SUMMER INSTITUTE IN MATHEMATICS FOR MATHEMATICS TEACHERS OF JUNIOR COLLEGES (PLUS TWO) OF KARNATAKA STATE

11th July 1988 - 30th July 1988

A REPORT

REGIONAL COLLEGE OF EDUCATION, MYSORE 570006

(National Council of Educational Research & Training, New Delhi)

RESOURCE PERSONS

1. *Dr. V.Shankaram (Academic Coordinator)*
2. *Dr. G. Ravindra*
3. *Dr. N. B. Badrinarayan*
4. *Mrs. S. Vasantha*

(All from Mathematics Faculty of Regional College of Education, Mysore)

A Report on the Summer Institute in Mathematics held at the Regional College of Education, Mysore for mathematics teachers of Junior (plus two) colleges of Karnataka, from 11.7.88 to 30.7.88

Introduction:

The New Education Policy (1986) recognises that the plus two stage is (i) a transition from general education to specialisation leading to academic and applied courses, (ii) terminal for a large number of pupils who take up a vocation which would ensure a livelihood for them, (iii) a sensitive stage from adolescence to youth and, (iv) exposed to the wind of change engulfing every sphere of life in the form of fast developing science and technology and information revolution. Rightly the document underlines the need for teacher training at this stage on a war footing, where the teachers - (i) get an awareness of the demands on them, (ii) get initiations to new techniques and methodologies, (iii) develop appropriate attitudes and interests to meet new challenges, and (iv) equip themselves with the content that should be transmitted to the pupils. Consequently, under a scheme of Improvement of Science Education in Schools, the Ministry of Human Resource Development (MHRD) has planned a number of summer institutes in various subjects for the benefit of teachers at the plus two stage. Some important elements suggested for training programmes are :

- a) Implications of NPE for Science Teaching
- b) Teaching Strategies suited to science (including mathematics) teaching
- c) Relevances of science to society and daily life
- d) Text book analysis
- e) Skills and techniques
- f) Modern curricular areas
- g) International system of units and measurement
- h) Computers as an aid to science teaching
- i) Continuous and Comprehensive Evaluation

Planning the Institute:

The Summer Institute in Mathematics conducted in R.C.E., Mysore from 11.7.88 to 30.7.88 noted these points at the stage of planning the institute. The planning was done at different levels - the college/ the science department/the mathematics section. The points kept in view while planning the institute were

1. Academic input

2. The administrative input

Academic input consisted of

i) organising a team of the faculty

ii) identification of the content areas suited to the needs of the participants

iii)identification of themes of general interests relating to the competencies which the teacher at the level ought to possess, and the demands on him.

iv) identification of the activities to serve the objectives of the institute, within reasonable limits.

Accordingly, the team consisted of -

Dr.V.Shankaram (Academic Coordinator)

Dr.G.Ravindra

Dr.N.B.Badrinarayan

and Mrs.S.Vasantha D. Nath

- all of mathematics faculty of the Science Department.

The criteria for the selection of the content were

a) needs of the classroom teacher in the light of the demands of the syllabus on him,

b) recommendations of the content in the light of the New Education Policy - 1986.

c) Local requirements, namely the syllabus of the state which deputed the teachers for the institute

d) enrichment of the content and

e) initiation to new areas

The content selected for the institute has been appended to the report. *(See Appendix)*

The resources available in the college were tapped for the benefit of the participants of the institute. These were the computer centre, the audio-video studio and the projector facilities of the college.

The participants for the institute were teachers of junior colleges of Karnataka state and there were eighteen participants. The list of participants is appended. (See Appendix)

Conduct of the Institute:

The institute commenced on 11.7.88 in the college. It was formally inaugurated by Dr.A.N.Maheshwari, the Principal of the college, who explained the aims and aspirations of the institute.

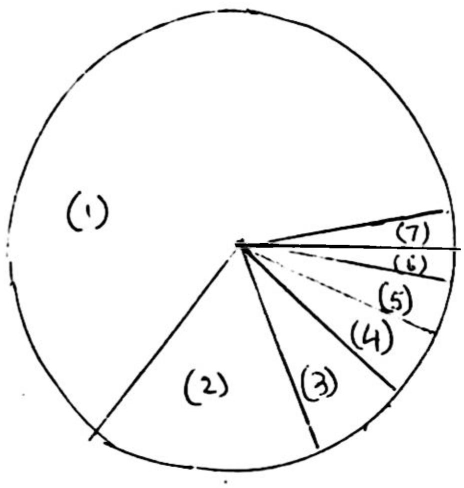
The institute functioned from 9 a.m. to 5 p.m. with an hour's lunch break. The time table copy is appended to the report. (See Appendix)

The institute's programme can be identified under the following heads:

a) Lecture cum discussion sessions
b) Participatory Programmes
c) Computer Awareness Programmes
d) Films
e) Library Consultancy
f) Tests
g) Problem Solving and assignments.

Topics from the identified content areas were discussed after initiation into the topics, in lecture cum discussions. Themes of general interest to a teacher were mooted in participatory programmes. In these sessions, participants were given wide scope to react from different angles. Participants visited the computer centre and had access to the computer knowledge. This experience was acclaimed by all the participants as a valued experience. The several films under the UGC programmes with mathematical themes were shown to the participants. In addition, many other ones too were shown. Each week end gave them opportunities to know how they faired in the institute. These were through short tests (of one hour duration) consisting of multiple choice and short answer type items. The participants were encouraged to consult the library to supplement what they got through lectures. Besides, they had enough time to do more problems and try new problems on their own.

	Total No. of man hours of the institute	109	(360°)
1.	Lecture cum discussions	70	(231.2)
2.	Participatory Programmes	15	(49.6)
3.	Library Consultancy	71/2	(24.8)
4.	Computer Awareness	6	(20.0)
5.	Tests	41/2	(14.4)
6.	Films	3	(10.0)
7.	Others	3	(10.0)



Under participatory programmes, the following themes were discussed - (a) textbook analysis, (b) construction of test items, (c) question paper review, (d) mathematical modelling and (e) New Education Policy 1986 and its implications to plus two stage higher education. The participation in these discussions was very wholesome.

Impressions of the faculty regarding the participants' participation in the institute:

A necessary condition for the success of the institute is the amount of seriousness with which those who run the institute conduct it. The seriousness is borne out of the attitudes which the faculty who are entrusted with the responsibility of the institute possess. This seriousness is reflected in the planning and conduct of the institute, a grasp of the requirements of the institute etc. But it will be still better if the seriousness and the effort of the faculty is complemented from the sides of the participants. The success of an institute largely depends also on the seriousness of purpose and attitudes of the participants. Taking into consideration the participation of the participants in the various programmes of the institute, their performance in the tests conducted every week, it may be said that their performance was not encouraging. It is suggested that teachers who feel the need for the institute and are willing to participate in the institute may be sent as the participants of the institute, so that they may be benefitted by the institute to the maximum extent.

The Participants' Impressions:

The questionnaire circulated among the participants besides informal discussions with them gave an opportunity for the organisers to know the impressions of the participants about the institute. The questionnaire contained questions which sought their impressions on the following aspects -

a) Objectives of the institute - adequacy
b) Topics - coverage, desirability of new topics, usefulness of the topics covered
c) The programmes liked by them
d) Physical facilities in the institute
e) Specific suggestions for the future institutes

While a majority of the participants answered the questions with due seriousness, there were some who were indifferent to these and yet some who did not appear to understand the questions.

To sum up their impressions -

a) *the objectives of the institute were achieved to a great extent.*

b) *they suggested a few additional topics and indepth coverage of some topics.*

c) *they liked greatly the programmes on computer awareness, films and appreciated the participatory programmes*

d) *they were critical of the lecture cum discussions in the sense they wanted less number of lectures*

e) *they wanted the institute to be conducted during the vacation.*

f) *they were satisfied with the physical facilities provided in the hostels where they stayed*

g) *they wanted daily allowance at higher rates; and*

h) *they have suggested more institutes of shorter duration (say two weeks) in a year.*

The Institute concluded on 30.7.1988 with the valedictory address given by Dr.A.N.Maheshwari, Principal, RCEM.

The Academic Coordinator, on behalf of his colleagues of the Institute expresses his gratefulness to the following for their unflinching cooperation.

