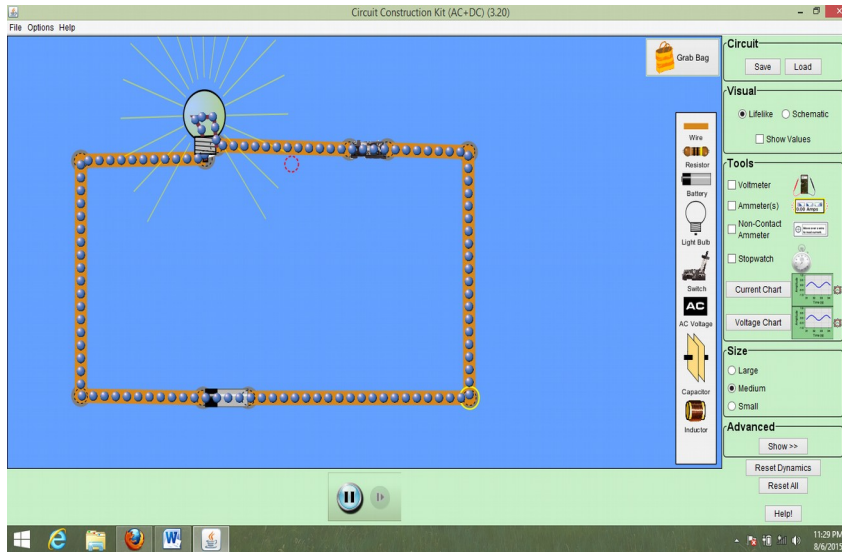
Power in Series & Parallel Combination

Open PhET simulation - <https://phet.colorado.edu/en/simulation/circuit-construction-kit-ac>

Take two bulbs A and B of different resistances and create different circuits using the components, whatsoever required for specific cases, and note down values of quantities mentioned in the table provided at the end.

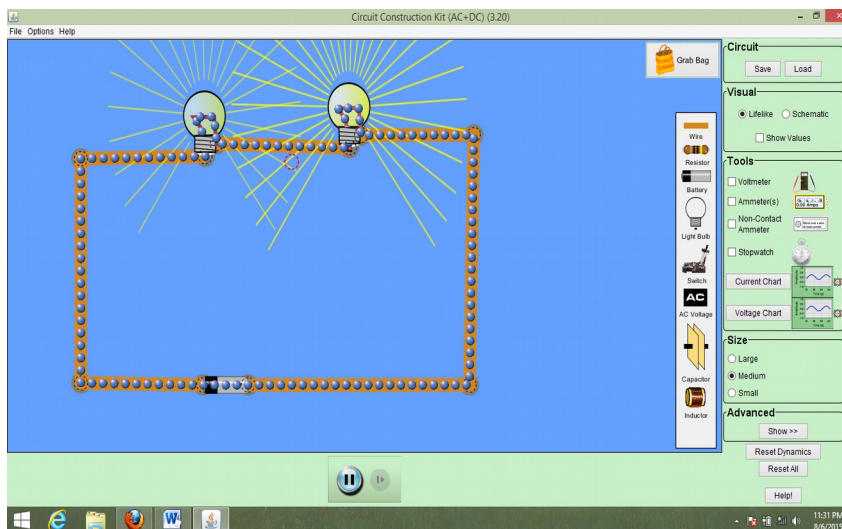
1. Simulation for individual bulbs A and B (Separate Simulation for each Bulb) –

Using “Circuit construction kit (AC + DC)” for two different bulbs A and B of different resistance find out the potential difference (individually) using voltmeter given in the tools and also find the current through the each bulb individually and record the observations in the worksheet.



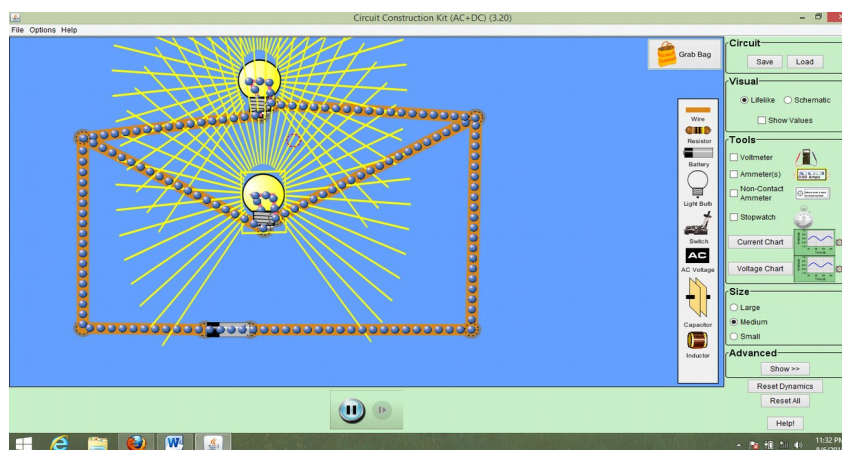
2. Simulation for Series combination –

Using “Circuit construction kit (AC + DC)” for two different bulbs A and B of different resistance in series combination find out the potential difference across the series combination using voltmeter given in the tools and also find the current through the series combination and record the observations in the worksheet.



3. Simulation for Parallel combination –

Using “Circuit construction kit (AC + DC)” for two different bulbs A and B of different resistance in parallel combination find out the potential difference across the parallel combination using voltmeter given in the tools and also find the current through the parallel combination and record the observations in the worksheet.

**Worksheet**

(For recording observation of simple activity and combination of bulbs)
Complete the Table and answer the below questions –

	Resistance (R)	Voltage (V)	Current (I)	$P = VI$	$P = I^2 R$	$P = V^2/R$
Bulb A						
Bulb B						
Bulb A and Bulb B in series						
Bulb A and Bulb B in Parallel						

- Compare the power of individual bulbs and combination of bulbs obtained from three different formulation, whether in three formulation result is same or different?
(Hint –Same)
- How the power of series combination is related to the power of individual bulbs?
(Hint - $P_S = (P_A \times P_B) / (P_A + P_B)$)
- How the power of parallel combination is related to the power of individual bulbs?
(Hint - $P_P = P_A + P_B$)

Electric Hazards and the Human Body

There are two known hazards of electricity—thermal and shock. A **thermal hazard** is one where excessive electric power causes undesired thermal effects, such as starting a fire in the wall of a house. A **shock hazard** occurs when electric current passes through a person. A classic example of this is the **short circuit**, a low-resistance path between terminals of a voltage source. We try to avoid them by following measures

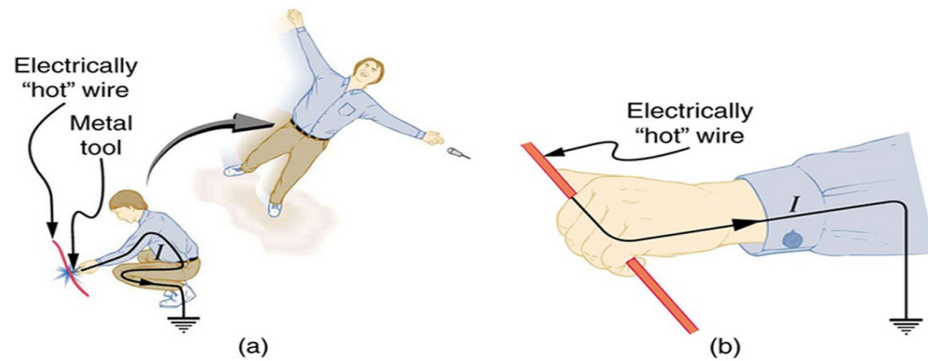


1. A fuse has a metal strip with a low melting point that, when overheated by an excessive current, permanently breaks the connection of a circuit to a voltage source.
2. A circuit breaker is an automatic but restorable electric switch. The one shown here has a bimetallic strip that bends to the right and into the notch if overheated. The spring then forces the metal strip downward, breaking the electrical connection at the points.

The major factors upon which the effects of electrical shock depend are

1. The amount of current I
2. The path taken by the current
3. The duration of the shock
4. The frequency f of the current ($f = 0$ for DC)

Effects on human body



An electric current can cause muscular contractions with varying effects. The victim is “thrown” backward by involuntary muscle contractions that extend the legs and toes. The victim can’t let go of the wire that is stimulating all the muscles in the hand. Usually the best way to help the victim is to give the fist a hard knock/blow/jar with an insulator or to throw an insulator at the fist. Those that close the fingers are stronger than those that open them.

Our bodies are relatively good conductors due to the water in our bodies. Ohm's law states that the current drawn depends on the resistance of the body. The resistance of human skin varies from person to person and fluctuates between different times of day. The NIOSH states "*Under dry conditions, the resistance offered by the human body may be as high as 100,000 Ohms. Wet or broken skin may drop the body's resistance to 1,000 Ohms,*" adding that "*high-voltage electrical energy quickly breaks down human skin, reducing the human body's resistance to 500 Ohms*".

Precautions from electric shock & safety limit

Wearing insulating shoes, a requirement in many professions, prohibits a pathway for electrons by providing a large resistance in that path. Whenever working with high-power tools (drills), or in risky situations, ensure that you do not provide a pathway for current flow (especially through the heart).

Very small currents pass harmlessly and unfelt through the body. This happens to you regularly without your knowledge. The threshold of sensation is only 1 mA and, although unpleasant, shocks are apparently harmless for currents less than 5 mA. A great number of safety rules take the 5-mA value for the maximum allowed shock.

4.6 Kirchoff's laws & Measurement devices

Learning Outcomes:

1. Derive the expression for internal resistance in terms of voltage and EMF of the battery.
2. Determine the expression for equivalent resistance and equivalent EMF's for the combination of given potential difference in an electrical circuit.
3. Verify Kirchoff's laws
4. Experimentally deduce the condition for balancing Wheatstone bridge.
5. Determine the unknown resistance using the meter bridge apparatus.
6. Compare EMF's using the Potentiometer.
7. Determine the internal resistance of a given voltage source using Potentiometer.

Activity: Group Discussion

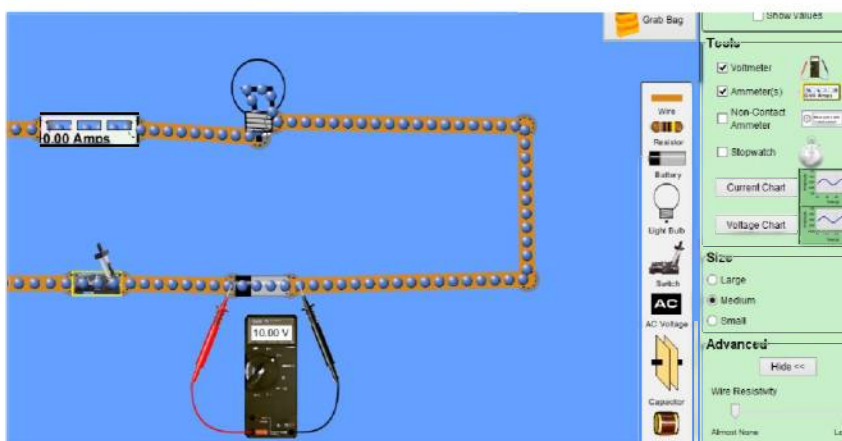
a) EMF & Internal Resistance :

Open the simulation -

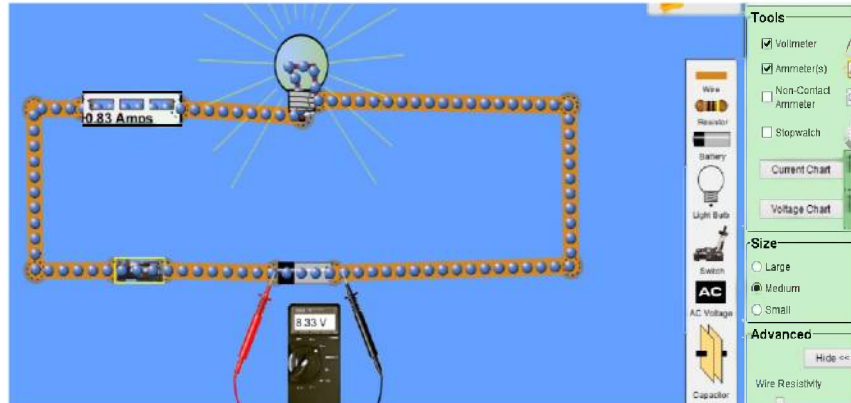
<https://phet.colorado.edu/en/simulation/circuit-construction-kit-ac-virtual-lab>

Procedure -

- i. Connect the circuit elements as shown in the picture below and measure as well as note the EMF and Voltage produced by the battery in 1st and 2nd cases respectively.



EMF



VOLTAGE

1. By right click on the cell adjust the values of EMF and internal resistance of cell (say $E=10\text{ V}$, $r = 2\ \Omega$). Measure the potential difference across the cell when key is opened (no current through the circuit) and when key is closed (there is a current flow in the circuit) using voltmeter available in the tool box at right side of the screen.

2. Potential difference across the cell decreases when there is a current flow due to voltage drop across the internal resistance of the cell equal to Ir . Then, $E - Ir$ is the net potential difference across the cell at the time of current flow. (EMF is the potential difference across the cell if there is no current flow and voltage is the potential difference across the cell if there is a current flow).

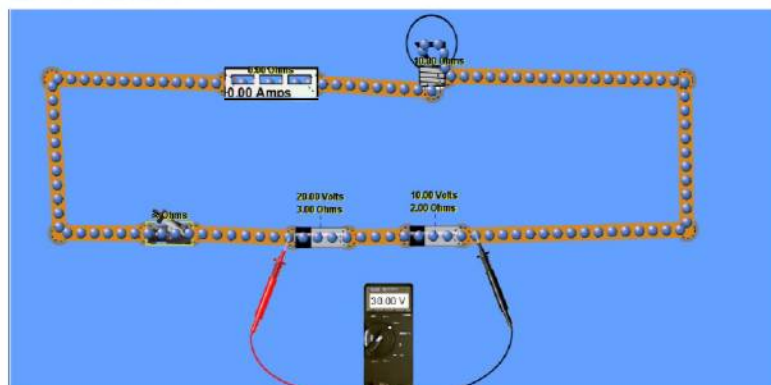
EMF(E) and voltage(V) can relate as $V = E - Ir$.

3. Measure the potential difference across the bulb if there is a current flow (according to ohms law $V=IR$). It is equal to $E - Ir = IR$.

We can calculate the value of internal resistance by measuring E , V and I

$$r = \frac{E - V}{I}$$

b) Series combination



Procedure –

1. Construct a circuit of cells are connected in series across a bulb with ammeter, voltmeter and key. Adjust the values of emf and internal resistance of cells (say $E_1 = 20\text{ V}$, $r_1 = 3\ \Omega$ and $E_2 = 10\text{ V}$, $r_2 = 2\ \Omega$).

