TRAINING PACKAGE FOR KRP's TO TRAIN TEACHERS IN CLASSROOM TRANSACTIONS IN MATHEMATICS FOR CLASSES VI TO IX

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Preface

Consequent to upgradation of the mathematics curriculum requires the development of skills and competencies of professional quality among teachers of mathematics. Although the recent curriculum reform efforts envision thoughtful and challenging instruction, most instructional practice is traditional with lecture like, formal recitation as the common teaching strategy. Teachers to succeed in enacting curriculum, require changes in their knowledge and beliefs about content pedagogy, learning and learners, as well as in their practices. This effort is not just a case of learning new strategies and/or techniques, but changing the overall perception of teaching and changing in their beliefs and attitudes that direct future practice. Professional development targeted at improving the content knowledge and pedagogical skills of teachers, are necessary for teaching and learning in schools.

Realising the importance of professional development of teachers, Karnataka State Education Authorities has initiated steps to organize training programmes for their teachers at secondary stage. Since the number of teachers to be trained in a face-to-face mode is too large, it is decided to follow a two-tier cascade model. In this model first the Key Resource Persons (KRPs) will be trained by RIE, Mysore and the KRPs will then train teachers within the state in a number of training programmes and in a number of cycles.

The training package for the teachers of Mathematics from Class VI to IX is prepared in the programme that has been organized in two phases, the first one from 27th to 29th September 2010. Sixteen teachers were deputed by the State Authorities. A simple test on pedagogy and content was conducted and then through discussions we tried to identify the difficult content areas. But later it is felt that a general approach in teaching and learning in mathematics was found necessary. Many of them were not aware of the different content areas and the strategies to transact them in a classroom.

So in the next phase of the programme which was conducted from 29th November 2010 to 3rd December 2010, in which seven teachers who were deputed by the Government were given training in categorizing the content in mathematics i.e. concepts, generalizations, facts and singular statements. Then analysis of a concept, the concept map, moves in teaching a concept, the components of unit plan, the ingredients of a lesson plan such as instructional objectives, teaching points, prerequisite knowledge, teaching aids, designing a learning experiences (activities), review and assignment were discussed . Many examples are being given in this training package.

Hope this package will help the KRP's to train teachers in their training programmes.

B Syam Prasad Raju
Programme Coordinator

Contents

Planning for Transacting Mathematics in a class room

Introduction

Every enterprise, a factory, an institution, a nation, a community needs proper planning for its growth and attainment of the desired goals. Plans for a teacher involve making tentative decisions regarding expectation for a given course and deciding how these expectations can best be accomplished. Plans are formulated for particular units of four or five sessions for each topic. The development of understanding and of competencies is possible through repeated opportunities to use the competencies in different situations and in a variety of ways. Plans for specific lessons involve objectives, strategies and the activities that can be altered or varied according to the levels of interest.

Carefully constructed plans by the teacher helps him/her in accomplishing the predesigned goals planning gives a direction and clarifies the thought. Planning helps teachers to becomes effective classroom leaders, since well-constructed plans facilitate efficient use of class time and instill confidence.
Now we will discuss about unit planning.

Unit Planning

Within the comprehensive scope of school curriculum, the individual teacher needs to chart a yearly course. Within this setting he/she decides, which parts of the year's work, if any, should be converted into units of learning.

Broadly speaking a unit is a division of a course. Unit includes all the materials that cover a topic. The division of a course into units helps to provide a framework for effective learning.

Definition of a unit plan:- A unit plan is a comprehensive series of meaningful learning experiences built around a central theme or idea and organized in such a way as to result in appropriate behavioural changes in pupils.

A unit plan may extend from a minimum of several days duration to one week or month or so depending on the content.

Characteristics of a unit:

(1) The unit must be unified.

(2) It should fit into content.

(3) It should be reasonably comprehensive.

(4) It should reflect the abilities and interests of the students.

(5) It should ascertain how to motivate the students.

(6) It should provide a variety of methods of teaching.

(7) It should contain problem solving activities.

(8) It should provide scope for evaluation.

(9) It should provide necessary resources.

Example of Unit Plan on Similarity

Introduction

On the other day in the morning by about 9 a.m. a few persons were measuring the shadows of a school building and a lamppost in front of the building. As I enquired, I was told that they were estimating the height of the building by knowing the lengths of the shadows of the building and the lamppost and the height of the lamppost. Thus to know the height of the building, one need not measure it directly. Often, those which cannot be directly measured, can be calculated. The height of a hill or light house, the distance of a boat from the foot of a light house or sea shore, the altitude of a kite above the ground level - are some instances. Some of these cannot be measured directly.