Dr.A.N.Maheshwari, Principal
Dr.S.S.Raghavan, Reader in Computer Education
Dr.(Miss) Lata Pande - Lecturer in A.V.Education
Dr.V.Ramachandra Rao - Coordinator, Extension
and the secretarial/administrative staff of the Science Department.

Thanks are due to all those who contributed to the success of the institute.

A REPORT OF LECTURE CUM DISCUSSION SESSIONS AND PARTICIPATORY PROGRAMMES HELD BY DR.V.SHANKARAM

The Relevance of NPE to the Teaching of Mathematics at the +2 Level:

At Pre-University level the objectives of teaching mathematics are two-fold - (1) students may proceed to colleges and university departments for studying higher mathematics, (2) students may cope up with mathematics required for professional courses like engineering. Also, it is expected that after undergoing mathematics courses at P.U. level, the students will be able to study maths themselves whenever required. At the P.U. level, mathematics is generally of conceptual and abstract. Keeping all these things in view, it is necessary that instead of lectures alone mathematics is taught through lecture-cum-discussion sessions so that students participate actively in the teaching-learning situations.

Currently there is a tremendous need to give a second look to the existing +2 curriculum so that it may meet the future needs. Keeping this in view, the following subjects are accommodated in the future +2 mathematics curriculum - (a) probability, (b) statistics, (c) linear programming, (d) algebraic structures and number system, (e) computers. The programme of this Summer Institute is planned so that the above need may be met to the possible extent.

SET THEORY:

It is through the language of sets that we study different mathematical structures. The elements of sets need not be numbers. The discussion on sets and relations was geared to prove and illustrate the theorem

> *Every equivalence relation on a set S determines a partition of the set S and conversely, every partition of the set S determines an equivalence on S.*

MATHEMATICAL LOGIC:

Here we are chiefly concerned with logic of statements. Statements are alternatively called sentences. A statement may be either true (T) or false (F), but not both at the same time.

Open and Closed Sentences:

A statement involving a variable x is called an open sentence. The solution set (or truth set) of an open sentence is a set consisting of all numbers which are solutions of the open sentence. For example, $x^2=1$ *is an open sentence, and the solution set here is* $\{-1, +1\}$. *Equations and inequations are open sentences.*

A statement which is not an open sentence is a closed sentence. A closed sentence is either true or false (but not both). All identities are necessarily true and so closed statements.

Negation and other Logical Connectives:

1. *The negation of a given statement is another statement which says that the given statement is not true. The negation of a statement p is denoted by* $\sim p$.
2. *The conjunction of two given statements p and q (denoted by* $p \wedge q$*) is true only if p and q are both true, and otherwise false.*
3. *The disjunction of two given statements p and q (denoted by* $p \vee q$*) is true if any of p and q is true or both are true.* $p \vee q$ *is false, if p and q are both false.*
4. $p \rightarrow q$ *read as "p implies q" or "If p, then q", is true if p is false or if p and q are both true.* $p \rightarrow q$ *is false if p is true and q is false.*
5. $p \leftrightarrow q$ *read as "p if and only if q" or "p implies and is implied by q" is true if p and q are together true or false.* $p \leftrightarrow q$ *is false if one of p and q is true and while the other is false.*

A table showing the truth-value of a statement p corresponding to the truth-value combinations of its component statements is called the truth table of the statement p. Now we give below the truth-table for the statements $\sim p$, $p \wedge q$, $p \vee q$, $p \rightarrow q$ *and* $p \leftrightarrow q$.

Truth-Table of $\sim p$

	$\sim p$
T	F
F	T

Truth Tables of Compound Statements

p	q	$p \wedge q$	$p \vee q$	$p \to q$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

These compound statements can be represented through Venn diagrams (sets).

With the help of the above truth tables which define $\sim p$, $p \wedge q$, $p \vee q$, $p \to q$ and $p \leftrightarrow q$, we can draw the truth tables of other statements. For example, the truth table of $\sim p \to q$ will be as follows.

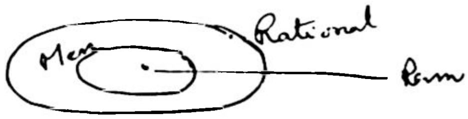
p	q	p	p q
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

Validity of Arguments:

Steps for deductive reasoning for arriving at a conclusion from the premises is called an argument. An argument which is correct according to our logic is called a valid argument. An argument may be valid, while the conclusion is false. Also, an argument may be invalid (not valid), while the conclusion is true. We do not attach the adjectives "true" or "false" with an argument.

In argument we generally use the rule of implication (or "modus ponens"). The rule states that "$(p \rightarrow q) \wedge p \rightarrow q$". Here is an example, which is a valid argument:- All men are rational. Ram is a man. So, Ram is rational. Here p stands for "x is a man". q stands for "x is a rational". Now we see that the above argument is reduced to the form "x is a man" $\rightarrow$ "x is rational". "Ram (for x) is a man". So, "Ram is rational".

The set theoretic representation will be as follows:



If p,q,r,..... lead to the conclusion s, then for a valid argument the statement $(p \wedge q \wedge r \wedge \ldots) \rightarrow s$ should be tautology (a statement which is true for all the truth-value combinations of its components.)

Sometimes, an argument, which is not valid is called a fallacy. Generally, in mathematics we come across the following two types of fallacies: (1) Hidden assumption, (2) Circular Arguments.

<u>Different Types of Implication</u>

The following are types of implication (as given in the truth-table).

p	q	$p \rightarrow q$	$q \rightarrow p$ converse	$\sim p \rightarrow \sim q$ inverse	$\sim q \rightarrow \sim p$ contrapositive
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

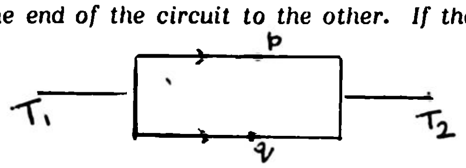
Indirect Proof:

Generally, the structure of a proof is as follows: "If B is true, then A is true; but B is true, so A is true". This structure of proof is called "the direct proof". Technically, this pattern is known as "modus ponens".

But many a time we may like to use a pattern of proof called "indirect proof". Proof of a statement by proving its contrapositive is called an <u>indirect proof.</u>

Application of Mathematical logic in Switching Circuits (Networks):

One application is to determine whether or not the current will flow from one end of the circuit to the other. If the network is the following,



then the compound statement corresponding to the network is p and will have the following truth-table.

p	q	p ∨ q
T	T	T
T	F	T
F	T	F
F	F	F

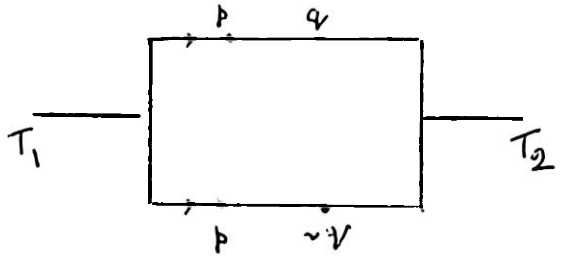
It follows that the current flows when both p and q are open, or when p is open and q is dosed, or when p is closed and q is open.

Another application is in finding the network, given the switches and the configurations when current should flow. The problems amounts to finding a compound statement corresponding to a given truth table. We consider the following configuration: Given switches are p and q; and the current flows when (1) both switches p and q are open; and (2) when the switch p is open and the switch is closed. Now we have to find the network.

Here, the Truth-Table of the unknown statement X is as follows.

p	q	X
T	T	T
T	F	T
F	T	F
F	F	F

We see (1) the statement $p \wedge q$ is true only when both p and q are both true, and false otherwise. (2) the statement $p \wedge \sim q$ is true <u>*only when*</u> *p is true and q is false. Now we construct the disjunction of the statements mentioned in (1) and (2) and get the required form of the statement X. So, X has the form $(p \wedge q) \vee (p \wedge \sim q)$. Now the network required is as follows:*



Algebraic Structures:

<u>*Introduction:*</u> *We select some suitable properties of different number system (e.g. integers, rationals, reals) and examine, whether these properties hold for general systems, the elements of which are not necessarily numbers. In course of such investigations, mathematicians created such systems called groups, rings, fields, etc. These systems are called algebraic structures or algebraic systems. Here we will discuss some important and interesting facts about the structure called rings.*

ALGEBRAIC SYSTEMS:

Here, the objective of the discussion was to prove and illustrate the following fundamental theorem concerning the ring homomorphism.