2. Measure the readings as in the previous activity using two cells separately and connecting them in series. Record the observations in the **Table 1**. In series combination current is same through each cell but voltage across each cell is different, so

$$V_1 = E_1 - Ir_1$$

$$V_2 = E_2 - Ir_2$$

$$V = V_1 + V_2$$

$$V = (E_1 + E_2) - I(r_1 + r_2)$$

Hence, if two cells are considered as single cell having EMF equal to 'E' and internal resistance 'r', then V is equal to E-Ir is the equivalent potential difference across the cell. Comparing above equations,

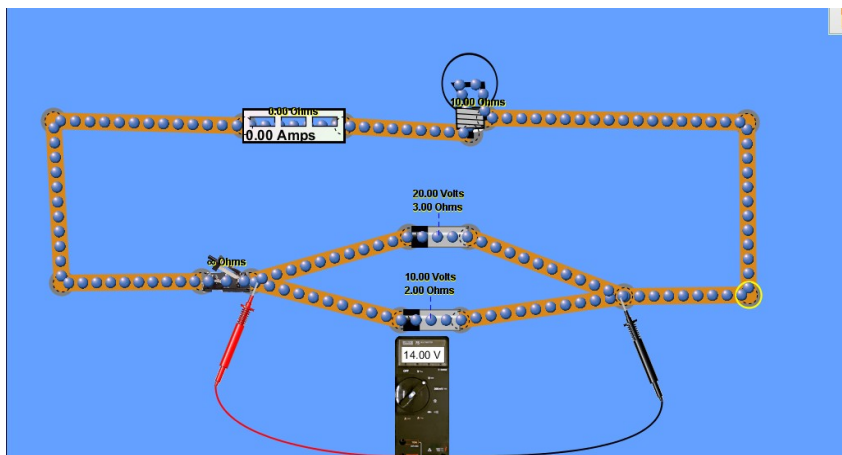
$$\text{Equivalent emf} \quad : \quad E = E_1 + E_2$$

$$\text{Internal resistance} \quad : \quad r = r_1 + r_2$$

We can calculate the value of effective internal resistance by measuring E, V and I as

$$r = \frac{E - V}{I}$$

c) Parallel combination



Procedure –

Construct a parallel combination of cells as shown in figure. Adjust the values of emf and internal resistance of cells (say $E_1 = 20\text{ V}$, $r_1 = 3\ \Omega$ and $E_2 = 10\text{ V}$, $r_2 = 2\ \Omega$).

Measure the readings as in the previous activity using two cells separately and connecting them in parallel. Record the observations in the **Table 1**.

In parallel combination, voltage is same across each cell but current through each cell is different

$$I_1 = E_1 - V / r_1$$

$$I_2 = E_2 - V / r_2$$

$$I = I_1 + I_2$$

$$I = \frac{E_1}{r_1} + \frac{E_2}{r_2} - V \left[\frac{1}{r_1} + \frac{1}{r_2} \right]$$

If two cells are considered as single cell having emf E and internal resistance r

$$I = \frac{E}{r} - V \left[\frac{1}{r} \right] \quad \text{is the net current due to two cells.}$$

Comparing above equations, we get equivalent EMF,

$$E = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

and equivalent internal resistance, $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$ or $r = \frac{r_1 r_2}{r_1 + r_2}$

We can calculate the value of effective internal resistance by measuring E , V and I as

$$r = \frac{E - V}{I}$$

TABLE - 1

Cell or cell combination	EMF (V)	Voltage (V)	Current (I)	Internal resistance $r = \frac{E - V}{I}$
Cell 1	$E_1 =$			r_1
Cell 2	$E_2 =$			r_2
Cells 1 and 2 in series	$E_s =$			$r_s =$
Cells 1 and 2 in Parallel	$E_p =$			$r_p =$

Worksheet

1. What is relation between emf and internal resistance of cells when cells are connected ?

- a. in series (Hint:emf and internal resistance is greater than the greatest value of emf and internal resistance of given cells)
- b. in parallel (Hint:emf in between the emf of given cells, internal resistance smaller than the smallest value of the internal resistance of given cells)

2. Measure Emf of each cell using voltmeter and potentiometer. Is there any difference? Why?

(Hint : value obtained from the voltmeter smaller than the value obtained from potentiometer because small current flowing through the cell and a potential drop is occurred across the internal resistance of cell)

3. What happened to internal resistance of cell if external resistance is increased?

(hint : increases)

4. What happened to the difference of emf and voltage If external resistance having

a) small value (hint : large)

b) large value (hint : small)

5. If two cells (having emf E_1 and E_2 internal resistance r_1 and r_2) are combined, is there any current flow through these cell if it is not connected to external resistance? If yes write its equation

a) When cells are connected in series (hint : no current flow)

b) When cells are connected in parallel (hint : there is a current flow $I = (E_2 - E_1) / (r_1 + r_2)$)

6. Two cells having same emf and internal resistance connected to an external resistor separately, combined in series and in parallel I is the current due to the single cell, I_s is the current when cells are connected in series and I_p is the current when cells are connected in parallel,

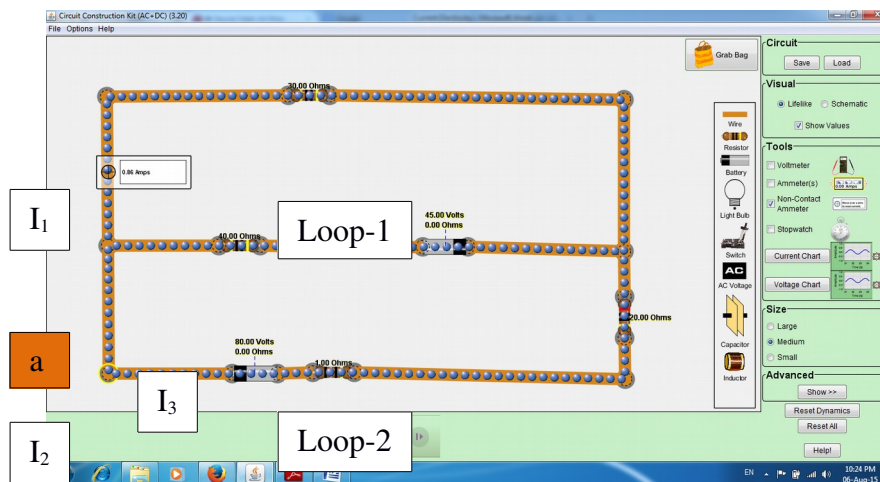
$I_s = \dots\dots\dots$ ($2I$, greater than $2I$, less than $2I$) (hint : less than $2I$ because internal resistance increases when cells are connected in series)

$I_p = \dots\dots\dots$ (I , greater than I , less than I) (hint : greater than I because internal resistance decreases when cells are connected in series)

Kirchoff Laws

Open the simulation - <https://phet.colorado.edu/en/simulation/circuit-construction-kit-ac-virtual-lab>

Using the Circuit Construction Kit available in PhET, we can construct the circuit given in NCERT Text (Fig3.22) with the same values. It looks like as given below.



1. Junction Rule

Consider the Junction **a** where three currents I_1 , I_2 and I_3 meet. We can measure the currents using Ammeter given on the right side of the window. What are the currents do you observe here? [$I_1 = 0.86\text{A}$, $I_2 = 2.58\text{A}$ and $I_3 = 1.72\text{A}$]

Can you find any relation between them? [$I_2 = I_1 + I_3$]

Kirchhoff's Junction rule is stated as – ‘**At any junction, the sum of the currents entering the junction is equal to the sum of currents leaving the junction**’.

2. Loop Rule

In the same circuit, we consider two closed loops. In loop 1, we measure the potential differences using the voltmeter given on the right side. What are the values you see here?

[45V (emf of the cell), 25.74V (across 30Ω), -69.02V (across 40Ω) and 1.73V (across 1Ω)] What conclusion can you make now? [The sum of the potential differences of the loop is zero]

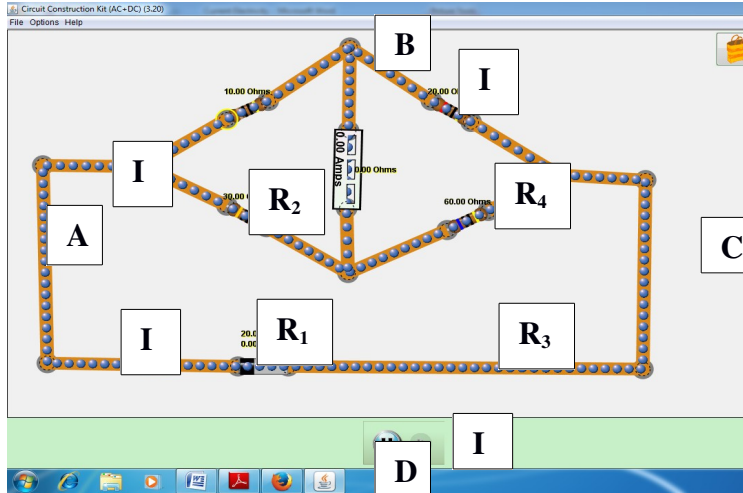
Kirchhoff's Loop rule is stated as – ‘**The algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero**’.

Wheatstone Bridge

Open the simulation -

<https://phet.colorado.edu/en/simulation/circuit-construction-kit-ac-virtual-lab>

Using the Circuit Construction Kit available in PhET, we can construct the circuit given in NCERT Text (Fig3.25) with the same values. It looks like as given below.



What do you observe?

[Currents are flowing through each arm including through the Galvanometer (Here, Ammeter)]

Now we want to make the current through the Galvanometer as zero. For that we continuously vary the values of resistances, and at a particular set of values the Galvanometer current becomes zero. Now the bridge is said to be balanced. [For example, $E = 20\text{V}$, $R_1 = 30\Omega$, $R_2 = 10\Omega$, $R_3 = 60\Omega$, $R_4 = 20\Omega$]

We apply Kirchhoff's Loop rule for the closed loop ABDA. Measure the potential drops across R_1 , Galvanometer and R_2 . [Here, the values obtained are 6.67V (across R_1), 0V (across Galvanometer, Galvanometer current is zero) and -6.67V (across R_2)]

The net potential is zero. That is, $I_1 R_1 - I_2 R_2 = 0$.

$$\text{Or, } I_1/R_1 = I_2/R_2 \quad (1)$$

In a similar manner, by applying Kirchhoff's rule to the closed loop BCDB, we obtain the relation:

$$I_3/R_3 = I_4/R_4 \quad (2)$$

Since no current is flowing through the Galvanometer, applying Kirchhoff's Junction rule to the junctions B and D, it can be shown that $I_1 = I_3$ and $I_2 = I_4$.

[We can check it by measuring the currents I_1 , I_2 , I_3 and I_4 using Ammeter. Here the values are $I_1 = 0.22\text{A}$, $I_2 = 0.67\text{A}$, $I_3 = 0.22\text{A}$ and $I_4 = 0.22\text{A}$. That is, $I_1 = I_3$ and $I_2 = I_4$]

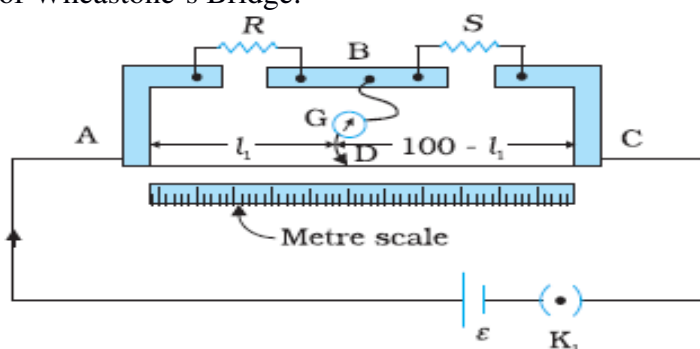
Using equations (1) and (2),

$$\frac{R_2}{R_1} = \frac{R_3}{R_4}$$

This is the balance condition of Wheatstone bridge or Principle of Wheatstone Bridge.

Metre Bridge

The students are divided into different groups. Each group is given the following materials. The Metre bridge apparatus, a Cell, two resistance boxes, a resistance wire, a Galvanometer, Connecting wires and a Jockey. The students are asked to find the resistance of the resistance wire, using Principle of Wheatstone's Bridge.



The connection is made as in figure. Jockey is moved from end A to C. At a particular point, the Galvanometer shows null deflection means no current is flowing through it. At this instant the Bridge is said to be balanced. Here we apply the principle of Wheatstone's bridge,

$$\frac{\text{Resistance across AD}}{R} = \frac{\text{Resistance across DC}}{S}$$

If R_{cm} is the resistance per unit cm, then above equation becomes –

$$\frac{l_1 R_{cm}}{R} = \frac{(100 - l_1) R_{cm}}{S}$$

$$\text{Or, } R = \frac{S \cdot l_1}{(100 - l_1)}$$

Potentiometer

Teacher introduces the Potentiometer apparatus by explaining its construction and uses. The primary circuit is connected. A torch bulb is connected at the place of secondary circuit along with a Jockey. The Jockey is moved along the resistance wire starting the end A to B.

Q. What do you observe?

[The intensity of light is increasing gradually].

Q. Why it is so?

[The potential drop is increasing with the length of Potentiometer wire].

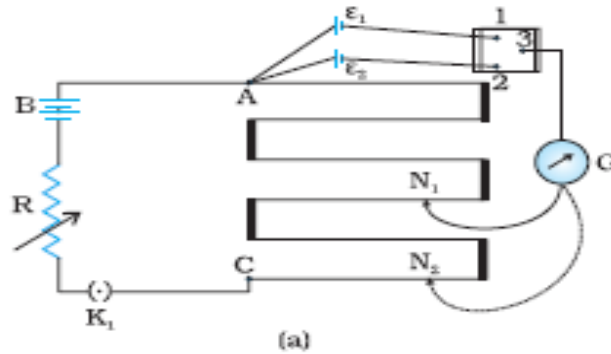
Q. What conclusion can you made here?

[Potential difference is directly proportional to the length of the wire].

Hence, $\epsilon \propto l$, **Or,** $\epsilon = \phi l$ (Principle of Potentiometer), where ϕ is the potential drop per unit length.

a) Comparison of EMFs of two cells using Potentiometer

Now connect the secondary circuit as shown below. The cell of EMF ϵ_1 is included in the circuit using the keys 1 and 3. Move the Jockey gradually along the resistance wire; the Galvanometer deflection can be brought to zero at a particular length say l_1 .



Now apply Kirchhoff's loop rule to the closed loop AN_1G31A ,
 $\Phi l_1 + 0 - \epsilon_1 = 0$ (1)

Again, do the same procedure using the second cell of emf ϵ_2 . Now the balancing length is l_2 . Applying Kirchhoff's rule to the closed loop AN_2G32A ,

$$\Phi l_2 + 0 - \epsilon_2 = 0$$
 (2)

From the above equations,

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

Now the students are given two cells and they are asked to do compare their emfs using Potentiometer.