The Idea that Helps One in all Such Situations is Similarity

Figures which are of the same shape (but not necessarily of the same size) are called similar figures. Similarity is an important concept and a very useful one. A mathematician sees this as an important concept since he is able to recognize order and pattern both in nature and in the mathematical world of abstraction. An architect and an engineer find the idea of similarity useful because of the aesthetic value it adds to the construction, besides the contribution of the idea to the strength and life of the structure. Looking at the structure of petals of flowers, leaves and their spread, a botanist perceives that the idea of similarity is useful in classification. A layman who visits temples and monuments can appreciate the worthwhileness of the idea of similarity as it adds to the beauty and aesthetics of decorative art and sculpture. The relationship between such algebraic notions as ratio and proportion and similarity helps a mathematician to establish mathematically, results concerning similarity, which remain, otherwise, only an empirical and intuitive idea based on imagery. Such an interaction between algebraic processes and geometrical ideas helps one to apply the results and principles in new situations to seek solutions of geometric problems and vice versa.

In contrast to the traditional treatment of similarity, where emphasis is on memorization of definitions and results the stress here is on understanding of the concept and its spread. The unit deals with the basic concept and the related ones. The approach to the study of similarity is based on notions of Euclidean geometry.

I. Instructional Objectives

The pupils acquire

1. the **knowledge** of similarity - terms, concepts, principles etc.

   Specifications

   (a) **Defines** the terms, concepts, principles etc.

   (b) **Explains**, the terms, concepts, principles etc.

2. **Understanding** of similarity - terms, concepts, principles, processes etc.

   Specifications

(a)	Illustrates	(g)	Interprets
(b)	Compares and Contrasts	(h)	Classifies
(c)	Discriminates between	(i)	Verifies
(d)	Sees relationship between	(j)	Generalizes
(e)	Cites examples of	(k)	Analyses
(f)	Constructs examples of	(l)	Follows known processes

   Proofs - in respect of the terms, concepts, principles, processes etc.

3. **Apply** the knowledge of similarity to new situations -

   Specifications: On Completion of the Unit, Student will be able to

   (a) Locates the problems involving the concept of similarity.

   (b) Analyses the problems.

   (c) Checks the adequacy of the data

   (d) Selects appropriate method

   (e) Estimates the result.

   (f) Suggests alternate (or new) methods of solving problems

   (g) Solves problems (new ones)

   (h) Formulates hypotheses from the data

   (i) Finds new applications

4. **Constructional and computational skills - in using concept of similarity**

Specifications

(a) Uses geometrical instruments appropriately and efficiently.

(b) Works out numerical problems accurately and with reasonable speed.

(c) Detects errors in a procedure.

(d) Make accurate drawings, measurements etc.

II. Major Concept

Two figures (or objects) having the same shape (not necessarily the same size) are similar.

III. Content Analysis - Terms and Concepts

(i)	Parallel lines	(vii)	Equiangularity
(ii)	Concentric circles	(viii)	Centre of similitude
(iii)	Congruent triangles	(ix)	Line of symmetry
(iv)	Ratio and Proportion	(x)	Idea of a scale
(v)	Ratio of similitude	(xi)	Perspective
(vi)	One-one correspondence	(xii)	Projection

Results for Understanding

(1) A straight line can be divided in a given ratio internally or externally at one and only one point.

(2) A straight line drawn parallel to a side (base) of a triangle divides the other sides in the same ratio.

(3) In a triangle if a line divides two sides of a triangle in the same ratio then that line is parallel to the remaining side.

(4) The internal and external bisectors of an angle of a triangle divide the opposite side in the ratio of the other sides internally and externally.

(5) Converse of 4.

(6) In two similar rectilinear figures, the angles of one are equal to those of the other.

(7) In two similar rectilinear figures, the sides of one are proportional to those of the other.

(8) Two or more equiangular triangles are similar.

(9) Equiangular polygons (other than triangles) are not necessarily similar.

(10) Congruent figures are similar but not conversely. Congruence is a special case of similarity.

(11) Two triangles are similar, if an angle of one equals an angle of the other and the sides about equal angles are proportional.

(12) In a right triangle, the perpendicular from the right angled vertex to the hypotenuse, divides the triangle into two triangles which are similar to each other and to the original triangle.

(13) The areas of similar rectilinear figures are proportional to the squares on the corresponding sides.

(14) Similar figures may be divided into the same number of similar triangles.

(15) If two (rectilinear) figures have their (corresponding) sides parallel or perpendicular each to each, then they are similar.

(16) If similar figures are constructed on the sides of a right triangle, then the area of the figure on the hypotenuse equals the sum of the areas of those on the remaining sides. (generalization of Pythagoras theorem).

IV. Content Development and Organisation

Among the terms and concepts are some which the student is supposed to know before studying a unit. The remaining ones are the new ones. The former set of terms and concepts form the prerequisites for the unit and therefore need to be recalled (or revised).