> *Let $\phi: R \rightarrow R'$ be a homomorphism of a ring R onto the ring R' with kernel K. Then K is an ideal in R and R' is isomorphic to the quotient ring R/K. More precisely the mapping $f: R/K \rightarrow R'$ defined by $f(a+K) = \emptyset\ (a)$ is an isomorphism of R/K onto R'.*

We can extend the concept of homomorphism to groups and linear spaces. In cases of groups, normal subgroups play the same role as ideals for rings.

The above discussion does not hold for fields, as any field F has only trivial ideals - the zero ideal $\{0\}$ and the field F itself.

PERMUTATION GROUPS

Permutation groups are of great significance in the development of finite groups. In fact, the study of finite groups can be made through permutation groups, which is so due to the following fundamental theorem of Cayley.

Cayley's Theorem: *Every finite group is isomorphic to a permutation group.*

The set of all permutations of n elements forms a group called the symmetric group of n and denoted by Sn. Sn has clearly n! elements. S_3 is the smallest non-commutative group and it has 6 elements. S_3 is isomorphic to the group of all symmetrics of an equilateral triangle.

THE REAL NUMBER SYSTEM:

The concept of the real line rests on the following important fact: To every point of the real line, there corresponds a real number and vice versa. That is, there is a one to one correspondence between the real line and the real number system. That is why, the real line is called so.

The following are important facts about the rationals(Q) .

1. *If we define* $p/q \simeq r/s \Leftrightarrow ps = qr \quad (q \neq 0,\ s \neq 0)$ *for all* $p/q \in Q,\ r/s \in Q$, *then* $\simeq$ *is an equivalence relation on Q.*
2. *Q forms an ordered field, with "$<$" as an order relation in Q.*
3. *There are infinite rationals between two rationals a and b, where $a < b$.*
4. *We can represent any rational by a terminating or recurring decimal number. Conversely, any terminating or recurring decimal number can be converted into the rational form p/q.*
5. *The set Q of all rationals is a countable (infinite) set.*

Real Numbers:

Though between any two rationals a and b with $a < b$, there is an infinity of rationals, there are numbers between a and b which are not rational. For example, it can be shown that $\sqrt{2}$ is not rational. Such numbers are called irrationals. The union of the set of all rationals and the set of irrationals form the set R of all reals. Following are some of the important properties of the real number system R.

1. *To every point of the number line, there corresponds a real number and vice versa. This is called the completeness property of R, the real number system. In this regard, R is different from Q.*
2. *The set R of all reals forms an ordered field, with "$<$" defined as an order relation on R.*
3. *Between any two reals there is an infinity of reals.*
4. *Any irrational number can be represented by a non-terminating non-recurring decimal number represents a real and vice versa.*
5. *The set of all reals is uncountable. So, the set of all irrationals is also uncountable.*

Algebraic and Transcendental Numbers:

A real which satisfies a polynomial equation with rational coefficients is called an algebraic number. A real number which is not an algebraic number is called a transcendental number. The following facts are worth mentioning here -

1. *Algebraic numbers may be both - rational and irrational. For example, all rationals, $\sqrt{2}$ and $\sqrt{3}$ are algebraic numbers.*
2. *All transcendental numbers are irrationals.*
3. *The set of all algebraic numbers is countable, while the set of all transcendental numbers is unaccountable. Still not many transcendental numbers are known. π and e are well-known examples of transcendental numbers.*

Nature of Mathematics

It is very important that a mathematics teacher knows about the nature of mathematics. Nature of mathematics has direct relevance to the method of teaching mathematics at school - particular so that the junior college (+2) level. This is so, because mathematics at this level is predominantly conceptual.

Mathematical method is essentially deductive, whereas scientific method involves inductive method predominantly. A mathematical proposition is analytic apriori, whereas a proposition in science is synthetic aposteriori. There is a misconcept in some circles that the principle of mathematical induction is inductive in nature. In fact, the method of mathematical induction is deductive in nature. It is unfortunate the phrase "induction" is used in connection with mathematical induction.

In mathematics mathematical method is also known as axiomatic method. In axiomatic method, we start with preconceived postulates or axioms and we arrive at different conclusions called theorems by applying the rules of inference to the axioms. The sequence of the steps through we arrive at a theorem from the axioms is called a proof of the theorem. We should note that the truth or falsity of a theorem is to be examined within an axiom system. For example, the theorem "The three angles of a triangle are together two right angle" holds for the axioms of plane geometry, but does not hold for the axioms of spherical geometry.

Axiomatic method gives strength to mathematics and mathematical method. Different sets of axioms give rise to different axiom systems. Modern mathematics study many of axiom systems or axiom structures. For example, in modern algebra we study many algebraic structures like group, ring and field. These algebraic structures are examples of axiom systems.

Mathematics gains strength from axiomatic approach in another way. Axioms in themselves do not say much. But the same axiom system may have different models or interpretations which hold good in different situations. These different models give different meanings to the same axiom system. The interpretations of an axiom system give rise to different applications of mathematics. Mathematics (or pure mathematics) studies the different axiom system and tries to find new theorems within the axiom systems without paying heed to practical world. But when mathematics is applied to practical situations of life (e.g. social science/economics) or to describe nature (e.g. physical laws) through the interpretation of axiom systems, applied mathematics is born. Mathematics (may be, pure mathematics), which is capable of being utilised in practical situations or otherwise through its suitable interpretation is called now-a-days applicable mathematics. Graph theory is a good example of applicable mathematics.

Here, it will not be out of place to know and discuss in some detail about the nature of mathematics at the secondary and +2 level. We give below a brief description of different branches of school mathematics and mathematics at +2 level.

a) Arithmetic: Arithmetic is the study of numbers and operation with them (e.g. addition, multiplication, etc.). In an extended sense, theory of numbers is a branch of mathematics at higher level. Arithmetic is made a part of primary education because of its practical utility. Applications of arithmetic give rise to different subjects like commercial arithmetic book-keeping and accounting.

b) Algebra: Generally, algebra is considered to be a sort of generalisation of arithmetic in that the domain of an algebraic variable is a set of numbers and the particular cases of an algebraic formula give rise to different arithmetical facts. Also, whereas in classical algebra we are concerned primarily with computational aspects, in modern algebra we study different abstract algebraic structures which are generalisations of different number systems.

c) *Geometry:* *Geometry deals with the measurement and properties of figures in a plane and space. Geometry is used in surveying, engineering, drawing and architecture. Modern geometry is study of different axiom systems in axiom-theorem format. Recently, many types of geometry (e.g. non-Euclidean geometry such as spherical geometry) are studied.*

d) *Trigonometry:* *Trigonometry started as a study of the properties of triangle in terms of the trigonometrical functions (such as Sin x, Cos θ etc.) which were originally defined in terms of a right angled triangle. But in modern treatment (due to circular measure) trigonometrical functions can be defined for any real number without the help of a right angled triangle. Trigonometry has been used in computing heights and distances. It has been helpful to develop many branches of higher mathematics - e.g. theory of Fourier series, complex analysis.*

e) *Mensuration:* *Mensuration is a study of lengths, areas and volumes relating to the figures and solids (in* R^2 *and* R^3*). In the treatment of mensuration, arithmetic, algebra, geometry and trigonometry are freely used. Mensuration is particularly useful in engineering.*

f) *Calculus:* *While studying the areas and volumes of curved surfaces and solids and planetary orbits, Newton (along with Leibnitz) invented calculus. Calculus is studied in two parts: Differential calculus and integral calculus.*

In differential calculus we study the derivatives dy/dx of real valued functions and its applications. Since a derivative is defined as a limit, the study of limiting process is an essential part of differential calculus. The derivative is used as a rate-measurer in different subjects such as physics and economics.

In the integral calculus, we study integration as the inverse process of differentiation and integral as a limit of sum. The fundamental theorem of integral calculus connect the two concepts of integration. We apply integration in finding lengths, areas and volumes and in other places where the limit of sum is involved.

In the past 150 years, while studying the foundations of calculus (in which the concept of limit is so significant) mathematicians have developed a branch of mathematics called mathematical analysis. In mathematical analysis are studied the different aspects of limit concept with great vigour and precision.

PARTICIPATORY PROGRAMMES:

1. Question paper analysis:

In this programme, participants evolved the criteria for analysis a question paper and one question paper of Karnataka P.U. Board was analysed.