How do you use Potentiometer to find the EMF of a cell, if you are supplied with two cells and the EMF of one of them is given?

b) Measurement of Internal resistance of a cells using Potentiometer

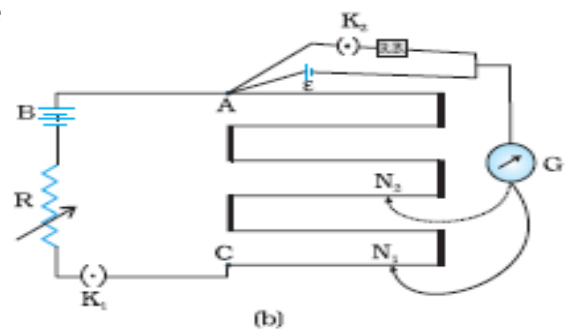
Connect the secondary circuit as in figure. If K_2 is open and let l_1 be the balance length, how do you express the Kirchhoff's rule?

$$\epsilon = \Phi l_1$$

If K_2 is closed, let V be the terminal pd and l_2 be the balance length, What will be the rule?

So,

$$\frac{\epsilon}{V} = \frac{l_1}{l_2}$$
 (1)



How do you relate emf of a cell (ϵ), current flowing through the circuit (I), internal resistance (r) and external resistance we introduced in the resistance box(R)?

$$\epsilon = I(r + R) = IR \left(\frac{r}{R} + 1 \right)$$

$$\epsilon = V \left(\frac{r}{R} + 1 \right)$$

$$\frac{\epsilon}{V} = \frac{r}{R} + 1 \dots\dots\dots (2)$$

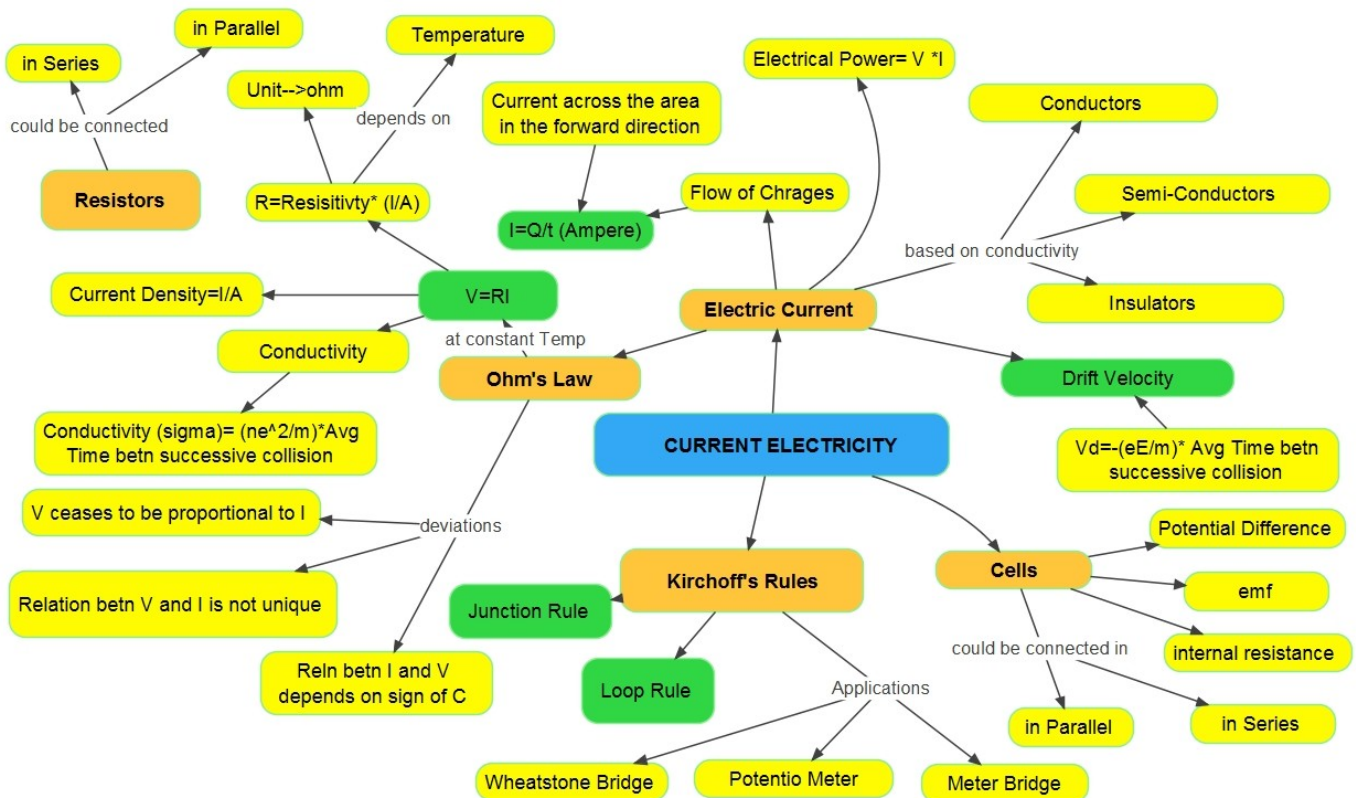
Comparing equations (1) and (2), how do you express r ?

$$\frac{r}{R} + 1 = \frac{l_1}{l_2}$$

$$r = R \left(\frac{l_1}{l_2} - 1 \right)$$

Q. What is the advantage of Potentiometer over voltmeter?

[It draws no current from the voltage source and is unaffected by the internal resistance of the source]



MOVING CHARGES AND MAGNETISM



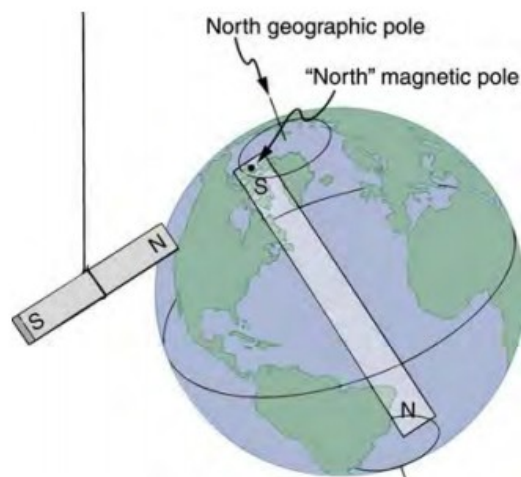
1.1 Introduction

Many of the historians of science believe that the compass, which uses a magnetic needle, was used in China as early as the 13th century B.C., its invention being of Arabic or Indian origin. The early Greeks knew about magnetism as early as 800 B.C. They discovered that the stone magnetite (Fe_3O_4) attracts pieces of iron. Legend ascribes the name *magnetite* to the shepherd Magnes, the nails of whose shoes and the tip of whose staff stuck fast to chunks of magnetite while he pastured his flocks. In 1269 a Frenchman named Pierre de Maricourt mapped out the directions taken by a needle placed at various points on the surface of a spherical natural magnet. He found that the directions formed lines that encircled the sphere and passed through two points diametrically opposite each other, which he called the *poles* of the magnet. Subsequent experiments showed that every magnet, regardless of its shape, has two poles, called *north* and *south* poles that exert forces on other magnetic poles just as electric charges exert forces on one another. That is, like poles repel each other, and unlike poles attract each other. The poles received their names because of the way a magnet behaves in the presence of the Earth's magnetic field. If a bar magnet is suspended from its midpoint and can swing freely in a

horizontal plane, it will rotate until its North Pole points to the Earth's geographic North Pole and its South Pole points to the Earth's geographic South Pole.¹ (The same idea is used in the construction of a simple compass.) In 1600 William Gilbert (1540–1603) extended de Maricourt's experiments to a variety of materials. Using the fact that a compass needle orients in preferred directions, he suggested that the Earth itself is a large permanent magnet. In 1750 experimenters used a torsion balance to show that magnetic poles exert attractive or repulsive forces on each other and that these forces vary as the inverse square of the distance between interacting poles.

The subjects of magnetism and electricity developed almost independently of each other until 1820, when a Danish physicist named Hans Christian Oersted discovered, in a classroom demonstration, that an electric current affects a magnetic compass. He saw confirming evidence that magnetism was related to electricity. Shortly thereafter, the French physicist Andre Marie Ampere proposed that electric currents are the source of all magnetic phenomena.

Misconception Alert!



The Earth acts like a very large bar magnet with its south-seeking pole near the geographic North Pole. That is why the north pole of your compass is attracted toward the geographic north pole of the Earth—because the magnetic pole that is near the geographic North Pole is actually a south magnetic pole! Confusion arises because the geographic term “North Pole” has come to be used (incorrectly) for the magnetic pole that is near the North Pole. Thus, “North magnetic pole” is actually a misnomer—it should be called the South magnetic pole

5.2 Magnetic Force on a Charged Particle & Current carrying conductor

Learning Outcomes:

- Explain the magnetic force acting on a charged particle.
- Describe the mechanism of magnetic force acting on a current carrying conductor.

Activity: Group Discussion

The charged particle at rest does not experienced by any force in a magnetic field. But, a force acts on a charged particle (like electron) when it moves in the magnetic field.

The force on a charged particle moving in the magnetic field is given by

$$F = qVB \sin \theta$$

In vector form,

$$\vec{F} = q [\vec{v} \times \vec{B}]$$

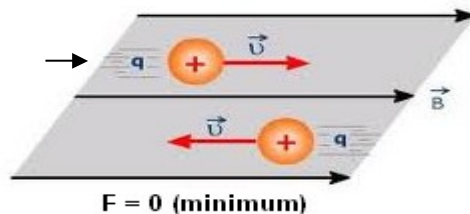
q = magnitude of charge, V = velocity of the particle, B = magnetic field strength, θ = angle between V and B

Case (i)

The force is minimum (zero), when the particle is at rest (i.e. $V = 0$) and also V is parallel or anti-parallel to B

i.e. When $\theta = 0^\circ$ or 180° $\sin \theta = 0$

$$\therefore F_{\min} = 0.$$



Case (ii)

The force on a charged particle is maximum, when V is perpendicular to B .

i.e. when $\theta = 90^\circ$ $\sin 90^\circ = 1$

$$F_{\max} = qVB$$

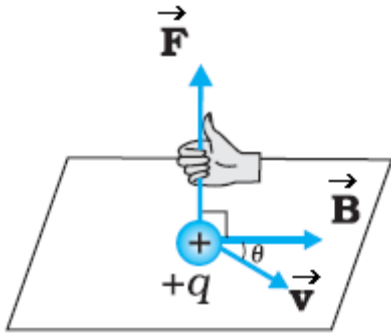
Lorentz force

The total force acting on a charged particle moving in the presence of both electric field $E(r)$ and magnetic field $B(r)$ is called Lorentz force. It is given by ,

$$\mathbf{F} = \mathbf{F}_{\text{electric}} + \mathbf{F}_{\text{magnetic}}$$

$$\mathbf{F} = q \mathbf{E}(\mathbf{r}) + q \mathbf{v} \times \mathbf{B}(\mathbf{r})$$

In vector form,



$$\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$$

This is Lorentz force.

Note (i)

Lorentz force in magnetic field is

$$\mathbf{F}_{\text{magnetic}} = q(\vec{v} \times \vec{B})$$

The direction of this force is always perpendicular to the plane containing V and B.

Also,
$$\mathbf{F} = qvB \sin \theta \hat{n}$$

Definition of the ‘B’

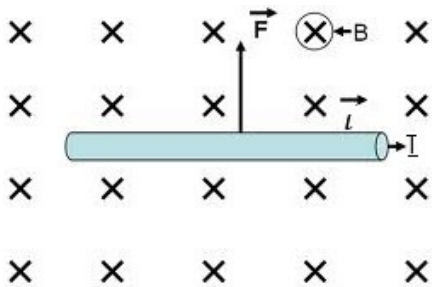
We know that,

$$\mathbf{F} = qvB \sin \theta$$

If $q = +1, v = 1$ and $\theta = 90^\circ$, then $F = B$

\therefore Magnetic field at a point is defined as “the force acting on unit +ve charge moving with unit velocity perpendicular to the direction of magnetic field”.

The SI unit of ‘B’ is tesla (T)



Note:

1. Another unit of ‘B’ is gauss, which is not in SI.
2. 1 gauss = 10^{-4} T.
3. Dimension of B is $[M^1A^{-1}T^{-2}]$

Magnetic force on a current carrying straight conductor:

Consider a conductor (or straight wire) of uniform area of cross section ‘A’ and length ‘L’. Since we have considered a conductor, the charge carriers are electrons. Let ‘n’ be the charge density of the electrons and ‘e’ be the charge of each electron.

\therefore The total charge $q = neAL$

Let ‘B’ be the external magnetic field.

If the study current ‘I’ flows through the conductor each charge has an average drift velocity ‘ V_d ’

The Lorentz force only in magnetic field is

$$\mathbf{F} = q(\mathbf{V} \times \mathbf{B})$$

here, $q = neAL$ and $V = V_d$

$$\mathbf{F} = neAL (\mathbf{V}_d \times \mathbf{B})$$

$$\mathbf{F} = neA V_d (\mathbf{L} \times \mathbf{B})$$

$$\mathbf{F} = I (\mathbf{L} \times \mathbf{B})$$

$$F = I L B \sin \theta$$

$$\mathbf{F} = I (\mathbf{L} \times \mathbf{B})$$

The direction of force F is perpendicular to both L and B . The magnitude of the force is

$$F = I L B \sin \Theta$$

Note:

If the rod or wire has arbitrary shape, then the force is given by

$$\vec{\mathbf{F}} = \sum I d\vec{\mathbf{l}} \times \vec{\mathbf{B}}$$

Special cases:

1. $F = I L B \sin \Theta$

When $\Theta = 0^\circ$ or $\Theta = 180^\circ$ ($\sin 0^\circ = 0$) $F = 0$

i.e. Current carrying conductor experience no force when placed parallel or anti parallel to the direction of magnetic field.

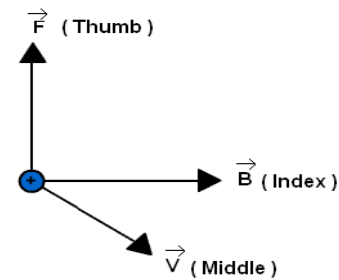
2. If $\Theta = 90^\circ$ ($\sin 90^\circ = 1$)

$$F = I L B$$

A current carrying conductor placed at right angles to the uniform magnetic field experience maximum force.

Left hand rule:

This rule states that, “If the first three fingers of the left hand are stretched so that they are mutually perpendicular to each other with the index finger representing ‘B’, middle finger representing ‘V’ and then thumb represents the force ‘F’ on the point positive charge.



Motion of charged particle in a magnetic field.

When a charged particle ‘q’ moving with a velocity ‘V’ enters the magnetic field ‘B’ which is perpendicular to ‘V’ (i.e. $\Theta = 90^\circ$), then the particle describes a circular path of radius ‘r’. The required centripetal force is provided by the magnetic force. This force only changes the direction of the charge continuously but not the magnitude of velocity. So, the particle describes uniform circular motion.