These are : (i) **Parallelism**

Concept (a) **Two lines are parallel if perpendicular distance of any point on one from the other is the same.**

This suggests the figure 6.1.

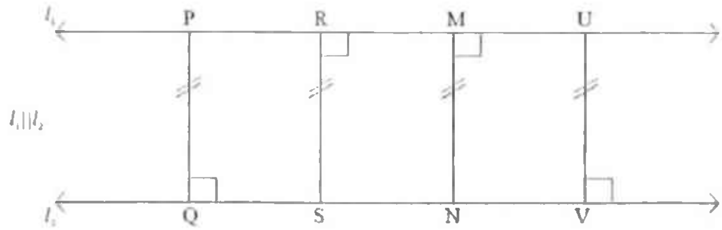


Fig. 6.1

(b) **Two flat surfaces are parallel** if the property in (a) is true here also.

Analogies (or Examples)

(i) The two rails of a railway track are parallel

(ii) The opposite edges of a door

(iii) The opposite (vertical) edges of a wall

(iv) The floor of a house is parallel to the flat roof

(v) Opposite walls of a house are parallel

(vi) In a rectangular box, the opposite edges are parallel and the opposite faces are parallel.

(ii) **Concentric Circles** are circles with the same centre.

Analogies (i) The outer and inner rims of a wheel are concentric circules (ii) The circular waves spreading out from a point in a pond are concentric circles.

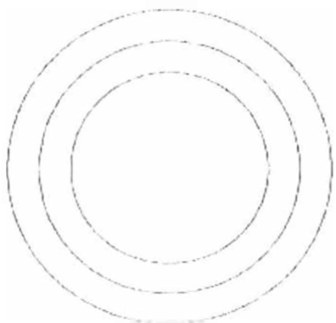


Fig. 6.2

(iii) **Congruent triangles** are the triangles in which the sides of one are equal to those of the other (and the angles of one are equal to those of the other).

Triangles are congruent under the following conditions:

(a) Two sides and the included angle of one are equal to two sides and the included angle of the other (SAS)

(b) Three sides of one are equal to those of the other (SSS)

(c) Two angles and a side of one are equal to two angles and a side of the other (AAS)

(d) If the triangles are right triangles, then they are congruent when the hypotenuse and a side of one are equal to those of the other.

(iv) **Ratio and Proportion: Ratio** is the quotient of the measures of two quantities measured in the **same** units. If the measures are a and b units then the ratio is written as a/b. a and b are respectively called the antecedent and consequent of the ratio a/b. If two ratios a/b and c/d are equal then we write $a/b = c/d$ or $a:b::c:d$ (read as a **is to** b as c **is to** d). Then a,b,c,d are said to be in proportion.

If $a/b = b/c$ a,b,c are said to be in continued proportion and b is called the **mean proportion** between a and c given by

$b2 = ac$ or $b = ac$

If AB is a line segment and P is a point on it (or extended portion) such that $AP^2=AB.\ BP$, P is said to divide AB in **extreme and mean** ratio.

(v) Division of a straight line segment in a given ratio.

Fig. 6.3

As in the above figure.

(a) If P is such that $\frac{AP}{PB}=\frac{a}{b}$ P is said to divide AB in the ratio a/b **internally**

(b) If Q is such that $\frac{AQ}{QB}=\frac{a}{b}$ b, Q is said to divide AB in the ratio a/b **externally**.

If the ratio is 1:1, AB **cannot be** divided in this ratio externally.

Development of New Concepts

Two figures having the same shape are said to be **Similar**

Any two **circles** are similar.

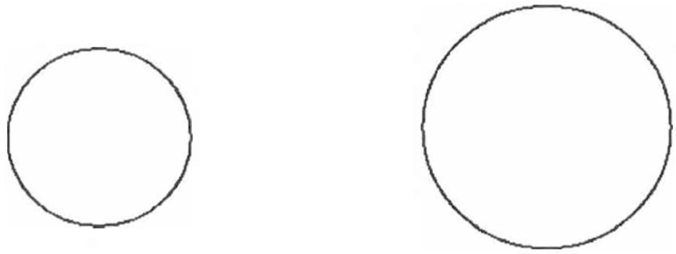


Fig. 6.4

Any two **squares** are similar.

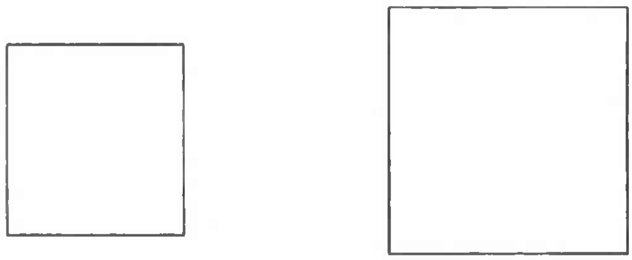


Fig. 6.5

Any two **equilateral triangles** are similar.