2. Constructing test items:

In the sessions of items construction, participants evolved criteria of a good test item and items constructed individually and in groups were modified after applying the criteria to the items.

A REPORT BY DR.G.RAVINDRA

Mathematical Modelling:

1. *Meaning of a Model : Models are simplified (but generally not perfect) representation of real situations.*

2. *Why models? (Often the motivation is economic, to save money or time, sometimes to avoid risk associated with tampering of real object).*

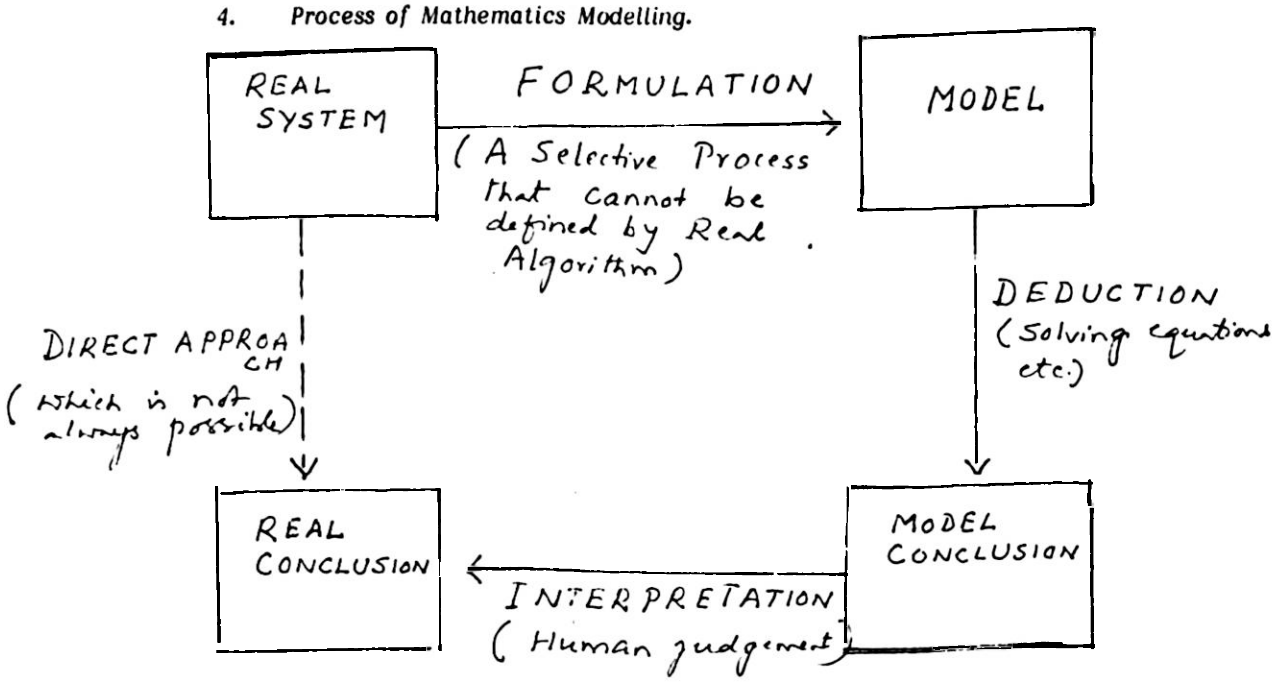
3. *Kinds of Models:*

i) *Iconic Models (These are images in a way of real situations - examples : photographs, drawing maps, model aeroplanes, planetorium, etc. These are specific and concrete.)*

ii) *Analogue Models: (These models use one set of property to represent another set of properties - Examples: Contour lines on a map, Graphs of relations. These models are specific but less concrete.)*

iii) *Symbolic Models : (Symbols are used to represent variables of a real situation and relationship between them - Examples, maxima.minima problems, linear programming problems, etc. These are abstract.)*

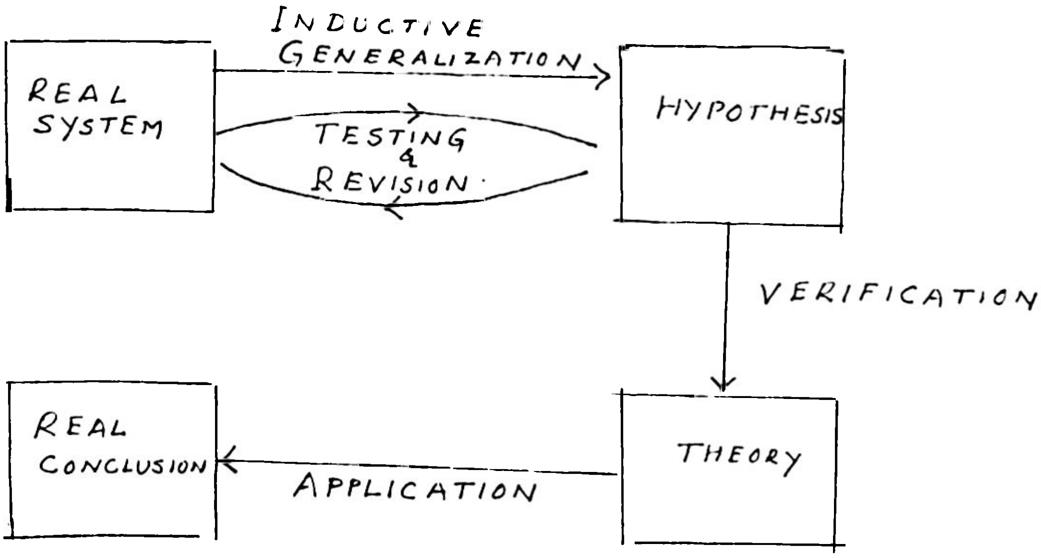
4. *Process of Mathematics Modelling.*



5. *Difference between a Model and Scientific Method:*

Models are invented and the scientific theories are discovered.

PROCESS OF SCIENTIFIC METHOD



6. *Examples of Mathematical Models*

a) *Maxima minima problem - selecting a sports field* etc.

b) *Linear programming problem*

c) *Picking largest of three numbers or 'n' numbers. (Using flow chart and computer oriented algorithm)*

d) *Chinese Postman problem (in which a postman wishes to deliver all his letters in such a way that he covers the least possible total distance).*

e) *the shortest path problem (using threads)*

f) *Travelling Salesmen problem (to visit several given cities and return to his starting point in such a way that he covers the least possible distance)*

g) *Colouring maps (using the famous four colour theorem)*

h) *Bar diagrams*

i) *Graphs of functions.*

LINEAR PROGRAMMING

Motivation through

a) Product Mix problem:

A typical representation of these problems is as follows (when restricted to two variables).

	Model (kind) A	B	Capacity
Labour (hour/unit)	a_{11}	a_{12}	b_1
Material (Kg/unit)	a_{21}	a_{22}	b_2
Profit (Rs/unit)	C_1	C_2	

The products could be industrial products or farm products and the problem is to find optimal mix.

b) Assignment problem (assigning resources to jobs in one to one way so as to minimise man hour or loss, or maximize profit).

c) Transportation problem.

2. *Mathematical Formulation of Linear Programming Problem (both in two variables and n-variables).*

3. *Solution of Linear Programming Problem in two variables (by graphical method)*

a) *Prerequisites:*

i) *Linear Equations and Inequations*

ii) *Graphs of Linear Equations and Inequations*

iii) *Identification of Graphs of linear inequations (i.e. hyperplanes and half spaces)*

iv) *Convex sets (examples, non-examples, illustrations)*

v) *Extreme points of a convex set*

vi) *The Feasible region (examples and non-examples)*

vii) *The set of feasible solutions of a Linear Programming Problem is convex.*

b) *Key result : If there exists an optimal (maximal or minimal) solution to a linear programming problem then atleast one of the extreme points (corners) of the feasible region of the problem will always qualify to be optimal solution.*

c) *Understanding of Max(z) = -Min (-z)*

4. *Problems Solving*

5. *References:*

i) *DANTZIG G B Linear Programming and Extensional, Princeton Univ. Press, 1973*

ii) *GASS S.I., Linear Programming, Addis*

iii) *G.HADLEY, Linear Programming, Addison Wesley 1982*

iv) *SASIENI.M., YASPEN A. AND FRIEDMAN L. - Operations Research (Methods and Applications) Wiley*

v) *KANTISWAROOP, GUPTA P.K. AND MANMOHAN - Operations Research Sultan Chand & Sons, 1984*

vi) *TAHA H.A. Operations Research, Collier MacMillan International Edition, 1976.*