\therefore Centripetal force = magnetic force

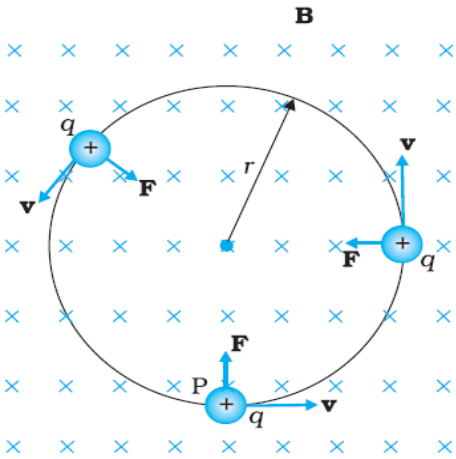
$$\frac{mv^2}{r} = qvB \sin \theta$$

$$\frac{mv}{r} = qB$$

(But $\Theta = 90^\circ$)

$$r = \frac{mv}{qB} \longrightarrow (1)$$

Here ‘m’ = mass of the particle.



This equation shows that, the larger the momentum (mV), larger is the radius and bigger is the circular path. The stronger the magnetic field (B), smaller is the radius and smaller is the circular path.

We know that,

$$V = r\omega = 2\pi \gamma r \tag{2}$$

ω = angular velocity.

From (1) and (2)

$$r = \frac{m (2\pi v r)}{q B}$$

$$\gamma = \frac{q B}{2\pi m}$$

Here, γ is the frequency of rotation of charged particle.

The period of rotation is,

$$T = \frac{1}{\gamma} = \frac{2\pi m}{q B}$$

Both, γ and 'T' are independent of velocity (V) or kinetic energy of the charged particle.

Using this principle, Cyclotrons (a system used to accelerate charged particles) are built.

A charged particle moving with a velocity 'V' enters the uniform magnetic field 'B' at an angle other than 90° (i.e. 0° < Θ < 90°)

The magnetic force acting on the particle produces a helical path. It can be explained as follows. If Θ is in between 0° and 90°, 'V' is not perpendicular to B, then velocity has a perpendicular component $V_{\perp} = V \sin \Theta$ and a parallel component $V_{\parallel} = V \cos \Theta$.

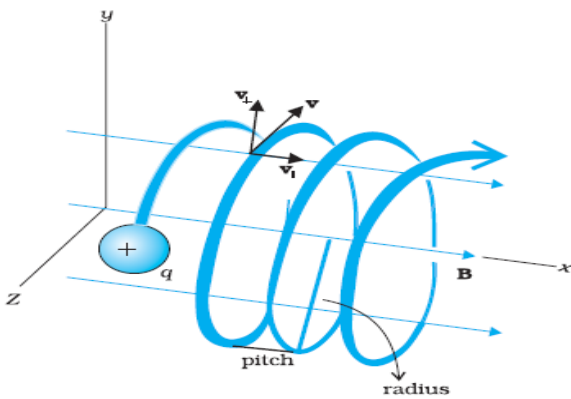
The V_{\perp} component makes the charged particle to move in a circular path ($\because F = qVB \sin \Theta$), While V_{\parallel} component makes the particle to move along a straight line in the direction 'B'. As a result a particle covers circular path as well as linear path. Therefore path followed by the particles is helix (helical path) as shown in figure.

This principle is applicable to the charged particles entering the earth's magnetic field from the distant universe.

Pitch of the helix (P):

The linear distance covered by the particle along the magnetic field in a time equal to the period of revolution is called pitch.

$$\therefore \text{Pitch} = \text{distance} = \text{Velocity} \times \text{time (period)}$$



$$P = V \sin \theta \times T = (V \cos \theta) \times T$$

The radius 'r' of circular motion is called the radius of the helix.

Motion of a charged particle in combined electric field and magnetic field and velocity selector

When a particle of charge 'q' is moving with velocity V in the presence of both electric E and magnetic field B, it experiences a force

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$

If 'E' is perpendicular to B and both are perpendicular to v as shown in figure, then

$$\vec{v} = v\hat{i}, \quad \vec{E} = E\hat{j}, \quad \vec{B} = B\hat{k}$$

∴ Electric force

$$\vec{F}_e = q\vec{E} = qE\hat{j}$$

Magnetic force

$$\vec{F}_B = q(\vec{v} \times \vec{B})$$

$$\vec{F}_B = q(v\hat{i} \times B\hat{k})$$

$$\vec{F}_B = qvB(-\hat{j}) \quad (\because \hat{i} \times \hat{k} = -\hat{j})$$

This shows that, the magnetic force acts along negative Y direction and electric force is along positive Y direction. Thus, F_E and F_B are in opposite directions.

The net force on a charge is given by

$$\vec{F} = qE\hat{j} - qvB\hat{j}$$

$$\vec{F} = (qE - qvB)\hat{j}$$

Suppose we adjust the values of E and B, such that $qE = qvB$ (i.e. $F_E = F_B$) Then the net force on the charge is zero (i.e. $F = 0$). Now, the charged particle moves straight (i.e. along X-axis) without change in direction.

$$\therefore qE = qvB$$

$$v = \frac{E}{B}$$

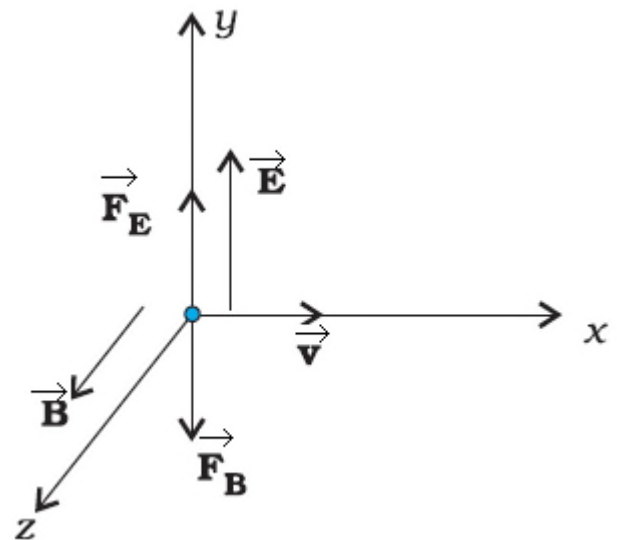
is called velocity selector.

Thus the velocity is independent of mass and charge.

Velocity Selector:

The chosen velocity under which the charged particle move undeviated through the crossed fields is called velocity selector.

The ratio (E/B) is called velocity selector.



Note:

1. This condition can be used to select the charged particles of a particular velocity in a beam containing charged particles with different speeds. J.J. Thomson used this method to find the ratio of charge to mass (e/m) of electrons. This principle is also used in mass spectrometer to separate the charged ions according to their charge to mass ratio.
2. Electric field and magnetic field perpendicular to each other are called crossed fields.

5.3 Ampere Circuital Law

Learning Outcomes:

- Explain the Ampere circuital law.
- Describe the mechanism of Ampere circuital law.
- Describe the applications of Ampere circuital law.
- Describe the force acting on two current carrying wires.

Activity: Group Discussion and Demo

Demo: Force between Two Current Loops

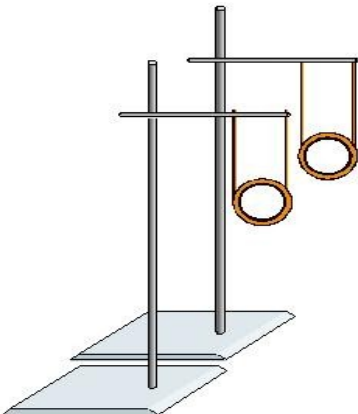
Materials:

1. 2 small-diameter coil
2. 2 30W or higher 5V DC power supply
3. 2 lab stands
4. 1 thin str aluminum foil
5. Wires with alligator clip leads 4
6. tape, 1 roll

*This demo was tested with a 2-inch diameter coil of 100 turns using 20 gauge magnet wire held together with duct tape.

Before presenting the demonstration, pose the problem of two short segments of wire aligned in a parallel fashion and have the students determine the direction of the magnetic field in each segment produced by the other segment. Then have them determine the direction of the magnetic force on each segment. Then consider many of these segments forming a loop in which each segment of the loop is attracted to the corresponding segment on the other loop. Have the students determine the magnetic moment vector of each coil to conclude that loops with parallel magnetic moments will attract.

Set up the demonstration as shown in the diagram above. Connect each loop to its own power supply, but do not complete the circuit until you are ready to start the demonstration as the wires may overheat. Do not

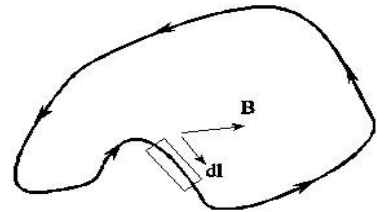


leave the circuits connected for more than a few seconds. Momentarily complete the circuit to both coils and watch them attract. Reverse the current in one of the loops to see them repel.

Students should be provided with additional information enabling them to revise their initial ideas for solving the challenge. The research aspect will consist of a demonstration on the force between two current loops and a formal lecture on Ampere's law and its applications.

Ampere's Law

Although the total magnetic flux through a closed surface must be zero, the sum of the magnetic field around a closed loop does not have to be zero. More specifically, if a closed loop is imagined in space, we can create a vector at each tiny segment called $d\mathbf{l}$ pointing in the direction of the loop with a magnitude equal to the length of the segment. Then we will look at the magnetic field at that point, and take the dot product $\mathbf{B} \cdot d\mathbf{l}$ and think of this as the magnetic field along that segment of the loop. The



integral of these dot products around the loop, $\oint_{loop} \mathbf{B} \cdot d\mathbf{l}$, is a mathematically precise way of considering the magnetic field around a closed loop.

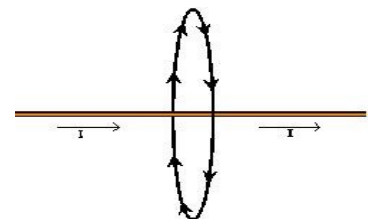
As you might imagine, the magnetic field around a loop would be greater if a wire carrying current was passing through the interior of the loop. In fact, the exact value of the field around the loop is

$$\oint_{loop} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

where I is the total current passing through the loop. This result is named Ampere's law, after its discoverer. Much like Gauss law, it is useful in determining the magnitude of the magnetic field in highly symmetric situations.

Example: Determine the magnetic field outside of an infinitely long straight wire.

Solution: Consider a closed circular loop of radius R around the wire centered on the axis of the wire, as shown in the diagram. You can see that the diagram is rotationally symmetric, so that the magnetic field must have a constant magnitude around the loop.



$$\oint_{loop} \mathbf{B} \cdot d\mathbf{l} = \oint_{loop} B dl = B \oint_{loop} dl = B(2\pi R)$$

Thus,

Now by Ampere's law,

$$\oint_{loop} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

$$B(2\pi R) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi R}$$

This result is in agreement with the Biot-Savart law. Note that this would appear to work for a finite segment of wire and give the same result, contradicting the result from the Biot-Savart law saying that

$$B = \frac{\mu_0 I}{4\pi R} (\sin \theta_2 - \sin \theta_1)$$

. Experimentally, the Biot-Savart law has been shown to be correct. This discrepancy can be understood by realizing that the segment of wire must be part of a larger circuit, breaking the symmetry and invalidating the use of Ampere's law, or that the current must be a non-steady flow from one conductor to another. From this, we can surmise that Ampere's law must be valid only for steady-state currents.

Toroid

A toroid is basically a solenoid that is bent into a circle, as shown. This doughnut shaped figure has an inner radius a , an outer radius b , a current I , and a total number of turns N . To determine the magnetic field using amperes law, we will imagine a circular loop of radius r sharing a center with the toroid in the plane of the toroid.

Case 1: $r < a$

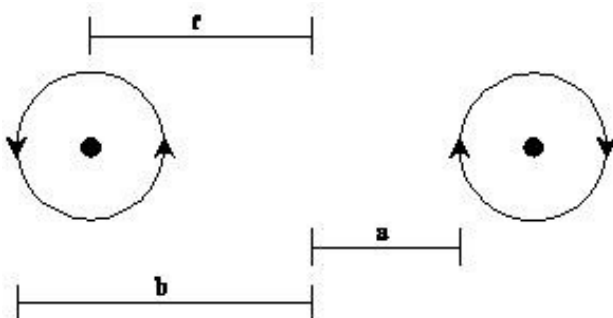
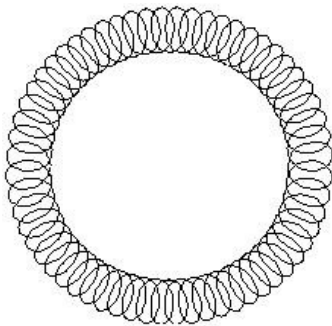
If $r < a$, then there is no current passing through the loop, so

$$\oint_{loop} \mathbf{B} \cdot d\mathbf{l} = 0$$

. Since the diagram possesses rotational symmetry, the magnetic field must have equal magnitude anywhere along the loop, so $B = 0$ everywhere inside the inner radius.

Case 2: $a < r < b$

In this case, each turn passes through the loop in one direction, as shown in the cross sectional view to the right.



$$\oint_{loop} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{total}$$

$$B \oint_{loop} dl = \mu_0 NI$$

$$B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r}$$

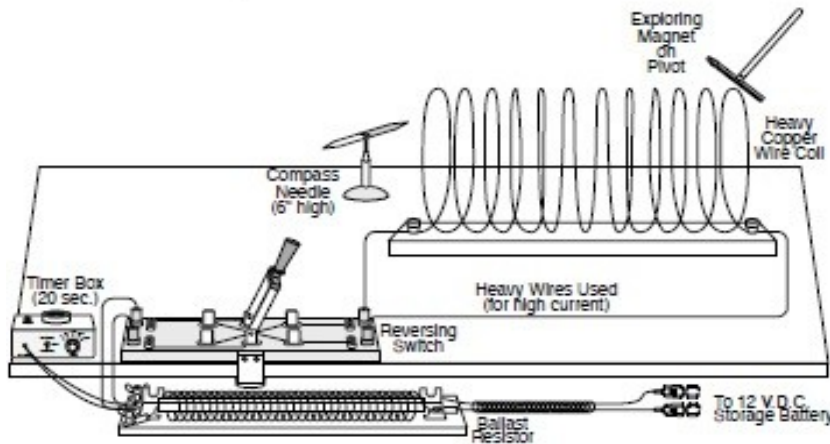
Case 3: $r > b$

In this case, for each inner turn passing current I through the loop in one direction, there is an outer turn passing current I through the loop in the other direction, so that the total current through the Amperian loop is zero, thus $B = 0$ everywhere outside the outer radius.

Teacher can guide the students to do the following projects.