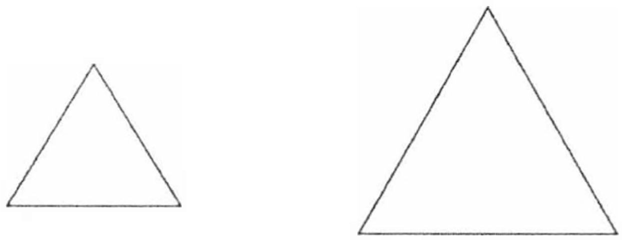


Fig. 6.6

Any two **line segments** are similar.

Fig. 6.7

Analogies and Examples

(a) Two different size photograph of the same object

(b) Any two maps of the same country, drawn on different scales

(c) An object and its shadow

(d) A picture and its projected image on a screen

1. **Similarity of Polygons**

(a) **Conditions for similarity**

(i) corresponding angles must be equal.

(ii) The corresponding sides of one must be proportional to those of the other.

(b) In the case of similar triangles, any one of the two conditions mentioned above is sufficient.

2. **One-one Correspondence**

When two similar triangles are considered, the pair of sides opposite to equal angles are described as **corresponding sides**. In turn, the pair of equal angles are **corresponding angles**.

These ideas are expressed as-

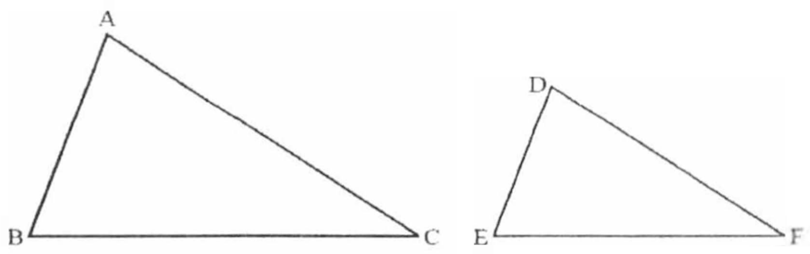


Fig. 6.8

The way in which the sides (or angles) of one triangle are associated with the sides (or angles) of another in a one-one manner is called a **one-one correspondence**. More generally, in case of two similar polygons ABCDEF... and PQRSTU... with pairs of sides $a,p;$ $b,q;$ $,c,r;$ etc. The one-one correspondences of sides is given by-

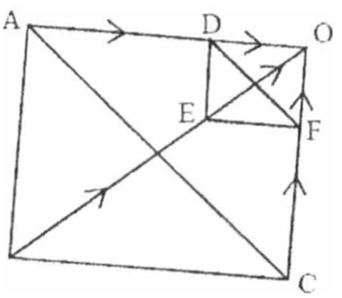
$a \leftrightarrow p$; $b \leftrightarrow q$; $c \leftrightarrow r$; $d \leftrightarrow s$; etc.

3. **Equiangularity**

This concept is implicit in the foregoing discussions and definition of similarity. Two similar figures (rectilinear) must necessarily be equiangular. In the case of similar triangles, the condition of equiangularity (the fact that the angles of one are equal to those of the other) is necessary as well as sufficient.

4. **Centre of similitude (or similarity) or Homothetic Centre**

Whenever similar rectilinear figures have their corresponding sides parallel if the corresponding vertices of figures are joined, the line joining the vertices of similar rectilinear and are concurrent at a point called the centre of similarity of the figures.



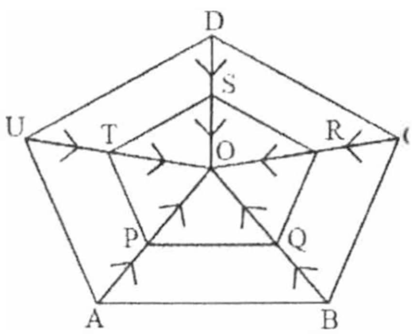


Fig. 6.9

O is the centre of similarity of the figures.

5. **Perspective**: Whenever similar figures are such that the lines joining their corresponding vertices meet at a point (the centre of similarity), the figures are said to be in **perspective**. For two figures to be in perspective, the centre of similarity must exist.

6. **Idea of Scale**: You know that, for a map of a country or the sketch of a figures to be a true representative must be similar to the original. This happens when the ratio of distance on the map to the actual distance is constant (which distance is the constant of proportionality), this ratio is called the **scale**. If a distance of 40 metre is represented by a length of 5cm, then the

scale is $\frac{5}{40\times100} = \frac{1}{800}$ called the **representative fraction**.

7. **Projection**: When an object is in front of a source of light, the shadow of the object is called the **projection** of the object.