NUMBER SYSTEM:

(Motivation through counting process)

1. *Number as a unique representation of elements in a set.*
2. *Counting process*
3. *Basic operations "+" "." on integers.*
4. *Identifying these operations as binary operations (integers)*
5. *Ring structure of integers*
6. *Division Algorithm (statement and illustration)*
7. *Euclidean Ring Structure of Integers*
8. *Formal definition and illustration of Rings and Euclidean Rings*
9. *Primes and composite number and misconceptions in these definitions.*
10. *Unique factorization Theorem (statement and illustration)*
11. *Significance of conditions of the theorem*
12. *Congruences, linear congruences and their solution*
13. *Problems solving*
14. *Verification that the integers do not satisfy multiplicative inverse property w.r.t. usual multiplication*
15. *The integers form a field w.r.t. to operations - "addition modulo p" and "multiplication modulo p".*

COMPLEX NUMBERS

(Motivation through roots of x^2+1)

1. Well-ordering principle
2. The reals form well ordered set
3. Roots of the polynomial x^2+1
4. Proving that the roots of the polynomial are not real
5. Extension of the reals to a field R(i) to accommodate the roots of the polynomial
6. Definition of complex numbers C as the set of ordered pairs (x,y), x and y being real numbers
7. Definition of "+" and "." ON C and identifying them as binary operations
8. Isomorphism between C and R(i)
9. C is not a well ordered set
10. The complex plane
11. The complex roots of unity are the vertices of a regular polygon of n sides inscribed in a circle $|Z| = 1$. (Application of de Moivre's Theorem)

POLYNOMIALS

1. MOTIVATION:

a) Insolvability of Quintic Polynomial (with glimpses of Galois Theory)

b) Impossibility of trisecting 60° with ruler and compass only

c) Sensitivity analysis of computer oriented algorithms

d) Application of roots of polynomials in finding maxima or minima of a function

e) Applications in Mathematical Modelling.

2. Definition, examples, non examples and illustration of polynomials "over a field F".
3. Significance of the phrase "Over a field F"
4. Roots of polynomial
5. Solvability by radicals
6. Statement of the fact that not all polynomials of degree atleast 5 are solvable by radicals
7. Definition, examples, non-examples, illustrations, significance of Irreducible polynomials over a field
8. Eisenstein Criterion for testing irreducibility over rationals of a polynomial with integer coefficients.
9. Understanding of difficulty for characterizing irreducible polynomials
10. Identification that the set of polynomials form Euclidean Domain.

A REPORT BY DR.N.B.BADRINARAYAN

During the course of Summer Institute in Mathematics, two topics were discussed for the benefit of the participants. Probability theory and Vector were the topics. While lecture cum discussion sessions for about 8 hours were on probability theory, about 8 hours were spent on vectors.

Under probability theory, the points of discussion were -

i) sample space and events
ii) probability of an event
iii) total probability (on sum of events) theorem)
iv) compound probability (on product of events)
v) conditional probability and independent events

Under vectors the points discussed were -

i) Vector - as an element of a vector space
ii) Position vectors in R^2 *and* R^3
iii) Scalar multiplication, sum and products (dot product and cross product) of two vectors
iv) Triple products (scalar and vector)
v) Vector approach to study geometry (coordinate geometry)

The discussions included problem solving.

Three tests were given on the topics discussed. Besides, there was a general theme of discussion. The theme was - Text Book analysis.

A REPORT BY MRS.S.VASANTHA

COMPUTER EDUCATION

The recently concluded Summer Institute for the +2 level teachers, included in its course content, a 'Computer Awareness' programme for its participants. It was mainly a participatory programme, which aimed at "hands on" experience for the teachers. Two aspects of 'Computer Education' were highlighted. Firstly, familiarity with computers, its capabilities and how to communicate with a computer. Secondly, how to use computer as an aid to the teaching of mathematics at the +2 level. Keeping these in view, the teachers were introduced to the following topics:

1. *Computer - its external features - CPU - brain of the computer, its keyboard, the input output devices - the cassette/disc system - VDU - the monitor, printer types of computers - main frame mini and micro.*

2. *Communicating with the computer - through a series of step by step instructions - algorithm - flow charts as a pictorial representation of algorithm - illustrations of flow charts.*

3. *What are the capabilities of a micro computer? - to understand this a study of 'utilities' (WELCOME) package, was made for a duration of one and a half hours.*

4. *The computer as a teaching aid - To understand the role of computer in the teaching/learning process, a study of computer based learning (CBL) packages was made by the participants.*

The list of CBL packages used:

a) *Rekha - a software package to draw line graphs and graphs of functions, also includes plotting of points in a plane coordinate system.*

b) *Geometric Transformations: aids understanding of transformations of plane figures and the inter-relationships among different transformations.*

c) *Tesselations: An excellent software for understanding symmetry groups, and drawing interesting tile patterns, aids understanding of combinations of transformations.*

d) Polyhedra : This software provides a library of polyhedral solids, which can be viewed and studied by students ultimately leading to the Euler's formula.

e) Calculus - Limits and Continuity :

Gives graphs of continuous as well as step functions, identifies points of discontinuity. A table of values of the function in a desired neighbourhood of the point of discontinuity is provided.

f) Astronomy: Software consists of a simulation of planetary motion, phases of moon, eclipses and rocket launching, etc.

How this course was organised?

The programme consisted of theory as well as practicals. The computer centre at the RCE, Mysore was utilized for providing practical experience to the participants.

DEMONSTRATIONS:

1. A Demonstration of the 'WELCOME' package was made before the participants. They made a study of this package for themselves (1 hour 45 minutes duration).

2. A demonstration of the CBL package on Matrix addition, multiplication and inversion was given by Dr.S.S.Raghavan, explaining at length the educational value of the software and the key points to be borne in mind while studying a CBL package.

***PRACTICAL SESSIONS:** (1 1/2 hour per session)*

The 18 participants were divided into six groups of three each and each group was provided with a BBC microcomputer system. Three sessions of practicals were planned as follows:

I Session: Study of Welcome package on BBC computer system)and keyboard practice). The sixteen programmes of this package gives an idea of what are the jobs that a microcomputer can do. It can even supervise the keyboard practice.

II Session and III Session: Study of CBL packages and preparing a note on the usefulness of the package they have seen. A set of questions were provided to the participants to aid the preparation of a report on the CBL package they saw.

Two theory sessions : In addition to the practical sessions, lecture/discussion on aspects like flow charts, history of computers, computer as a teaching aid were held for two sessions for 1 1/2 hours duration each.

A Report on the film shows arranged during the summer institute:

The following films on mathematical topics were screened for the participants to supplement the theoretical discussions in the respective topics.

1. *Why O.U. film duration 24 minutes (Calculus)*
2. *Binomial theorem O.U. Film 24 minutes duration (Algebra)*
3. *Networks and matrices O.U. film - duration 24 minutes (Algebra)*
4. *Sin $(\alpha + \beta)$ - O.U. Film (Trigonometry/geometry)*
5. *Symbols, equations and computers - duration 24 minutes*
6. *Ratio and proportion*
7. *Meaning of π*
8. *Congruence of triangles*

A note on Calculus Course:

The lectures/discussion on differential and integral calculus were utilized for the study, discussion and clarification of the important concepts of calculus as given below:

Functions and their graphs, types of functions, limits,continuity, types of discontinuity, differentiability, one sided limits and differentiability, relation between differentiability and continuity, applications of derivative - as a rate measurement for tangents and normals, maxima minima problems and applications to physics and other subjects. Integration-meaning, integrable functions - relation between processes of differentiation *and integration - Fundamental theorem - Primitives - Applications to calculation of areas, volumes, etc.*

Attention was paid to the following aspects during the discussions.

1. *Use of a variety of graphs for understanding concepts of Calculus.*
2. *Examples and counter-examples in the clarification and illustration of concepts of Calculus.*
3. *Use of applications in medicine, population models, economics (supply and demand) physical sciences, Biological models, study of fossils. Growth of Bacteria, etc.*

A large number of examples from these disciplines were discussed during the course.

The topics of maxima and minima was discussed at length and a large number of problems from physics, chemistry, etc. were worked out.