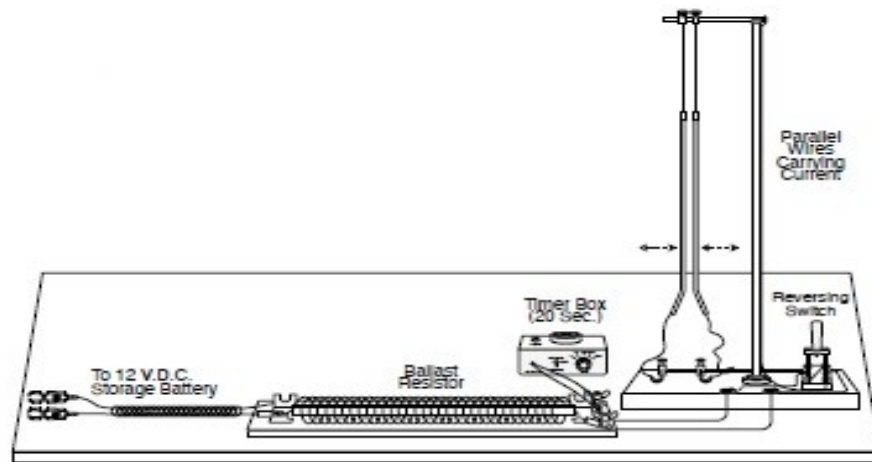
Project 1 - Magnetic field around a solenoid with pivoting probe magnet. A large D.C. current (about 100 Amps), is sent through a simple solenoid coil made of heavy copper wire. The magnetic field produced is explored by a hand-held 'exploring magnet', or with a compass needle on a stand.

NOTE: The current is large, so a Timer box is set for 20 seconds to remind one to turn the demo off. Also, double wires are used from the switch to the coil in order to handle the large current.



Project 2 – Ampere Law

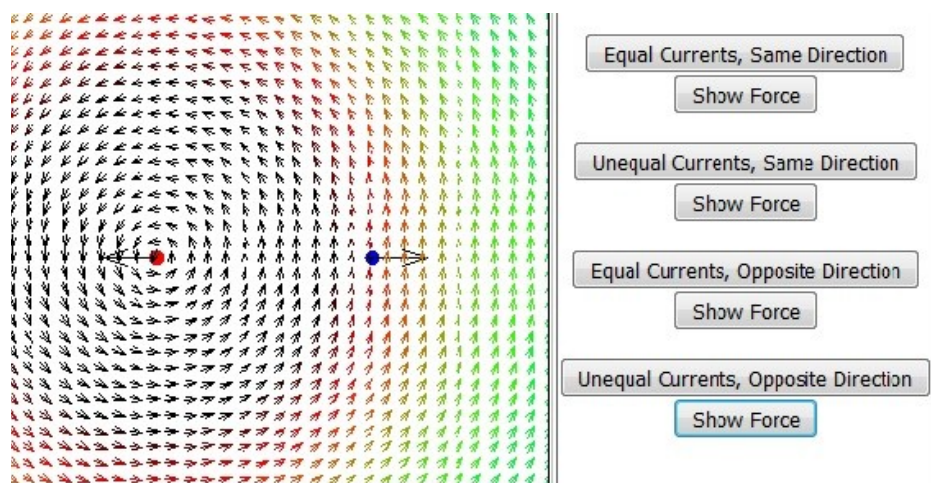
Approximately 120 amps of current is sent through two vertical parallel wires. When the reversing switch is thrown one way, the current in both wires is flowing the same direction, causing attractive magnetic fields that make the wires jump together. When the switch is thrown the other way, current in one wire flows in a direction opposite to that in the other wire, causing repulsive magnetic fields that make the wires jump apart. 12 V.D.C. (from a car storage battery) is connected through a .09 ohms series ballast resistor to limit the current to 120 amps. A Timer box, set for 20 seconds, is attached across the resistor. It beeps to warn the demonstration operator to turn off the power to the apparatus, to avoid melting of the wires.



Force Between Two Parallel Currents

Teacher can conduct a problem session using the animation given in the following link –

http://physics.bu.edu/~duffy/semester2/c14_forcewires.html



Two parallel current-carrying wires will exert forces on one another. Let's say wire 1 carries a current I_1 out of the page. Wire 2, a distance d to the right of wire 1, carries a current I_2 out of the page. Assume that both wires are long. Let's assume that I_1 is a lot larger than I_2 . Which wire exerts more force on the other?

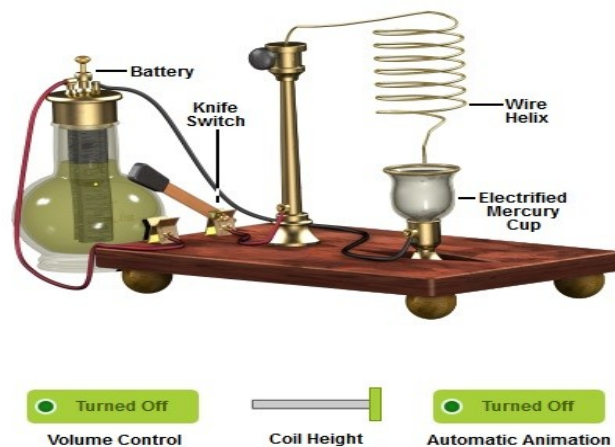
1. Wire 1 exerts more force on wire 2 than wire 2 exerts on wire 1
2. Wire 2 exerts more force on wire 1 than wire 1 exerts on wire 2
3. Wire 1 exerts the same magnitude force on wire 2 as wire 2 exerts on wire 1

Roget's Spiral For Attraction Between Parallel Currents

This topic can be handled using the animation provided in the following link –

<https://nationalmaglab.org/education/magnet-academy/watch-play/interactive/contracting-helix>

This nifty device, a kind of precursor to the Slinky, demonstrates how parallel wires attract. A current-carrying wire generates around it a circular magnetic field in a way described by the right hand rule. That is to say, if you had a wire pointing from the bottom to the top of this page, the magnetic field to the left of that wire would be coming out toward you, while on the right side of the wire that circling field would be heading into the page. As a result, when you put two such parallel wires together, with current traveling in the same direction, those wires will attract. At the point at which their respective magnetic fields intersect, they are traveling in opposite directions, and opposites attract. An English doctor and inventor named Peter Mark Roget (of thesaurus fame) devised a whimsical contraption in 1835 that exhibited this principle in action. Known both as a **contracting helix** and **Roget's spiral**, it is demonstrated in the tutorial below. Because it turns electricity into motion, the contracting helix is a type of primitive motor, akin to Faraday's motor.



A **Wire Helix** hangs from a metal frame so that its bottom tip dips into a metal-bottomed glass **Cup** containing mercury, a liquid which readily conducts current. When a **Battery** (in the form of a grenet cell in this tutorial) supplies current to the wire, the long series of parallel wires inherent in the form of the helix attract each other, causing the helix to contract. As it does, the end is lifted out of the mercury, breaking the circuit and discontinuing the current. As there is no longer current in them,

the parallel wires no longer attract each other, the spiral relaxes, its tip falls back into the mercury bath, and the whole cycle begins anew. The result is a constant contracting and relaxing of the coil, producing a Slinky-like effect. In addition, whenever the coil tip is lifted from the mercury, a **spark gap** is created, creating a brief spark. You can interrupt this cycle by disconnecting the circuit with the red **Turn Off** button to open the **Knife Switch** or by adjusting the **Coil Height** slider.

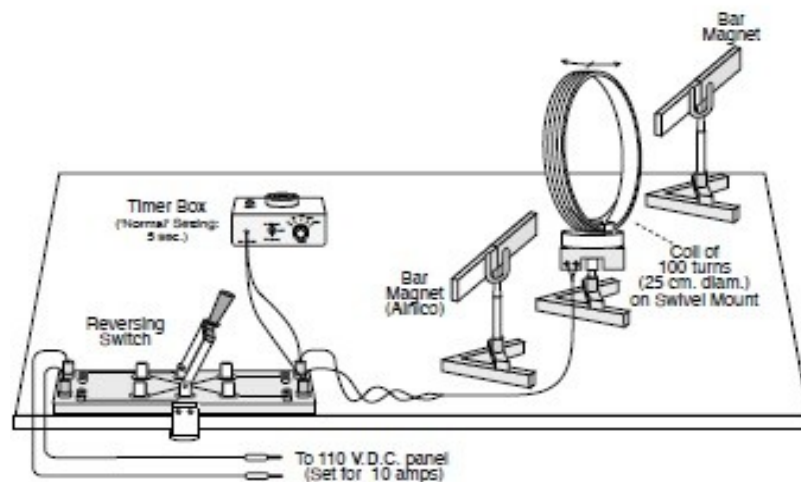
5.4 Torque on Coil Suspended Between Two Magnets

Learning Outcomes:

- Explain the torque on coil suspended between two magnets.

Activity: Group Discussion

This topic can be made more interesting for students by developing a project on it. Given as follows.



This is a simple galvanometer. Throwing the 'reversing switch' one way sends about 10 amps through a coil of wire mounted on a swivel base. The magnetic field generated by the coil is perpendicular to the plane of the coil. The north end of the coil field is attracted to the south end of the bar magnet field, and so the coil swings through an angle and stays there until the current is turned off. Throwing the switch in the opposite direction causes the coil to swing in the opposite direction. The Timer box reminds the demonstration operator to turn off the apparatus in 5 seconds to avoid melting the wires.

5.5 Electric Motor

Learning Outcomes:

- Explain the mechanism of Motor(AC and DC)

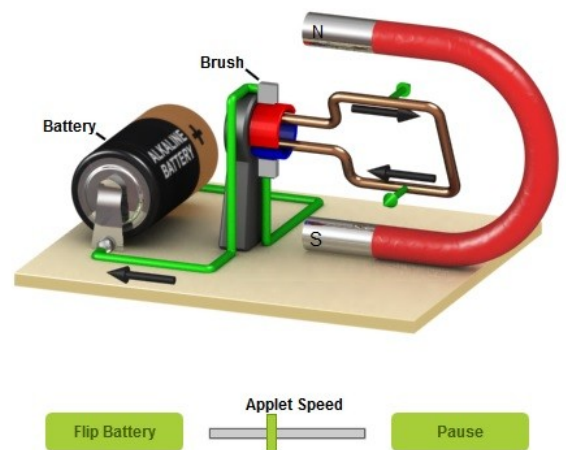
Activity: Group Discussion

This topic can be handled using the animation provided in the following link –

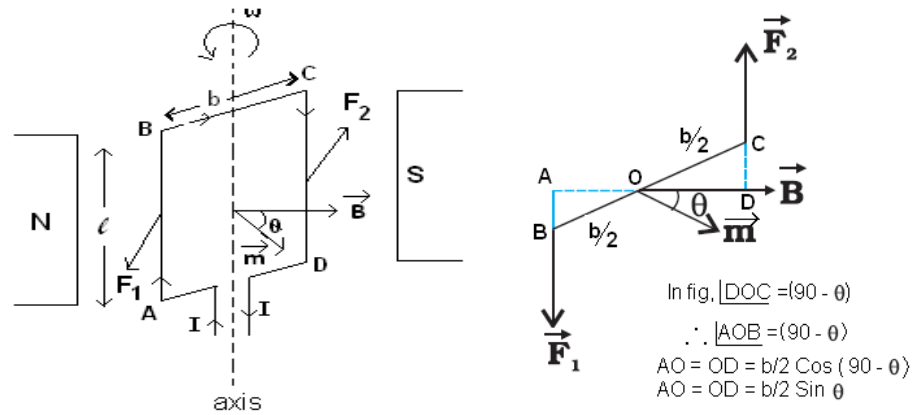
<https://nationalmaglab.org/education/magnet-academy/watch-play/interactive/dc-motor>

Electric motors turn electricity into motion by exploiting electromagnetic induction. The motor features a permanent horseshoe magnet (called the stator because it's fixed in place) and a turning coil of wire called an armature (or rotor, because it rotates). The armature, carrying current provided by the battery, is an electromagnet, because a current-carrying wire generates a magnetic field; invisible magnetic field lines are circulating all around the wire of the armature. The key to producing motion is positioning the electromagnet within the magnetic field of the permanent magnet (its field runs from its north to south poles). The armature experiences a force described by the left hand rule. This interplay of magnetic fields and moving charged particles (the electrons in the current) results in the torque (depicted by the green arrows) that makes the armature spin. Use the Flip Battery button to see what happens when the flow of current is reversed. Take advantage of the Applet Speed slider and Pause button to visualize these forces better.

A single, 180-degree turn is all you would get out of this motor if it weren't for the split-ring commutator - the circular metal device split into halves (shown here in red and blue) that connects the armature to the circuit. Electricity flows from the positive terminal of the battery through the circuit, passes through a copper brush to the commutator, then to the armature. But this flow is reversed midway through every full rotation, thanks to the two gaps in the commutator. This is a clever trick: For the first half of every rotation, current flows into the armature via the blue portion of the commutator, causing current to flow in a specific direction (indicated by the black arrows). For the second half of the rotation, though, electricity enters through the red half of the commutator, causing current to flow into and through the armature in the opposite direction. This constant reversal essentially turns the battery's DC power supply into alternating current, allowing the armature to experience torque in the right direction at the right time to keep it spinning.



Derivation



Consider a rectangular current loop ABCD of length l and breadth ' b ' placed in a uniform magnetic field B . Let I be the current flowing in the loop and θ be the angle between the B and normal to the plane of the loop along which magnetic moment (m) is present.

The force acting on side AB is

$$F_1 = BI l \quad (\because F = BI l \sin \theta)$$

The direction of this force is perpendicular to the plane of the paper, directed inwards.

The force acting on side CD is

$$F_2 = BI l$$

The direction of this force is also perpendicular to the plane of the paper, directed outwards. The forces F_1 and F_2 constitute the couple. The torque

on the loop is $\zeta =$ Torque due to force F_1 about 'O' + Torque due to F_2 about 'O'

$$\zeta = F_1 \times \perp^{\text{lar}} \text{ distance AO} + F_2 \times \text{OD}$$

$$\zeta = BI l \times AO + F_2 \times OD$$

$$\tau = BI l \frac{b}{2} \sin \theta + BI l \frac{b}{2} \sin \theta$$

$$\tau = \cancel{2} BI l \frac{b}{2} \sin \theta$$

$$\tau = BI l b \sin \theta$$

$$\tau = B I A \sin \theta$$

Where $l \times b = A$ is the area of the loop.

For ' N ' number of turns

$$\tau = N B I A \sin \theta$$

Also, $NIA = m =$ magnetic moment. Hence,

$$\tau = \mathbf{m}B \sin \theta$$

In vector form,

$$\vec{\tau} = \vec{\mathbf{m}} \times \vec{\mathbf{B}}$$

Note:

1. If $\theta = 0^\circ$, then $\zeta = 0$ and if $\theta = 90^\circ$, then $\zeta = NIAB = \text{maximum}$.
2. The torque on a current loop in magnetic field is analogous to the electrostatic case. i.e. $\zeta = P_e \times E$
3. The SI unit of magnetic moment is 'Am²'

5.6 Moving coil Galvanometer:

Learning Outcomes:

- Explain the mechanism of Moving coil Galvanometer
- Explain the mechanism of Conversion of galvanometer into Ammeter.
- Explain the mechanism of Conversion of galvanometer into voltmeter

Activity: Group Discussion

Galvanometer is a device is used to detect or to check the current flowing in a circuit. The galvanometer consists of a coil of many turns wound on a cylindrical soft iron core. The coil is free to rotate about a fixed axis in a uniform radial magnetic field produced by concave magnetic pole pieces.