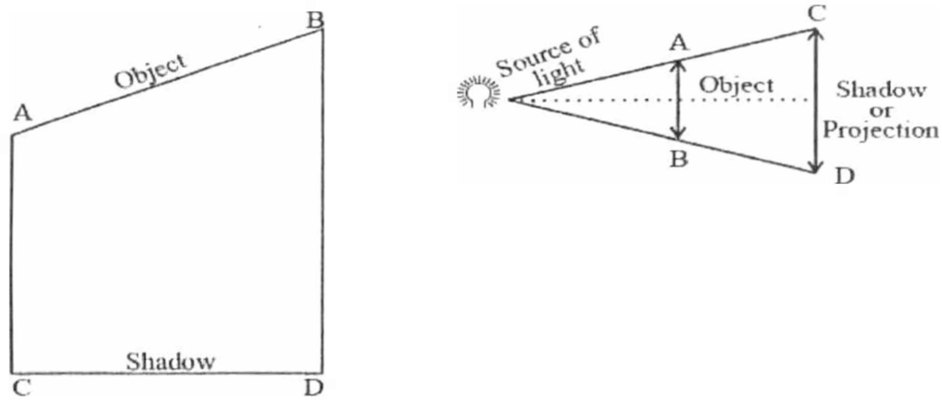


Fig. 6.10

8. When an object is held before a mirror, the image or reflection of the object is similar to the object. If the mirror is plane, the object and the reflection are congruent. The line representing the mirror is the **line of symmetry** as the object and the reflection are symmetrical.

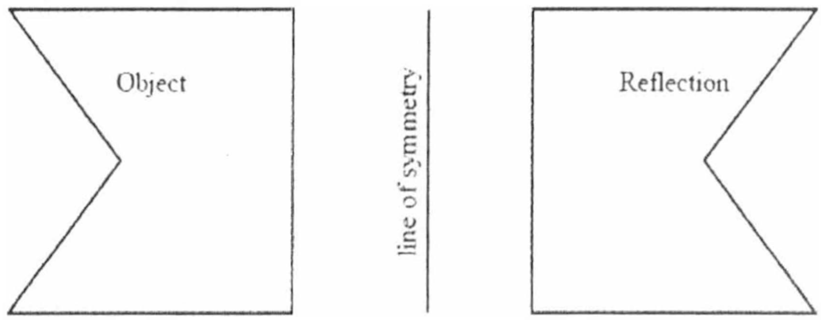


Fig. 6.11

V. Learning Experiences: (Activities and Techniques)

1. Recalling a number of analogies and examples of objects and situations involving the idea of similarity. Eliciting more such ones from the pupils.
2. Explaining the usefulness of the idea and asking the pupils to narrate or list some more uses of the idea of similarity.
3. Asking them to reason out their inferences.
4. Giving practice in drawing similar figures and encouraging them to make models of similar figures and objects.
5. Let them identify one-one correspondence between the components of similar figures.

6. Asking them questions regarding-
   i) Conditions under which two figures are similar.
   ii) Conditions when similar figures are congruent
   iii) Construction of similar figures.
7. Letting them to construct similar figures, centre of similarity.
8. Asking them to determine whether the figures are in perspective.
9. Asking them to reason out why two given figures are **not** similar.
10. Encouraging them to know more about similarity and the applications of the idea.
11. Helping them to construct alternate proofs to known results.
12. Helping them to solve (a) new riders (b) new problems and (c) problems in new and practical situations, involving the notion of similarity.
13. Allowing them to classify objects and figures on the basis of similarity and examine similar figures to know their related properties.
14. Encouraging them to construct and improvise models of objects/figures which are similar and which display the properties of similar figures.
15. Providing them challenges by way of tests, puzzles, problems, brain busters involving the idea of similarity.
   (a) Some of the activities can be spread over a period of time.
   (b) Can be conducted both inside and outside the classroom.
   (c) Can be conducted as group activity for different groups which are based on homogeneous grouping.

Teaching Hints: The concept of similarity may be introduced in a variety of ways - through (a) analogies, (b) examples, (c) situations (d) problems (e) practical problems (f) by appealing to the pupil's aesthetic sense etc., The advantages of such an introduction are many. The one important advantage is that it provides the necessary motivation. The pupils know how far the idea is worthwhile, useful, from how many angles, the concept can be studied, the wide ramification and depth of the idea. All this gives them a sense of satisfaction and the healthy impression that whatever they are learning is meaningful and worth learning.

The accent in teaching as for the methods and techniques must be on learning by doing and not teaching by talking or teaching by doing all by the teacher himself. The idea of similarity gives a wide scope for individual and group participation.

Evaluation at convenient intervals, at the end of units/sub units is very useful. It builds confidence in the good students, tells the less capable their deficiencies and provides feed back to the teacher who can alter his techniques.