Effort was also made to remove the likely errors and misconceptions which may creep in the minds of students/teachers while studying concepts of Calculus. A detailed discussion of how to present these concepts particularly to the students to minimise such errors was carried out.

APPENDIX 1

Summer Institute in Mathematics
(Regional College of Education, Mysore)
July 11 - 30, 1988

SYLLABUS

COURSE 1

Algebra - I

Number System:

i) Natural numbers-Divisibility, H.C.F. and L.C.M. Prime factorisation theorem. Co-primes, Mathematical Induction.

ii) Integers - as an ordered ring, Congruence modular relation on the set of integers, their group structure, modular arithmetic.

iii) Rational numbers - as an ordered field. Density property, representation on the number line. Irrational numbers - irrationality of $\sqrt{2}$, e and π

iv) Real numbers - definition by Dedikind-cuts. Least upper bound to greatest lower bound of a set of real numbers. Completeness, Intervals - open and closed. Real number - as a complete ordered field.

v) Complex numbers - Irreducibility of x^2+1 in the field of real numbers. Definition of a complex number as an ordered pair (x,y) and in the form x+iy. Addition and multiplication. Complex numbers as a field. The modulas and argument of a complex number. Representation of a complex number in Argand's diagram.

vi) Polynomials with real coefficients - as a ring. Factorisation of a polynomial. A quadratic polynomial, a quadratic equation and its roots.
A cubic polynomial and symmetric functions of its roots.

COURSE 2

Algebra - II

1. Vectors:

The Cartesian products R^2 and R^3 the plane and space. Vector as an ordered pair in R^2 and an ord. trible in R^3. The unit vectors : $i = (1,0)$, $j = (0,1)$ in R^2. $i = (1,0,0)$, $j = (0,1,0)$, $k = (0,0,1)$ in R^3. Linear combination of vectors i, j and k. The position vector of a point (x,y) in R^2, a point (x,y,z) in R^3. Properties of vectors - forming a vector space - general definition of a vector as an element of a vector space. Linear combination of vectors,Linear dependence and independence of vectors. Dimension and basis of a vector space. Inner (scalar) product of two vectors. The dot and cross products of two vectors, geometrical meaning.

2. *Matrices and Determinants:*

Matrix Algebra - sum and product. Types of matrices. Adjoint of a matrix. Inversible matrices. Determinant of a real function. Properties of determinants, Rank of a matrix - Related theorems solution of matrix equations : $AX = B$. Consistency and inconsistency of a set of linear equations - Cramer's Rule for solving equations. Condition for the existence of non-trivial solutions of a system of Lin. Hom. equations : $AX = 0$.

3. *Coordinate Geometry:*

In R^2: Vector equation of a straight line and different cartesian equations. Vector equation of a circle and equation of a tangent - corresponding center in equations.

In R^3: Vector equation of a plane - Cartesian equation vector equation of a line - Cartesian equation vector equation of a sphere - Cartesian equation vector equation of a circle - cartesian equation the vector equation of a tangent plane to a sphere - condition of tangency of a plane to a sphere.

COURSE 3

Calculus:

1. *Sets - operations on sets, Relations on sets - Equivalence Relation and Equivalence classes. Ordered relations - partial and total, Binary operations on sets.*

2. *A real function of a single variable domain and range - single valued and many valued functions. Inverse function of a function. Quadratic functions and their zeros. The exponential function e^x and logarithmic function $\log_e x$ and their graphs.*

   Trigonometric functions - sin x, cos x and Tan x and their graphs.

3. *Limit of a function at a point - theorem continuity of a function at a point. Properties of continuous functions - scalar multiple sum and product of continuous functions.*

4. *Derivative of a function at a point. Geometrical meaning. Differentiability and continuity. Roll Theorem - M.V.Theorems, Extreme values of a function.*

5. *Limit,continuity and differentiability of a function Z = f(x,y) (of two variables) at a point. Partial derivatives and the total derivative - the Chain Rule.*

6. *Integral of a function on the anti-derivative. Definite integral - Reimann Integrability and definition of a definite integral as a limit of a fundamental theorem of integral calculus. Evaluation of definite integrals -Theorems on definite integrals.*

COURSE 4

Logic: *True and false statements, Truth values, Truth sets of open sentences. Composition of statements, conjunction, disjunction and negation. One way and two way implications, Theorem, inverse, converse and contrapositive. Logical validity of a statement. Tantologies and Contradictions. Venn diagrams for logical inferences. Permutation.*

Probability: *Permutation, combination and B-theorem, Ramdom experiment and the associated sample space. Definition of an element, mutually exclusive events, probability of an event. Addition theorem. Conditional probability and independent events.*
Ramdom variable. Probability distribution of a Ramdom variable.

Linear Programming: *Equations and inequations in two unknowns (x,y). Geometrical representation convex sets (polygon). A problem of optimisation - mathematical formulation. Solution of a maximisation (minimisation) problem - feasible solutions and optional solutions by graphical method.*

Computing: *(As prescribed under Maths curriculum at the +2 stage.)*

APPENDIX 2

REGIONAL COLLEGE OF EDUCATION, MYSORE 6
MATHEMATICS SUMMER INSTITUTE
(11.7.88 to 30.7.88)

Date	Time	Programme Particulars
11.7.1988	10.00 - 11.30 a.m.	Registration and Inauguration (ANM)
	11.30 - 1.00 p.m.	Implications of New Education Policy to +2 stage (VS)
	2.00 - 3.30 p.m.	Linear Programming (GR)
	3.30 - 5.00 p.m.	Functions (SV)
12.7.1988	9.00 - 10.00 a.m.	Library Consultation/Self Study
	10.00 - 11.30 a.m.	Linear Pogramming (GR)
	11.30 - 1.00 p.m.	Limits of functions (SV)
	2.00 - 3.30 p.m.	Permutations (NBB)
	3.30 - 5.00 p.m.	Sets (VS)
13.7.1988	9.00 - 10.00 a.m.	Linear Programming (GR)
	10.00 - 11.30 a.m.	Combinations (NBB)
	11.30 - 1.00 p.m.	Limits (SV)
	2.00 - 3.30 p.m.	Sets (VS)
	3.30 - 5.00 p.m.	Mathematical Modelling (GR)
14.7.1988	9.00 - 10.00 a.m.	Library Consultation/Self study
	10.00 - 11.30 a.m.	Linear Programming (GR)
	11.30 - 1.00 p.m.	Limits (SV)
	2.00 - 3.30 p.m.	Introduction to Probability (NBB)
	3.30 - 5.00 p.m.	Mathematical Modelling (GR)
15.7.1988	9.00 - 10.00 a.m.	Library Consultation/Self study
	10.00 - 11.30 a.m.	Probability (NBB)
	11.30 - 1.00 p.m.	Permutation Group (VS)
	2.00 - 3.30 p.m.	Linear Programming (GR)
	3.30 - 5.00 p.m.	Week End Test I
16.7.1988	9.00 - 10.00 a.m.	Mathematical Logic (VS)
	10.00 - 11.30 a.m.	Continuity of a function (SV)
	11.30 - 1.00 p.m.	Linear Programming (GR)
	2.00 - 5.00 p.m.	Self Study/Completing Assignment in Permutation Group
17.7.1988	Library Consultancy, Self Study and Completing Assignment in Probability / Mathematical modelling.	