When current flows through a coil, a torque acts on the coil. The torque is given by

$$\tau = NBI A \sin \theta$$

Where the symbols have their usual meaning. The value of $\theta = 90^\circ$ for radial field.

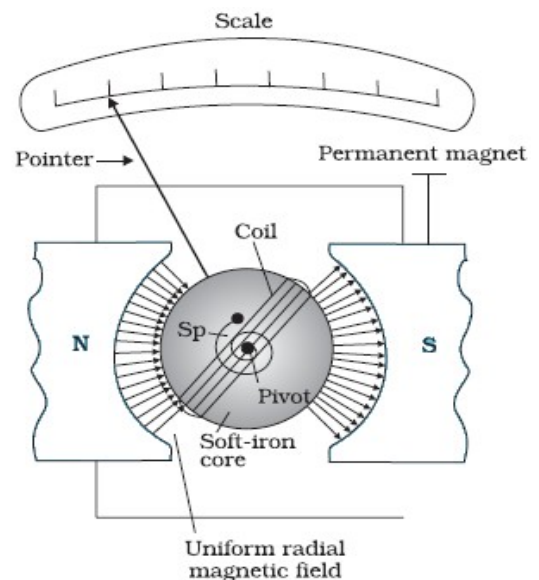
The magnetic torque $NIAB$ tends to rotate the coil through an angle of deflection ' Φ '. The spring 'S' provides a counter torque which is proportional to ' Φ '

$$\begin{aligned} \text{i.e. } \zeta_s &\propto \Phi \\ \zeta_s &= k \Phi \end{aligned}$$

Where 'K' is a torsional constant of the spring.

i.e. the restoring torque per unit twist.

$$\text{In equilibrium, } NIAB = k \Phi$$



$$\Phi = \left(\frac{NAB}{k} \right) I$$

The deflection ' Φ ' can be recorded on the zero centered scale by a pointer attached to the spring. The quantity in bracket is a constant for a given galvanometer.

Current sensitivity of the Galvanometer:

The deflection produced per unit current is called current sensitivity of the galvanometer.

Current sensitivity is given by

$$\frac{\Phi}{I} = \frac{NAB}{k}$$

Galvanometer is said to be more sensitive if the deflection produced is more per unit current. By increasing the number of turns and decreasing the value of ' K ', the sensitivity can be increased.

Drawbacks of MCG:

1. MCG is a very sensitive device, it gives a full – scale deflection for a current of the order of μA . So it cannot be used to measure current directly.
2. The MCG has to be connected in series in the circuit to measure the current. The resistance of the coil of the MCG is large. It may alters the current flowing in the circuit.

Conversion of Galvanometer into an Ammeter

Ammeter is a device used to measure the current in a circuit. It should be connected in series in a circuit. Its introduction should not alter the current in the circuit. Hence it must have a very low resistance. The resistance of an ideal ammeter is zero.

A galvanometer is converted into an ammeter by connecting a very small resistance in parallel with it.

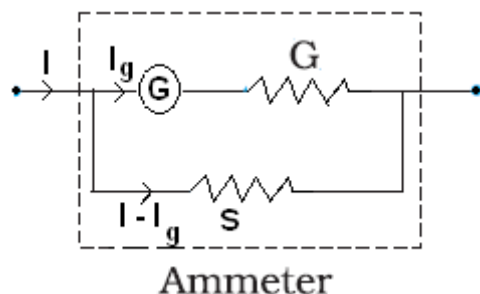
A small resistance connected in series with the galvanometer is called shunt.

Let ' S ' is shunt resistance and ' G ' be the resistance of the galvanometer. Let ' I ' be the main current and I_g be the current through ' G '.

The effect resistance of this arrangement is (For parallel connection)

$$R_{\text{effective}} = \frac{G S}{G + S}$$

Also, P.d across ' G ' = P.d across ' S '



$$I_g G = (I - I_g) S \quad (\because V = IR)$$

$$S = \frac{I_g G}{(I - I_g)}$$

This is the expression for shunt resistance. Also,

$$I_g = \frac{IS}{G + S}$$

Thus, $I_g \propto I$, Therefore, the scale can be graduated to give the main current directly.

Conversion of galvanometer into a voltmeter (MCG as voltmeter):

Voltmeter is a device used to measure the potential difference across two ends of a circuit element.

It should be connected in parallel to the circuit element. Its introduction should not alter the potential difference to be measured. i.e. the voltmeter should not draw current. Hence it must have a very high resistance.

The resistance of an ideal voltmeter is ' ∞ ' (infinity).

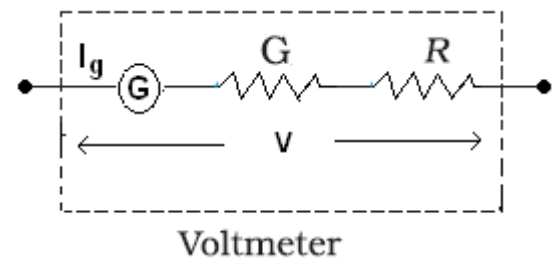
A galvanometer can be converted into voltmeter by connecting a high resistance in series with the coil of the galvanometer.

Let ' R ' is the high resistance is connected in series with the galvanometer resistance ' G ' and ' V ' is the potential difference to be measured. If I_g be the current required to produce full scale deflection, then

$$V = I_g(G + R) \longrightarrow (1)$$

$$G + R = \frac{V}{I_g}$$

$$R = \frac{V}{I_g} - G$$



This is the expression for high resistance to be connected in series. Also, $I_g \propto V$. \therefore The scale can be graduated to read potential difference directly.

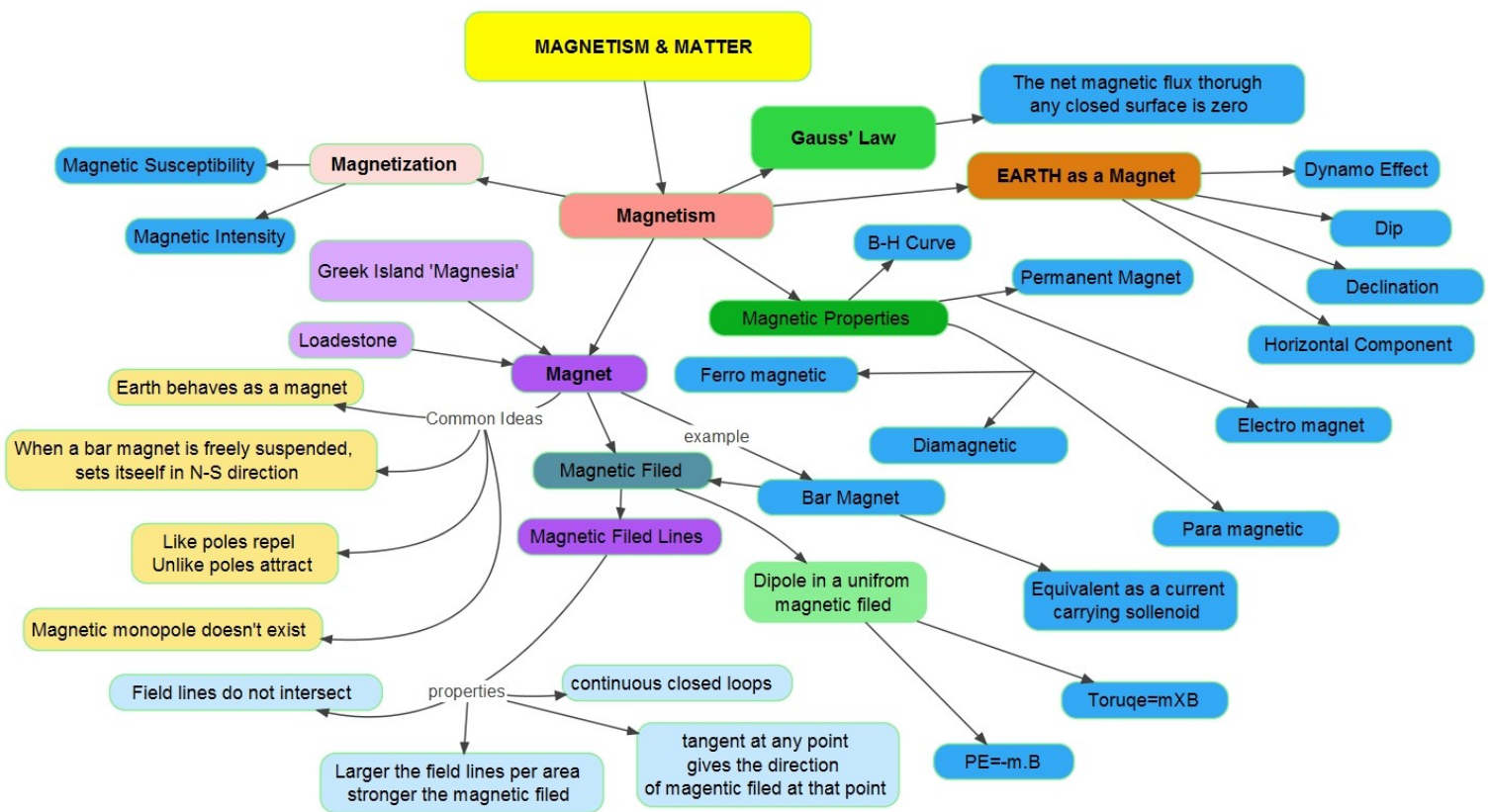
Note:

The deflection produced per unit voltage is called voltage sensitivity of the galvanometer. It is given by

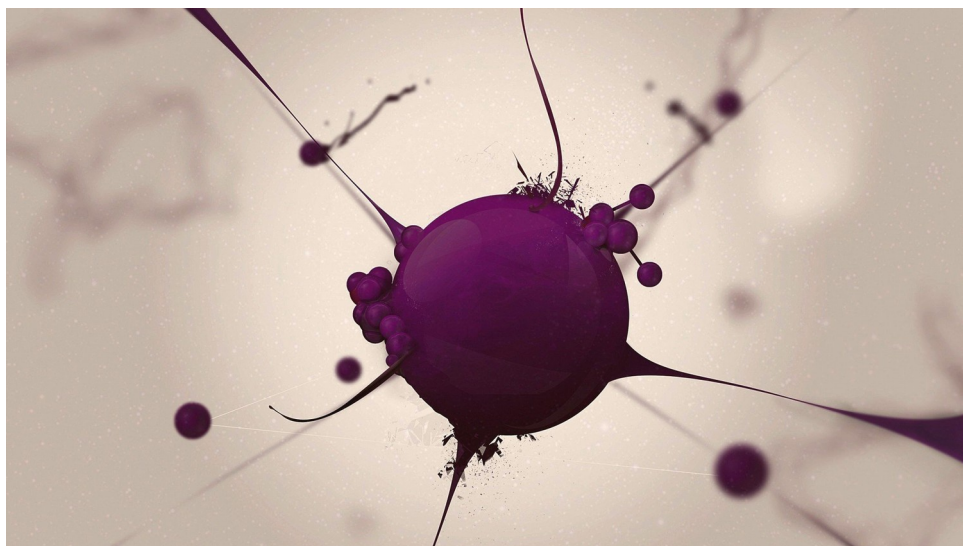
$$\frac{\Phi}{V} = \left(\frac{NAB}{k} \right) \frac{1}{R}$$

Distinguish between Ammeter and Voltmeter:

	Ammeter		Voltmeter
1	It is used to measure current	1	It is used to measure voltage
2	It is always connected in series with the circuit.	2	It is always connected parallel in the circuit.
3	Resistance of ammeter is very low	3	Resistance of voltmeter is very high
4	Resistance of an ideal ammeter is zero.	4	Resistance of an ideal voltmeter is infinity
5	Galvanometer is converted in to ammeter by connecting a very low resistance in parallel to it.	5	Galvanometer is connected into voltmeter by connecting a very high resistance in series with it.
6	Shunt resistance is	6	High resistance is



ATOMS



Atoms on a whole are electrically neutral. Therefore, an atom must also contain some positive charge to neutralise the negative charge of the electrons. But what is the arrangement of the positive charge and the electrons inside the atom? In other words, what is the structure of an atom?

An atom is the smallest particle of an element that has the properties characterizing that element. It is a complex arrangement of negatively charged electrons arranged in some shells about a positively charged nucleus. This nucleus contains most of the atom's mass and is composed of protons and neutrons (except for common hydrogen which has only one proton). Atoms are the basic building blocks of matter that make up everyday objects. A desk, the air, even you are made up of atoms!

6.1 Earlier atomic theories

Activity: General Discussion

With the help of a Power point presentation, teacher introduces different Atomic theories before Rutherford.

1. "Earth, air, fire and water" vs "Atomos"

The first theories of matter were put forward by Empedocles in 450 BC, he proposed that all matter was composed of four elements - Earth, air, fire and water.

Later, Leucippus and Democritus suggested matter was made up of tiny indestructible particles continuously moving in empty space. They called them "atomos", which is where we get the modern word "atom".

2. Dalton's atomic theory

John Dalton, an English schoolmaster, proposed the first modern atomic theory.

- He suggested that all elements are composed of tiny, indestructible particles, he called atoms. All matter was made up of small, indivisible particles called atoms.
- Atoms of a given element possess unique properties and weight.
- Three types of atoms exist: simple (elements); compound (simple molecules) and complex (complex molecules).

3. Thompson's atomic model

In 1897, J.J. Thompson discovered the electron, Thompson held that atoms were spheres of positively charged matter in which electrons were embedded, known as the plum-pudding model.

See Appendix 1: Models of Atom: a Historic Perspective

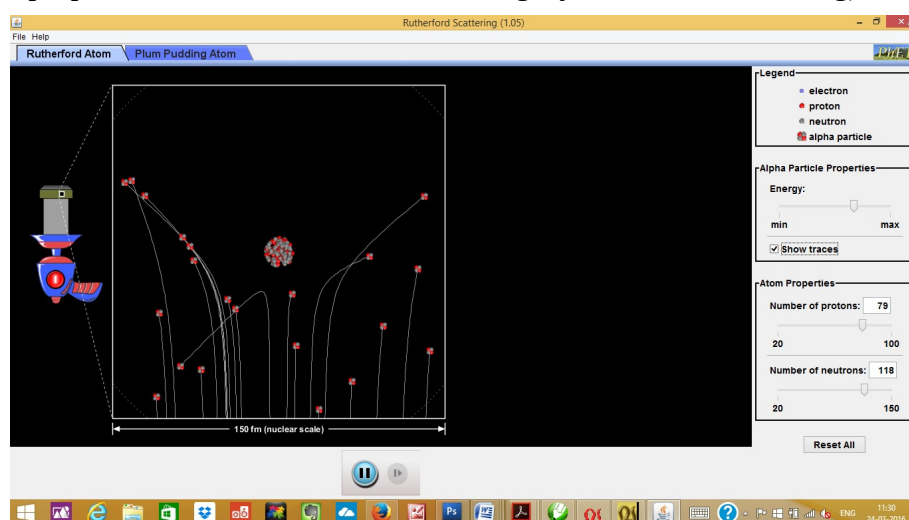
6.2 Alpha particle scattering Experiment and Rutherford's Nuclear model of Atom

- Alpha particle scattering
- Concept of atomic nucleus

Activity: **General discussion, ICT – Animated Alpha particle scattering experiment**

Teacher plays the PhET animation of Rutherford Experiment

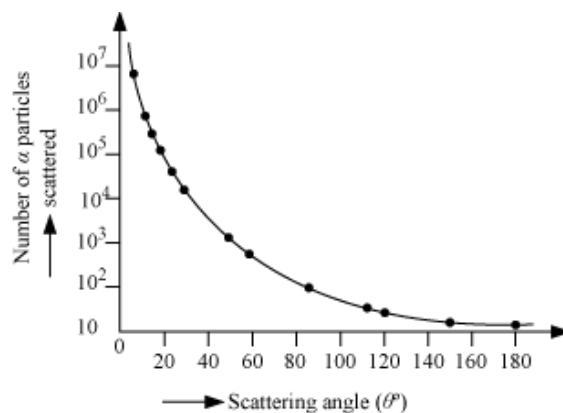
(<http://phet.colorado.edu/en/simulation/legacy/rutherford-scattering>)



What do you observe here?