VI. Theorems and Results

1. A line segment can be divided in a given ratio internally/ externally at one and only one point.

2. If a pair of transversals cut a set of parallel lines, the intercepts on the pair are proportional

i.e. $\frac{PQ}{P'Q'} = \frac{QR}{Q'R'} = \frac{RS}{R'S'}$ =Etc.

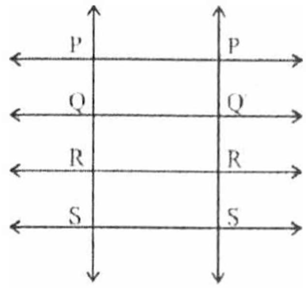


Fig. 6.12

3. A line parallel to side of a triangle divides the other sides in the same ratio.

i.e. $\frac{\overline{AX}}{\overline{XB}} = \frac{\overline{AY}}{\overline{YC}}$

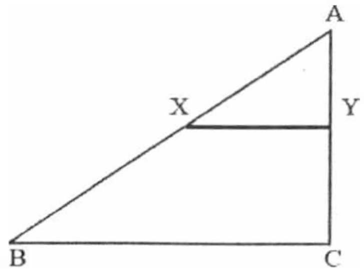


Fig. 6.13

4. The internal/external bisector of an angle of a triangle divides the side opposite to the angle internally/externally in the ratio of the other sides of the triangle.

i.e. $\frac{AB}{AC} = \frac{BP}{PC}$ and $\frac{AB}{AC} = \frac{BQ}{QC}$

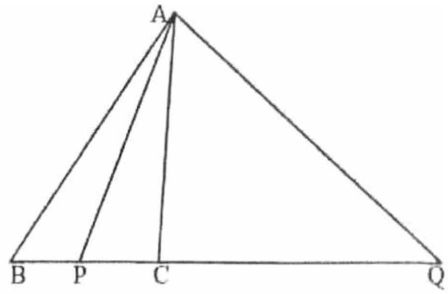


Fig. 6.14

5. Two triangles are similar (i) if they are equiangular, or (ii) if the sides of one are proportional to those of the other, or (iii) if an angle of one equals an angle of the other and the sides about equal angles are proportional.

6. If a perpendicular is drawn from the right angled vertex of a right triangle to the hypotenuse, the two triangles got are similar and similar to the given triangle.

7. The areas of similar triangles are proportional to the squares of the corresponding sides.

8. The perimeters of similar triangles are proportional to the sides of the triangles.

9. The perimeters of similar figures are proportional to the sides and the areas of the figures are proportional to the squares on the sides.

10. Similar figures with parallel corresponding sides are in perspective i.e. the lines joining corresponding vertices are concurrent (at the point called the **Centre of similarity**).

11. If similar figures are constructed on the sides of a right triangle, then the area of the figure on the hypotenuse equals the sum of the areas of the figures on the other sides.

12. The bisectors of the base angles of a triangle meet the opposite sides at X and Y. If XY is parallel to the base, the triangle is isosceles.

13. AD is a median of ΔABC and the bisectors of the angles ADB and ADC meet at E and F. EF is parallel to BC.

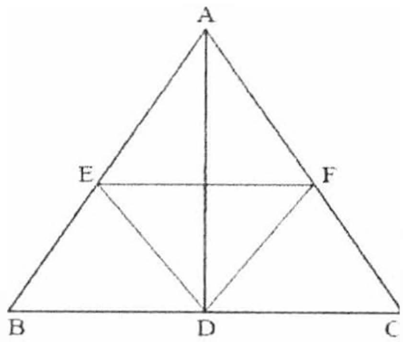


Fig. 6.15

14. The bisectors of the angles of A, B and C of ΔABC meet the sides BC, CA and AB at D,E,F respectively. Then BD.CE.AF = DC.EA.FB.

15. In the quadrilateral ABCD, BC=DC. The bisectors of the angles ACB and ACD meet AB and AD at E and F respectively. Then EF || BD Fig 6.16.

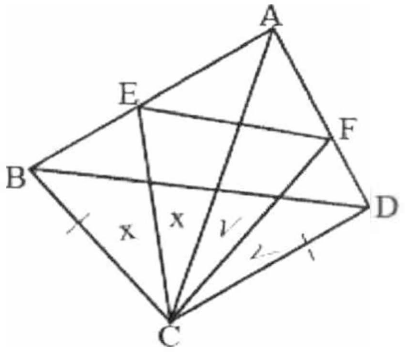


Fig. 6.16

16. If the sides of a triangle are parallel to the sides of another triangle respectively, then the triangles are similar Fig. 6.17.