Date	Time	Programme Particulars
18.7.1988	9.00 - 10.00 a.m.	Number System - Integers (GR)
	10.00 - 11.30 a.m.	Mathematical Logic (VS)
	11.30 - 1.00 p.m.	Probability (NBB)
	2.00 - 3.30 p.m.	Differentiability of a function (SV)
	3.30 - 5.00 p.m.	Constructing test items (VS)
19.7.1988	9.00 - 10.00 a.m.	Mathematical Logic (VS)
	10.00 - 11.30 a.m.	Applications of derivative as a rate measurement (SV)
	11.30 - 1.00 p.m.	Library consultancy/self study
	2.00 - 3.30 p.m.	Probability (NBB)
	3.30 - 5.00 p.m.	Textbook Analysis (NBB)
20.7.1988	9.00 - 10.00 a.m.	Probability (NBB)
	10.00 - 11.30 a.m.	Mathematical Logic (VS)
	11.30 - 1.00 p.m.	Derivatives (SV) - Tangents and Normals
	2.00 - 3.30 p.m.	Library consultation/Self study
	3.30 - 5.00 p.m.	TextbookAnalysis (NBB)
21.7.1988	9.00 - 10.00 a.m.	Library consultation/self study
	10.00 - 11.30 a.m.	Algebraic Structures (VS)
	11.30 - 1.00 p.m.	Maxima and Minima (SV)
	2.00 - 3.30 p.m.	Vectors (NBB)
	3.30 - 5.00 p.m.	Construction of Tests (VS)
22.7.1988	9.00 - 10.00 a.m.	Library consultation/Selfstudy
	10.00 - 11.30 a.m.	Vectors (NBB)
	11.30 - 1.00 p.m.	Question Paper Analysis (VS)
	2.00 - 3.30 p.m.	Maxima and Minima (SV)
	3.30 - 5.00 p.m.	Week-end Test II
23.7.1988	9.00 - 10.00 a.m.	Algebraic Structures (VS)
	10.00 - 11.30 a.m.	Maxima MinimaApplications (SV)
	11.30 - 1.00 p.m.	Vectors (NBB)
	2.00 - 5.00 p.m.	Self Study/Completing the assignments in mathematical logic/algebraic structures.
24.7.1988	Self Study and Completing assignments in mathematical logic/ algebraic structures.	
25.7.1988	Self Study and completing assignments in mathematical modelling.	

Date	Time	Programme Particulars
26.7.1988	9.00 - 10.00 a.m.	Rational Numbers (VS)
	10.00 - 11.30 a.m.	Rings (GR)
	11.30 - 1.00 p.m.	Integration (SV)
	2.00 - 3.30 p.m.	Vectors (NBB)
	3.30 - 5.00 p.m.	ComputerAwareness (SV)
27.7.1988	9.00 - 10.00 a.m.	Polynomials (GR)
	10.00 - 11.30 a.m.	Real Numbers (VS)
	11.30 - 1.00 p.m.	Integration (SV)
	2.00 - 3.30 p.m.	Vectors (NBB)
	3.30 - 5.00 p.m.	ComputerAwareness (SV)
28.7.1988	9.00 - 10.00 a.m.	Library Consultation/Self Study
	10.00 - 11.30 a.m.	Films
	11.30 - 1.00 p.m.	Integration (SV)
	2.00 - 3.30 p.m.	Vectors (NBB)
	3.30 - 5.00 p.m.	Final Test
29.7.1988	9.00 - 10.00 a.m.	Polynomials (GR)
	10.00 - 11.30 a.m.	Mathematical Modelling or Question Paper Analysis (GR)
	11.30 - 1.00 p.m.	Library Consultation/Self Study
	2.00 - 3.30 p.m.	Integration (SV)
	3.30 - 5.00 p.m.	Nature of Maths (VS)
30.7.1988	9.00 - 10.00 a.m.	Complex Numbers (GR)
	10.00 - 11.30 a.m.	Nature of Mathematics (VS)
	11.30 - 1.00 p.m.	Valedictory Function

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APPENDIX 3

SUMMER INSTITUTE IN MATHEMATICS - 1988

I Test

1. *Tick the right answer:*

i) *The region formed by the inequations* $ax+by \leq c$, $ax+by \geq d$, $x,y \geq 0$ *(where a,b,c,d are positive real numbers and* $d \leq c$*)*

a) is empty　　b) non empty

c) empty or non empty　　d) non empty with more than four vertices

ii) *If a region represened by m linear inequations is a polygonal region with n sides, then*

a) $m < n$　　b) $m \leq n$

c) $m = n$　　d) $m \geq n$

iii) *If a set S has 6 elements, the number of subsets of S is*

a) 62　　b) 63

c) 64　　d) 32

iv) *If* $f : R \rightarrow R$ *is a function, then the inverse function* f^{-1} *exists if*

a) $f(x) = \sqrt{x^2 + 9}$　　b) $f(x) = x^2+9$

c) $f(x) = x+3$　　d) $f(x) = \frac{1}{x+5}$

v) *If R is the relation "*$a \equiv b \pmod 7$*" over Z, the set of integers, then one of the equivalence classes is*

a) $\{3,10,17,24,\ldots\}$　　b) $\{0,7,14,21,28,\ldots\}$

c) $\{x \mid x = 7a+2, \text{ where } a \in Z\}$

d) None of the above

vi) *If* $f : R \rightarrow R$ *is defined by* $f(x) = + x^2-4$, *then the range of f is a subset of*

a) Z, the set of integers　　b) Q, the set of rationals

c) R^+ the set of positive reals　　d) R, the set of reals

vii) *If* $f(x) = e^{\sin^{-1}x}$ *then f(0) is*

a) 0　　b) 1

c) -1　　d) e

viii) $f(t) = \text{Cos } 2\pi kt$ *is a periodic function with period*

a) $1/2\ k$　　b) $1/k$

c) $1/2\pi k$　　d) $1/2\pi$

ix) Every even function on R^1 can be written as

a) the sum of an even function and an odd function.

b) the product of two even functions.

c) product of an even function and an odd function

d) the sum of two odd functions.

x) ^nC and ^np are related by

a) $^nC_r = r!\,^nP_r$ b) $^nP_r = r!\,^nC_r$

c) $^nP_r = r\,^nC_r$ d) $^nC_r = \,^nP_r$

xi) There are 10 points no three of which are collinear in a plane. The number of triangles with vertices at given points is

a) 10! b) $10P_3$

c) $10C_3$ d) None of the above

xii) The probability of getting the sum of the numbers shown up equals 10 when two die numbered 1 to 6 are tossed is

a) 1/12 b) 17/36

c) 1/9 d) 1/2

2. Let ABCD be a square in the first quadrant of xy-plane $x+y=1$ is the equation of the side AB, find the equation of the sides BC CD and DA.

3. Write the inequations whose intersection represents the square ABCD in the above problem.

4. A contractor has 6 six-ton lorries, 4 ten-ton lorries with 9 drivers available. He has contracted to move a minimum of 288 tons of cement from a cement factory to awarehouse daily. The six ton lorries can make 8 journeys and ten-tor lorries can make 6 journeys a day. How should the contractor organize the use of his lorries to run at a minimumcost of six ton lorries cost Rs.250 a day and ten ton lorries Rs.400 a day. Formulate this as a l.p.p. model.

5. Let R be a relation on Z, the set of all integers. Let a R, b R and (a,b) R mean "a divides b". Is R an equivalence relation? Why?

6. Give an example of a relation, which is symmetric, but neither reflexive nor transitive.

7. Let a/b, c/d belong to Q, the set of rationals. Let $(a/b, c/d) \in R$ if ad = bc. Show that R is an equivalence relation on Q. What are the equivalence classes of Q?

8. Sketch the graph of $f(x) = \begin{cases} x^2, & x < 0 \\ -1, & 0 < x < 2 \\ x, & x > 2 \end{cases}$
Specify the domain and the range.

9. Evaluate $\lim_{\theta \to 0} \frac{7 \sin^{-1} \theta}{5 \theta}$

10. Find three functions f, g and h such that (f o g o h) (x) = F(x0
where $F(x) = \frac{1}{|x| + 3}$

11. How many numbers between 2000 and 4000 can be formed using 0,1,2,3,4 if the digits are not to repeat?

12. If ${}^nP_4 = {}^nC_5 \times (4!)$, find n.

13. Describe asample space for the following experiments.
a) Drawing two balls from a bag containing 3 red, 2 green and 1 white balls.
b) Tossing a dice numbered 1 to 6 together with a coin.

REGIONAL COLLEGE OF EDUCATION, MYSORE 6

Summer Institute in Mathematics

II Test

1. i) If $p \to q$ is false and $q \vee r$ is true, which of the following statements is true?
   a) $(p \wedge r) \to q$
   b) $(p \vee r) \to q$
   c) $(q \wedge r) \to p$
   d) $(p \leftrightarrow r) \to q$

   ii) Which of the following statements is equivalent to $p \to q$?
   a) $\sim p \vee q$
   b) $p \vee q$
   c) $p \wedge q$
   d) $p \vee \sim q$

   iii) Which of the following is a tautology?
   a) $(p \wedge q) \to (p \wedge \sim q)$
   b) $(p \wedge q) \to (\sim p \wedge \sim q)$
   c) $(p \wedge q) \to (p \to \sim q)$
   d) $(p \wedge q) \to p$

   iv) If $P(AB) = 0$, then A and B are
   a) independent events
   b) any two events
   c) mutually exclusive events
   d) sure and impossible events

   v) $P(A+B) = P(A) + P(B)$ hold for
   a) any two events A and B
   b) any twoindependent events A and B.
   c) any two identical events only
   d) any two mutually exclusive events

2. List all the elements of the group of symmetries of a rectangle which is not a square.

3. Is the following argument valid? - "Because there is no real number greater than 1 and 2 is a real number,2 is not greater than 1". Give reasons.