1. Most of the alpha particles pass straight through the gold foil.
2. Only about 0.14% of incident α -particles scatter by more than 1° .
3. About one α -particle in every 8000 α -particles deflects by more than 90° .

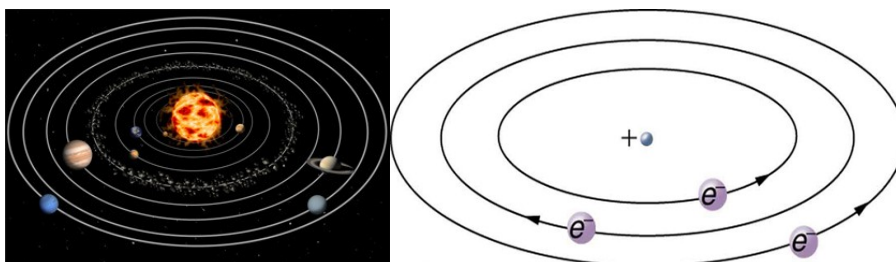
Observations – A graph is plotted between the scattering angle θ and the number of α -particles $N(\theta)$, scattered at $\angle\theta$ for a very large number of α -particles.



What conclusions can be arrived using the experiment.

1. Atom is made up of a large amount of empty space
1. Almost the entire mass of an atom and the whole positive charges are concentrated in a small region of space called nucleus.
2. The electrons would be moving in orbitals about the nucleus just as the planets do around the sun.

Teacher asks the students to compare between Planetary orbits and Orbits of electrons



Alpha particle trajectory

Activity: General Discussion, Simple Experiment

Teacher asks the students about the force comes in to play between alpha particle and the target nucleus.

The trajectory of an alpha particle can be computed employing Newton's second law of motion and Coulomb's law for electrostatic force of repulsion between the alpha particle and the positively charged nucleus.

What is the magnitude of the force?

[Hint:

$$F = \frac{1}{4\pi\epsilon_0} \frac{(2e)(Ze)}{r^2}$$

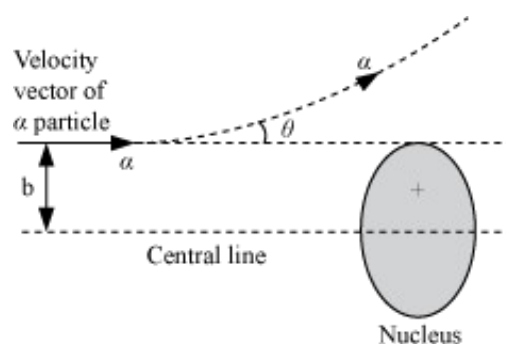
Where,

Ze – Charge of gold nucleus

$2e$ – Charge on alpha particle

r – Distance between α -particle and the nucleus

Teacher introduces the concept impact parameter (b) The impact parameter is the perpendicular distance of the initial velocity vector of the α -particle from the centre of the nucleus.



How does the impact parameter change with the force?

[Hints: For large impact parameters, force experienced by the alpha

particle is weak because $F \propto \frac{1}{(\text{Distance})^2}$

Hence, the alpha particle will deviate through a much smaller angle. When impact parameter is small, force experienced is large and hence, the alpha particle will scatter through a large angle.]

The Chinese Hat

This activity is a physical analogue of the actual alpha particle experiment done by Rutherford. From this activity, students will get an idea of the trajectory of the alpha particles and how is it affected in the presence of external potential.

The 'Chinese hat' analogue provides a practical way for students to get a feel for the physics of alpha-scattering. Roll a marble past the hat to see it deflected. You can change the speed and impact parameter. Students can change these parameters systematically and observe the effects.



The hat is designed so that, as the slope of the 'hat' gets steeper, the component of gravity parallel to the slope opposing the motion of the ball also gets larger. The actual shape is such that at any position on the slope a distance r from the centre of the 'hill', the component of a particle's weight parallel to the slope $\sim 1/r^2$. (Revision of the relationship between $1/r$ potential and $1/r^2$ force can be done here if desired.)

If possible, it's worth getting several Chinese hats so that students can work with them in small groups. (The 'hat' can also be used when studying gravity. Turn it upside down to become a 'potential well' so that you can demonstrate orbits, and discuss the difference between bound and unbound 'particles'.)

Electron Orbits

Activity: General discussion on Alpha particle trajectory, Electron orbits

What force is required for electron to revolve around the nucleus?

[Hint: Centripetal Force]

Which agency provides this force?

[Hint: The electrostatic force of attraction between electron and nucleus provides the necessary centripetal force]

How do you obtain a relation for orbital radius?

[Hints:

F_c – Centripetal force required to keep a revolving electron in orbit

F_e – Electrostatic force of attraction between the revolving electron and the nucleus

Then, for a dynamically stable orbit in a hydrogen atom,

$$F_c = F_e$$

$$\frac{mv^2}{r} = \frac{(e)(e)}{4\pi\epsilon_0 r^2} \quad \dots(i)$$

$$r = \frac{e^2}{4\pi\epsilon_0 r^2}$$

$$r = \frac{e^2}{4\pi\epsilon_0 mv^2} \quad \dots(ii)$$

Teacher asks the students to develop the expressions for Kinetic Energy, Potential Energy, Total Energy of an electron

1. KE of electron in the orbit

$$K = \frac{1}{2}mv^2$$

From equation (i),

$$K = \frac{e^2}{8\pi\epsilon_0 r}$$

2. PE of electron in the orbit

$$U = \frac{(e)(-e)}{4\pi\epsilon_0 r} = \frac{-e^2}{4\pi\epsilon_0 r}$$

Teacher explains that the Negative sign indicates that revolving electron is bound to the positive nucleus.

3. Total Energy of electron in the orbit

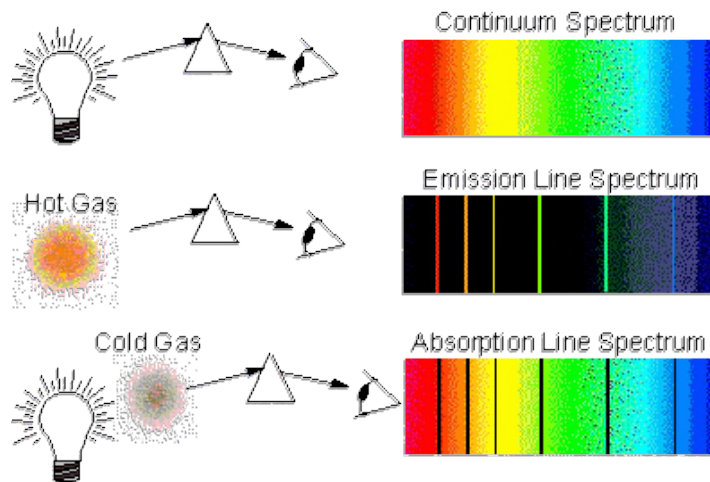
$$E = k + U = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r}$$

$$E = -\frac{e^2}{8\pi\epsilon_0 r}$$

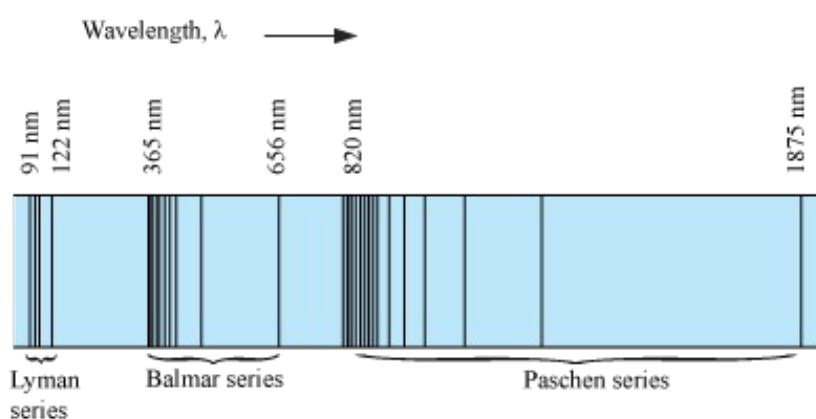
6.3 Atomic spectra

Activity: General discussion on Atomic spectra

Teacher explains Emission and absorption spectra

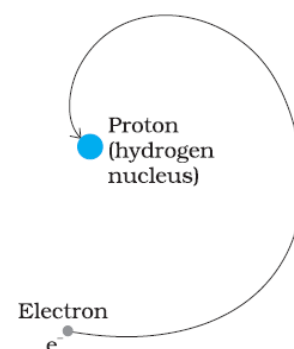


- Each element emits a characteristic spectrum of radiation.
- In the excited state, the atoms emit radiations of a spectrum, which contains certain specific wavelengths only. This spectrum is termed as emission line spectrum and it consists of bright lines on a dark background.
- The spectrum emitted by atomic hydrogen is shown in the figure below.



Failure of Rutherford's Atom Model

According to classical electromagnetic theory, an accelerating charged particle emits radiation in the form of electromagnetic waves. The energy of an accelerating electron should therefore, continuously decrease. The electron would spiral inward and eventually fall into the nucleus



6.4 The Bohr Model of Hydrogen atom

Activity: General discussion, Group Discussion

Bohr suggested a new model for the atom as Rutherford's atom model was unstable. He introduced the concept of stationary orbits.

Postulates of Bohr's Atom Model

1. An electron in an atom could revolve in certain stable orbits without the emission of radiant energy,
2. The electrons revolve around the nucleus only in those orbits for which the angular momentum is the integral multiple of

$$\frac{h}{2\pi}$$

$$L = \frac{nh}{2\pi}$$

3. Electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy difference between the initial and final state.

$$h\nu = E_i - E_f$$

$$E_i > E_f$$

Energy levels

Energy levels (also called electron shells) are fixed distances from the nucleus of an atom where electrons may be found. Electrons are tiny, negatively charged particles in an atom that move around the positive nucleus at the center. Energy levels are a little like the steps of a staircase. You can stand on one step or another but not in between the steps. The same goes for electrons. They can occupy one energy level or another but not the space between energy levels. They can jump from a lower to the next higher energy level if they absorb this amount of energy. Conversely, if electrons jump from a higher to a lower energy level, they give off energy, often in the form of light.

A hydrogen atom consists of an electron orbiting its nucleus. The electromagnetic force between the electron and the nuclear proton leads to a set of quantum states for the electron, each with its own energy. These states were visualized by the Bohr model of the hydrogen atom as being distinct orbits around the nucleus. Each energy state, or orbit, is designated by an integer, n . Spectral emission occurs when an electron transitions, or jumps, from a higher energy state to a lower energy state. The energy of an emitted photon corresponds to the energy difference between the two states. Because the energy of each state is fixed, the energy difference between them is fixed, and the transition will always produce a photon with the same energy.

4. **Angular momentum** is given by,

$$L = mvr$$

According to Bohr's 2nd postulate,

$$L_n = mv_n r_n = \frac{nh}{2\pi}$$

n → Principal quantum number

v_n → Speed of moving electron in the n^{th} orbit

r_n → Radius of n^{th} orbit

$$v_n = \frac{e}{\sqrt{4\pi\epsilon_0 m r_n}}$$



$$\therefore v_n = \frac{1}{n} \frac{e^2}{4\pi\epsilon_0} \frac{1}{\left(\frac{h}{2\pi}\right)}$$

$$\therefore r_n = \left(\frac{n^2}{m}\right) \left(\frac{h}{2\pi}\right)^2 \frac{4\pi\epsilon_0}{e^2}$$

For $n = 1$ (innermost orbit),

$$r_1 = \frac{h^2 \epsilon_0}{\pi m e^2}$$

This is called Bohr radius, represented by the symbol a_0 .

$$a_0 = \frac{h^2 \epsilon_0}{\pi m e^2}$$

Substitution of values of h , m , ϵ_0 and e gives $a_0 = 5.29 \times 10^{-11}$ m.

5. Total energy

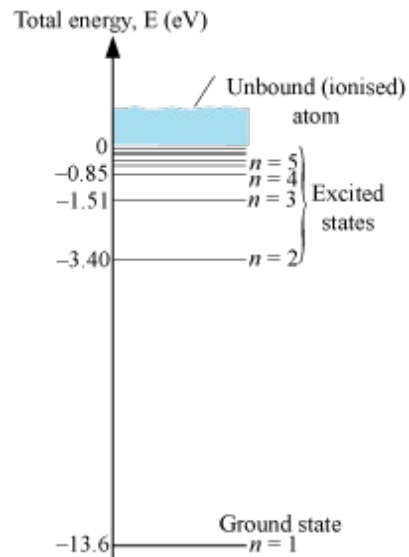
$$E_n = -\left(\frac{e^2}{8\pi\epsilon_0}\right) \left(\frac{m}{n^2}\right) \left(\frac{2\pi}{h}\right)^2 \left(\frac{e^2}{4\pi\epsilon_0}\right)$$

$$\text{or } E_n = -\frac{me^4}{8n^2\epsilon_0^2h^2}$$

$$E_n = -\frac{2.18 \times 10^{-18}}{n^2} \text{ J}$$

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

6. Energy level diagram for a hydrogen atom:



6.5 Line Spectra of Hydrogen Atom

Activity: General Discussion

Teacher starts the discussion by brushing up the previous knowledge about Bohr's model of Hydrogen atom. She leads a General discussion on Line spectra of H atom.

How do you express Bohr's third postulate mathematically?

[Hint: $h\nu_{if} = E_{ni} - E_{nf}$]

What are the expressions for E_{n_i} and E_{n_f} ?

[Hint: $E_n = - \frac{m e^4}{8 n^2 \epsilon_0^2 h^2}$]

Well, then how do you find the frequency ν_{if} ?