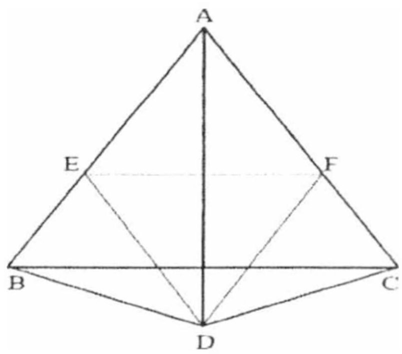


Fig.6.17

VII. **Evaluation** (Sample test items)

1. Which conditions(s) for similarity is not satisfied for the following pair of figures?

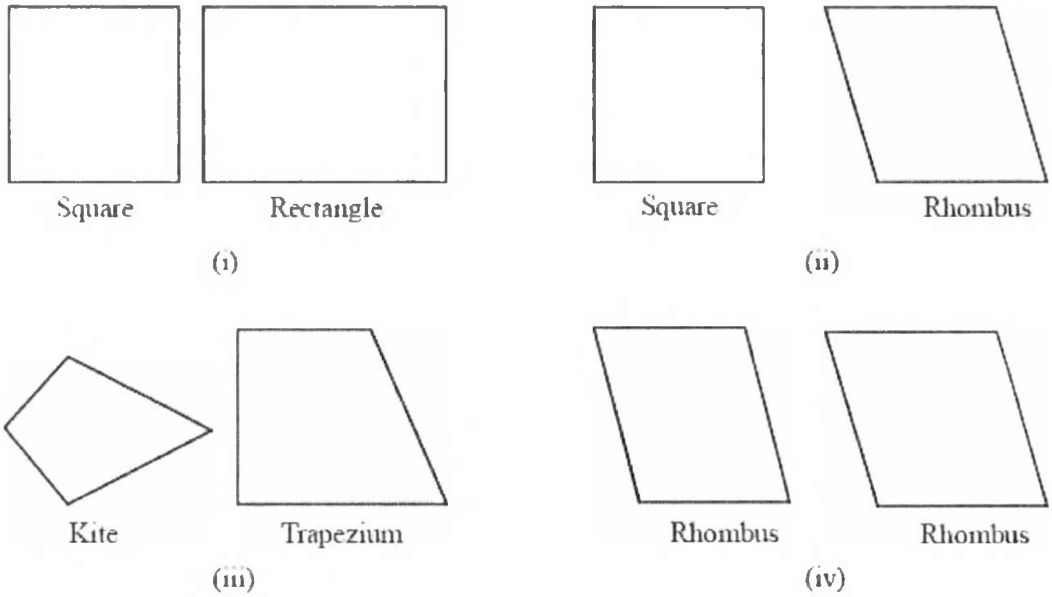


Fig. 6.18

2. Which of the classes are of similar figures/objects?

(i)	Squares	(ii)	Parallelograms
(iii)	Equilateral triangles	(iv)	Isosceles triangles
(v)	Rectangles	(vi)	Circles
(vii)	Rhombuses	(viii)	Kites

3. Given a triangle, is it possible to divide it into a number of similar triangles? If so how can you do it.

4. If you are given a loaf of bread in the shape of a cube describe how you would slice all of them into similar triangular pieces.

5. P divides AB = 10cm in the ratio 5:2. Find AP if P divides AB (a) internally (b) externally.

6. In the triangle ABC, AB = 6cm, AC = 4cm. If AD divides BC in the ratio 3:2, then which of the statements is true?

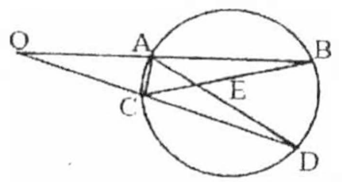


Fig. 6.19

(i) AD is a median
(ii) AD is an altitude
(iii) AD is an angular bisector, of the angle BAC.

7. In the figure 6.20, write down the pairs of similar triangles.

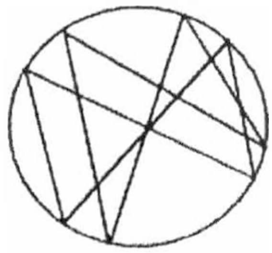


Fig. 6.20

8. The figure 6.21, which statement below holds?

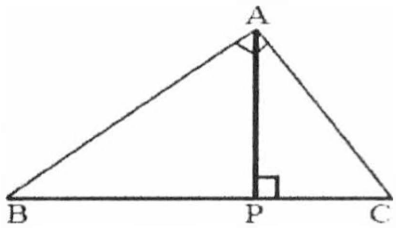


Fig. 6.21

(1) PA.PB = PC2
(2) PB.PC = PA2
(3) PA.PC = PB2

9. How do you inscribe a square in a given triangle?

10. Which statement is always valid?

(i) When the vertices of similar figures are joined in pairs, the lines joining them are concurrent.

(iii) When the corresponding vertices of similar figures are joined, the lines joining them are concurrent.

(iii) When the corresponding vertices of similar figures with corresponding sides parallel, are joined, the lines joining are concurrent.