4. Show that $f(x) = |x|$ is not differentiable at $x = 0$.

5. Show that the maximum rectangle inscribed in a circle is a square.

6. Prove that $\text{Sin} \frac{1}{x} = f(x),\ x \neq 0,$ and $= 0$, $x = 0$ is continuous for $x \neq 0$.

7. Using 0,2,4,5,6 a three digital number is formed at random. What is the chance that the number formed is divisible by 5?

8. Find the conditional probability $P(B/A)$ given $P(AB) = 0.25$ and $P(A) = 0.75$.

9. Two families F_1 and F_2 contain 2 boys + 1 girl and 1 boy + 2 girls. One child is picked at random from each family. What is the probability that of these one is a boy and the other is a girl?

REGIONAL COLLEGE OF EDUCATION, MYSORE 6

Summer Institute in Mathematics

Final Test

I. Tick (✓) the correct answer:

i) Which of the following sets is an ideal in the ring Z of all integers?

a) $\{ \ldots, -5, -2, 1, 4, 7, \ldots \}$

b) $\{ \ldots, -4, -1, 2, 5, 8, \ldots \}$

c) $\{ \ldots, -6, -3, 0, 3, 6, \ldots \}$

d) None of (a), (b) & (c).

ii) Which of the following is a ring but not a field?

a) $\{0, 2, 4, 6, 8, \ldots\}$

b) $\{2, 4, 6, 8, \ldots\}$

c) $\{0, \pm 2, \pm 4, \ldots\}$

d) $\{0, -2, -4, \ldots\}$

iii) Given the coplanar vectors, $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{c}$ can be expressed in the form $\vec{c} = m\vec{a} + n\vec{b}$, $m,n \in R$ if $\vec{a}$, $\vec{b}$ are

a) unit vectors

b) collinear vectors

c) non collinear vectors

d) any two vectors

iv) Which of the following vectors is a unit vector?

a) $\hat{i} + \hat{j}$

b) $1/2\,\hat{i} + 1/2\,\hat{j}$

c) $1/4\,\hat{i} + 1/4\,\hat{j}$

d) $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$

v) Which of the following is <u>not defined.</u>

a) $(\vec{a} \cdot \vec{b}) \times \vec{c}$

b) $(\vec{a} \times \vec{b}) \cdot \vec{c}$

c) $(\vec{a} \times \vec{b}) \times \vec{c}$

d) $(\vec{a} \cdot \vec{b}) \cdot \vec{c}$

vi) Given i) $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$ ii) $(\vec{a} \cdot \vec{b})\ (\vec{c} \cdot \vec{d})$ iii) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})$ and iv) $(\vec{a} \cdot \vec{b})\ (\vec{c} \times \vec{d})$, which of the statements given below is correct?

a) (i), (ii), (iii) are scalars and (iv) is a vector.

b) (i), (ii) are scalars and (iii), (iv) are vectors.

c) (i) is a scalar and (ii), (iii), (iv) are vectors.

d) (i), (ii), (iii), (iv) - are all vectors.

vii) p isa prime number if,

a) P is an integer with only factors 1 and P.

b) P is a real number such that its only factors are 1 and P.

c) P is an integer such that it has exactly two distinct factors.

d) P is a real number such that it has exactly two factors.

viii) The unique factorization theorem states that

a) every integer > 1 can be expressed uniquely as a product of distinct primes.

b) every natural number can be expressed uniquely as product of primes.

c) every integer > 1 can be expressed uniquely as a product of primes.

d) every integer can be expressed uniquely as product of distinct primes.

2. Prove : $|\vec{a} - \vec{b}| \geq |\vec{a}| - |\vec{b}|$.

3. Show that the position vector of the centroid of the triangle with vertices whose position vectors are $\vec{a}$, $\vec{b}$, $\vec{c}$ is $\dfrac{\vec{a} + \vec{b} + \vec{c}}{3}$

4. Show by vector method that in a rhombus the diagonals bisect at right angles each other.

5. Prove that an integer is divisible by 3 if the sum of its digits is divisible by 3.

6. If a and b are natural numbers such that $a^2 - b^2$ is a prime then show that $a^2 - b^2 = a + b$.

7. Prove or disprove: If a divides bc, then a divides b or a divides c.

8. Let f be defined on [-1, 1] by $f(x) = 1,\ 0 \leq x \leq 1$
$= 0,\ -1 \leq x < 0$
Does f have a primitive?

9. State the fundamental theorem of IntegralCalculus and illustrate it geometrically.

10. Express $\int_0^a x^2 dx$ as the limit of a sum. $a > 0$ (but finite)

11. List all the elements of Z/I, where Z is the ring of all ~~ideals~~ reals and $I = \{4x \mid x \in Z\}$

12. Are 7/15 and 7/20 expressible as terminating decimal numbers? Give reasons.

13. Prove that $\sqrt{3}$ is an irrational number.

APPENDIX 4

SUMMER INSTITUTE IN MATHEMATICS

Questionnaire for Participants

Name (optional) :

1. What, in your opinion, should be the objective/s of this Summer Institute?

2. Is the expected objectives, in your opinion, achieved in this Summer Institute?

3. List the topics done in the institute which, you should think, get more coverage.

4. Mention the additional topics, if any, that you wish to be included in the future summer institutes.

5. Which aspects of summer institute were useful to you? List them according to their usefulness.

6. Was/Were there any aspect/s of the Summer Institute that you particularly liked? Why did you like them?

7. Do you find the physical facilities provided in the summer institute satisfactory? What additional facilities do you suggest?

8. Did you face any difficulty regarding your deputation, T.A., D.A., etc.

9. Any specific suggestion for the future Summer Institutes.

APPENDIX 5

List of Participants who are attending the Summer Institute in Mathematics being held in the College from 11-30th July 1988:

1. Mr.K U Surendrappa,Lecturer
   G.Channappa P.U.College
   Anagodu, Davanagere Taluk

2. Mr H M Shivappa, Lecturer
   Govt. Mothi Veerappa P.U.College
   Davanagere

3. Mr H N Srikantaiah,Lecturer
   Govt P.U.College
   Chickamagalur

4. Mr. M T Ananthamurthy, Lecturer
   Govt. P.U. College
   K.R.Pet 571 426, Mandya District

5. Mr M S Venkatasubba Rao, Lecturer
   Government P.U. College
   Kanalawadi,Doddaballapur Taluk 561 203

6. Mr. N H Chincholi,Lecturer
   Govt. Junior College for Boys
   Bidar

7. Mr. S K Goudar, Lecturer
   R.M.G. Junior College
   Mudhol,Bijapur District

8. Miss.Indukala D. Kotnoor, Lecturer
   Government Junior College
   Aland, Gulbarga District

9. Mr.Sundararaja Iyengar, Lecturer
   Government P.U. College
   Adichunchanagiri, Mandya District

10. Mr.K Venkataramana Aithal,Lecturer
    National P.U.College
    Barkur 576 210 (D.K.)

11. Mr.R B Pujari, Lecturer
    Government P.U. College
    Nagamangala, Mandya District

12. Mr. Gururaj, Lecturer
    Government P.U.College
    Virajpet, CoorgDistrict

13. Mr.Goure Basavaraj,Lecturer
    Government P.U.College
    Ponnampet,Coorg District

14. Mr.V Mudlagiri, Lecturer
Government P.U.College
Nelamangala, Bangalore District

15. Mr. N G Shivanna, Lecturer
Government P.U.College
Amruthur, Tumkur District

16. Mrs.B N Uma, Lecturer
New Vani Vilas Junior College for Women
V.V. Puram, Bangalore 560004

17. Mrs. N L Swarna Kumari, Lecturer
Government P.U. College
H.D. Kote

18. Mr.K Upendra, Lecturer
Government P.U. College
Panchanahalli, Wadur Taluk
Chikkamagalur District