[Hint: $\nu_{if} = \frac{m e^4}{8 \epsilon_0^2 h^3} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$]

Teacher explains that this formula is known as Rydberg formula, and the constant $\frac{m e^4}{8 \epsilon_0^2 h^3}$ is called Rydberg constant.

Can you find the numerical value of Rydberg constant?

[Hint: $1.03 \times 10^7 \text{ m}^{-1}$, very close to the value obtained from empirical Balmer formula]

Now, we are going to draw the Energy level diagram and diagram showing Line spectra.

Hope you remember the value of E_n in eV.

[Hint: $-13.6/n^2 \text{ eV}$]

Teacher draws the table given below on the board. The students are asked to complete the table.

Energy Level (n)	E_n
1	-13.6 eV
2	
3	
4	
5	
6	
7	
8	
9	
10	

Now, you may draw the Energy level diagram using the above data in your activity log book.

[Hint: Fig 12.8, NCERT Text]

Mark all possible transitions and make them different groups according to the lower level.

[Hint: Fig 12.9, NCERT Text]

Teacher explains the names given to each spectral series.

Lyman series ($n_f=1$), Balmer series ($n_f=2$), Paschen series ($n_f=3$), Brackett series ($n_f=4$) and Pfund series ($n_f=5$)

The spectral lines are grouped into series according to n_1 (the lower energy state). Lines are named sequentially starting from the longest wavelength/lowest frequency of the series.

Rydberg formula for hydrogen

$$\frac{1}{\lambda_{\text{vac}}} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Where

λ_{vac} is the wavelength of electromagnetic radiation emitted in vacuum,
 R is the Rydberg constant, (approximately $1.097 \times 10^7 \text{ m}^{-1}$)
 n_1 and n_2 are integers greater than or equal to 1 such that $n_1 < n_2$,
 corresponding to the principal quantum numbers of the orbitals
 occupied before and after the 'quantum jump'.

n_i	n_f	Series name	Wavelength
1	$2 \rightarrow \infty$	Lyman series	91.13 nm (UV)
2	$3 \rightarrow \infty$	Balmer series	364.51 nm (Visible)
3	$4 \rightarrow \infty$	Paschen series	820.14 nm (IR)
4	$5 \rightarrow \infty$	Brackett series	1458.03 nm (Far IR)
5	$6 \rightarrow \infty$	Pfund series	2278.17 nm (Far IR)
6	$7 \rightarrow \infty$	Humphreys series	3280.56 nm (Far IR)

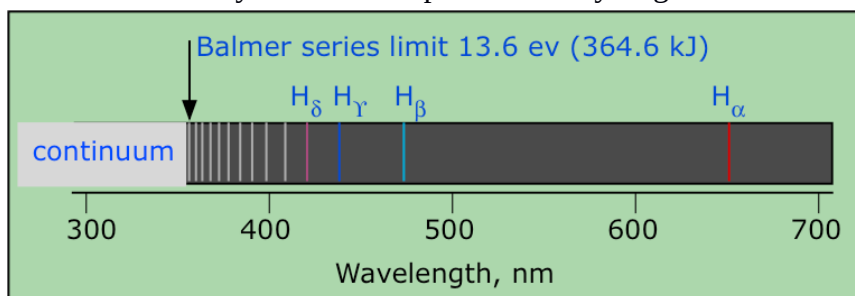
For example, the $2 \rightarrow 1$ line is called "Lyman-alpha" ($\text{Ly-}\alpha$), while the $7 \rightarrow 3$ line is called "Paschen-delta" ($\text{Pa-}\delta$).

Lyman series ($n_1 = 1$ and $n_2 = 2 \rightarrow \infty$)

The series is named after its discoverer, Theodore Lyman, who discovered the spectral lines from 1906–1914. All the wavelengths in the Lyman series are in the ultraviolet band.

Balmer series ($n_1 = 2$ and $n_2 = 3 \rightarrow \infty$)

Named after Johann Balmer, who discovered the spectral lines in the year 1885. Balmer lines are historically referred to as "H-alpha", "H-beta", "H-gamma" and so on, where H is the element hydrogen. Four of the Balmer lines are in the technically "visible" part of the spectrum, with wavelengths longer than 400 nm and shorter than 700 nm. Parts of the Balmer series can be seen in the solar spectrum. H-alpha is an important line used in astronomy to detect the presence of hydrogen.



Paschen series (Bohr series $n_1 = 3$ and $n_2 = 4 \rightarrow \infty$)

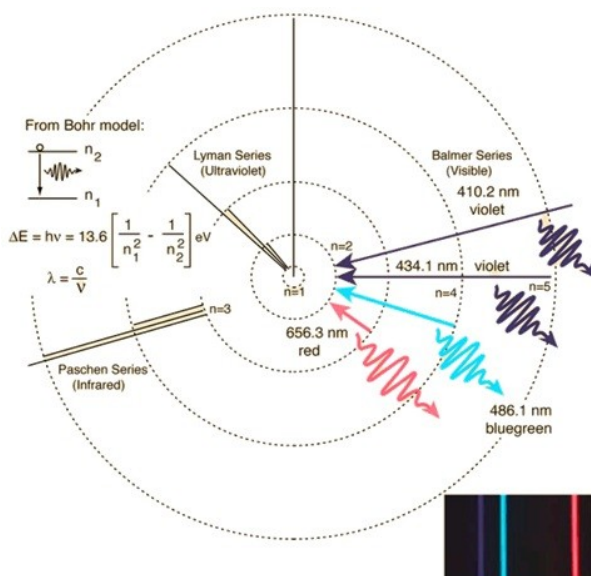
Named after the German physicist Friedrich Paschen who first observed them in 1908. The Paschen lines all lie in the infrared band. This series overlaps with the next (Brackett) series, i.e. the shortest line in the Brackett series has a wavelength that falls among the Paschen series.

Brackett series ($n_1 = 4$ and $n_2 = 5 \rightarrow \infty$)

Named after the American physicist Frederick Sumner Brackett who first observed the spectral lines in 1922.

Pfund series ($n_1 = 5$ and $n_2 = 6 \rightarrow \infty$)

Experimentally discovered in 1924 by August Herman Pfund.



Teacher consolidates the discussion by reminding Energy level diagram and different Spectral series. The students are asked to complete the worksheet No.3

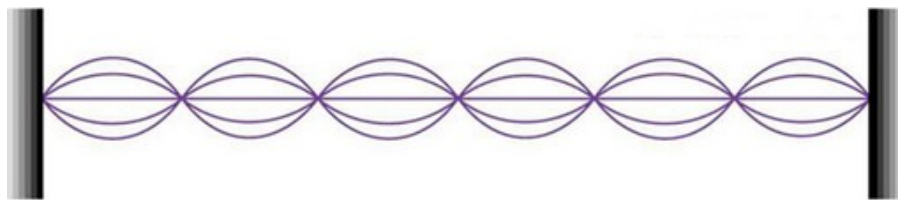
6.6 De Broglie explanation of Bohr's Second postulate of Quantization

Activity: General Discussion

- Among the Bohr's postulates the most puzzling is his second postulate. It states that the angular momentum of the electron orbiting around the nucleus is quantized.
- De-Broglie's hypothesis that electron has a wavelength $\lambda = h/mv$ gave an explanation for Bohr's quantised orbits by bringing in the wave particle duality.
- Louis de Broglie argued that the electron in its circular orbit, as proposed by Bohr, must be seen as a particle wave. In analogy to waves travelling on a string. ($L_n = nh/2\pi$; $n = 1, 2, 3 \dots$). Then the question arises- Why should the angular momentum have only those values that are integral multiples of $h/2\pi$?
- The French physicist Louis de Broglie explained this puzzle in 1923, ten years after Bohr proposed his model. Louis de Broglie

argued that the electron in its circular orbit, as proposed by Bohr, must be seen as a particle wave. In analogy to waves travelling on a string, particle waves too can lead to standing waves under resonant conditions.

- we know that when a string is plucked, a vast number of wavelengths are excited. However only those wavelengths survive which have nodes at the ends and form the standing wave in the string. It means that in a string, standing waves are formed when the total distance travelled by a wave down the string and back is one wavelength, two wavelengths, or any integral number of wavelengths. Waves with other wavelengths interfere with themselves upon reflection and their amplitudes quickly drop to zero.

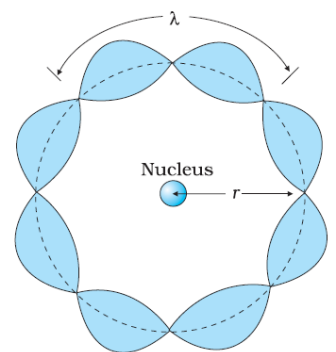


- For an electron moving in n th circular orbit of radius r_n , the total distance is the circumference of the orbit, $2\pi r_n$. Thus $2\pi r_n = n\lambda$, $n = 1, 2, 3\dots$. Bohr's model, involving classical trajectory picture (planet-like electron orbiting the nucleus), correctly predicts the gross features of the hydrogenic atoms, in particular, the frequencies of the radiation emitted or selectively absorbed.
- Orbits correspond to circular standing waves in which the circumference of the orbits equal whole number of wavelength.

$$2\pi r_n = n\lambda$$

$$\lambda = h/mv$$

$$2\pi r_n = n h/mv_n \text{ or } m v_n r_n = nh/2\pi$$






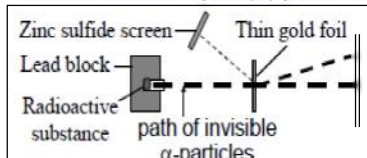

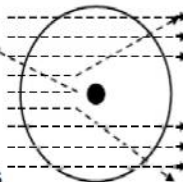
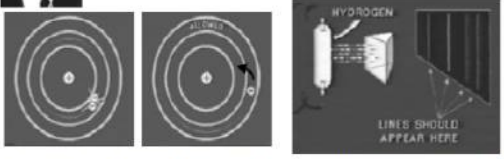


Limitations of Bohr Atom Model

- The Bohr model is applicable to hydrogenic atoms. It cannot be extended even to mere two electron atoms such as helium. Difficulty lies in the fact that each electron interacts not only with the positively charged nucleus but also with all other electrons.
- While the Bohr's model correctly predicts the frequencies of the light emitted by hydrogenic atoms, the model is unable to explain the relative intensities of the frequencies in the spectrum. Experimental observations depict that some transitions are more favoured than others.

Appendix 1: Models of Atom: a Historic Perspective

Models of the Atom: a Historical Perspective

<p>Early Greek Theories</p>  <p>Democritus</p> <ul style="list-style-type: none"> • 400 B.C. - Democritus thought matter could not be divided indefinitely. • This led to the idea of atoms in a void.   <p>Aristotle</p> <ul style="list-style-type: none"> • 350 B.C - Aristotle modified an earlier theory that matter was made of four "elements": earth, fire, water, air. • Aristotle was wrong. However, his theory persisted for 2000 years. 	<p>John Dalton</p> <ul style="list-style-type: none"> • 1800 -Dalton proposed a modern atomic model based on experimentation not on pure reason.  <ul style="list-style-type: none"> • All matter is made of atoms. • Atoms of an element are identical. • Each element has different atoms. • Atoms of different elements combine in constant ratios to form compounds. • Atoms are rearranged in reactions. <ul style="list-style-type: none"> • His ideas account for the law of conservation of mass (atoms are neither created nor destroyed) and the law of constant composition (elements combine in fixed ratios).
<p>Adding Electrons to the Model</p> <p>Materials, when rubbed, can develop a charge difference. This electricity is called "cathode rays" when passed through an evacuated tube (demos). These rays have a small mass and are negative. Thompson noted that these <u>negative</u> subatomic particles were a fundamental part of <u>all</u> atoms.</p> <ol style="list-style-type: none"> 1) <u>Dalton's "Billiard ball" model</u> (1800-1900) Atoms are solid and indivisible. 2) <u>Thompson "Plum pudding" model</u> (1900) Negative electrons in a <u>positive</u> framework. 3) <u>The Rutherford model</u> (around 1910) Atoms are mostly empty space. Negative electrons orbit a positive nucleus. 	<p>Ernest Rutherford (movie: 10 min.)</p> <ul style="list-style-type: none"> • Rutherford shot alpha (α) particles at gold foil.   <p>Most particles passed through. So, atoms are mostly empty.</p> <p>Some positive α-particles deflected or bounced back!</p> <p>Thus, a "nucleus" is positive & holds most of an atom's mass.</p> 
<p>Bohr's model</p> <ul style="list-style-type: none"> • Electrons orbit the nucleus in "shells" • Electrons can be bumped up to a higher shell if hit by an electron or a photon of light.  <p>There are 2 types of spectra: continuous spectra & line spectra. It's when electrons fall <u>back down</u> that they release a photon. These jumps down from "shell" to "shell" account for the line spectra seen in gas discharge tubes (through spectroscopes).</p>	<p>Atomic numbers, Mass numbers</p> <ul style="list-style-type: none"> • There are 3 types of subatomic particles. We already know about electrons (e^-) & protons (p^+). Neutrons (n^0) were also shown to exist (1930s). • They have: no charge, a mass similar to protons • Elements are often symbolized with their mass number and atomic number E.g. Oxygen: ${}^{16}_8\text{O}$ • These values are given on the periodic table. • For now, round the mass # to a whole number. • These numbers tell you a lot about atoms. # of protons = # of electrons = atomic number # of neutrons = mass number – atomic number • Calculate # of e^-, n^0, p^+ for Ca, Ar, and Br.

Worksheets

Worksheet 1

Vocabulary Cross word

Left to Right

1. Positively charged parts of an atom
6. Negatively charged parts of an atom
7. Atoms are the building blocks for...
8. The number of electrons in atoms determines an element's ____ properties
9. Neutral particles of an atom
10. A chart which lists all of the known elements

[Protons, electrons, molecules, chemical, neutrons, periodic table]

Top to Bottom

2. Protons and neutrons are found in this part of an atom
3. Force that repels the protons of an atom
4. Area of science that studies tiny particles like atoms
5. The word *atomos* comes from this language

[Nucleus, electromagnetic, quantum mechanics, Greek]

Worksheet 2

Complete the Table

[Hint: Orbital radius of electron is $n^2 a_0$]

n	r	v	E_n	Standing Wave
1	$r_1 = a_0 = 0.529 \times 10^{-10} \text{m}$	$v_1 =$	$E_1 = -13.6 \text{eV}$	
2				
3				
4				
5				

Draw the orbital diagram of an atom with 5 energy levels. Take $a_0 = 1 \text{mm}$.

Worksheet 3

Transition		Energy Difference	Frequency	Name of the series	Range of the radiation emitted
From	To				
2	1			Lyman	Ultraviolet
3	1				
4	1				
5	1				
∞	1				
3	2				

4	2				
5	2				
∞	2				
4	3				
5	3				
6	3				
∞	3				

1. What are the lowest and highest frequencies of Lyman Series?
[Hint: $\nu = \text{Energy difference}/h$]
2. What is the frequency range of Lyman series?
3. What is the frequency range of Balmer series?
4. Is there any Energy gap between Lyman and Balmer series? If yes find its value?