12. Describe **five** situations in life, where similarity idea comes into play.

Evalution Items

1. Explain the purpose of introduction in unit planning.
2. What is the role of content analysis in lesson/unit planning.
3. How does a unit plan differ from the presentation and development of subject matter in a text book.
4. Discuss the role of evaluation in unit planning. Prepare a unit plan on one unit each in (i) Arithmetic (ii) Algebra (iii) Geometry.

Now we will discuss about Lesson Plan

Need for Planning

Educators have always agreed on the need for an intelligent planning of every lesson. To plan is to act with a purpose; a plan is a blue print which helps in the efficient economical and for smooth conduct of any activity. If teaching is to be effective in terms of learning by students, it is necessary to ensure this through careful advance planning which would involve visualising the entire teaching-learning situation as it is likely to develop in the classroom. Every teacher has before him/her some very specific purposes in teaching a topic of a unit. He is conscious to achieve these purposes during the course of the lesson. He needs to think about the best possible manner in which he can realise these purposes with the maximum efficiency and the minimum waste of available resources. Whereas this applied to all teachers, the need for such advance planning is all the greater for the beginning teachers. This is because the newness of teaching, subject matter, students and the school. Careful planning help develop the teacher's mathematical competence that enhances the confidence of the teacher in front of the students, and eventually provide the context for developing creative talent in teaching.

Planning includes right from the selection of the content, structuring the content, sequencing the content so as to make the lesson easy to follow, making the lesson easy to understand, thinking for finally the ways to make the students to remember, and identify the important things that are worth remembering, ways to develop reasoning abilities among the students.

Advantages of Planning

(1) Planning clarifies thought
(2) Planning infuses confidence
(3) Planning gives a direction and makes smooth sailing through entire period
(4) Planning makes the teacher to show himself to the students that a teacher is well organised, authority figure in learning.

Now we consider some basic elements in lesson planning.

Content Categories in Mathematics

Content in mathematics exists in three primary forms- **facts, concepts** and **generalisations**.

Fact : (Singular statements):

Fact is defined as the type of content which is singular in occurrance, which has occurred in the past or exists in the present, but has no predictive value. Facts are acquired solely through the process of observation.

Example: (1) π is an irrational number
(2) The base of the common logarithmic function is 10.

Concept : Concept is defined as the type of content which results from the categorisation of a number of observations.
Or

a decision rule which when applied to the description of an object, specifies whether or not a name can be applied.

Example: (1) triangle
(2) prime number

Generalisation: It is an inferential statement which expresses a relation between two or more concepts, applies to more than one event and has predictive and explanatory value.

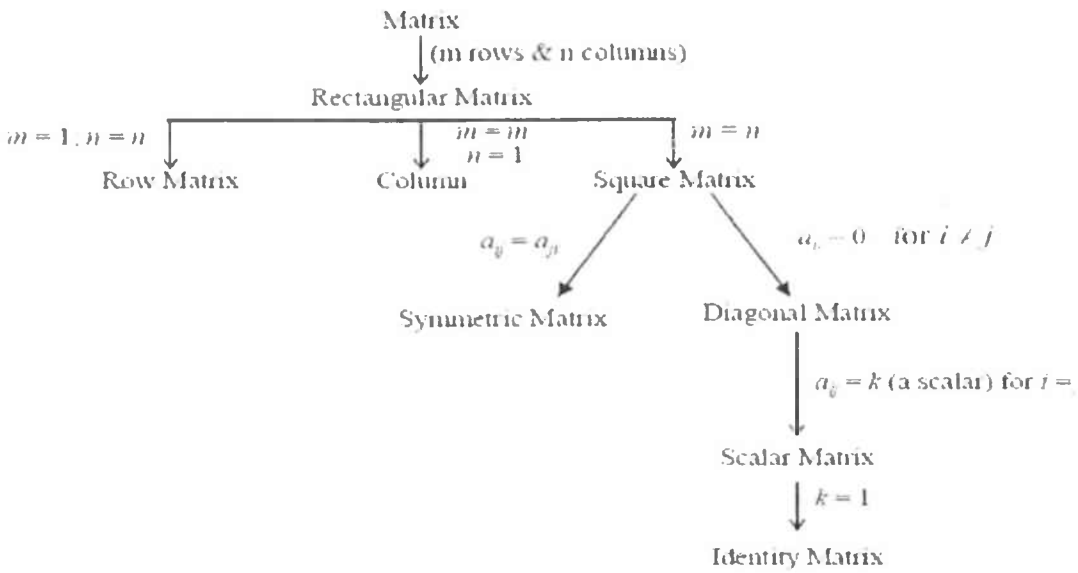
Example: (1) The diagonals of a rectangle bisect each other.
(2) The slope of a linear function is constant.

Selection of Content

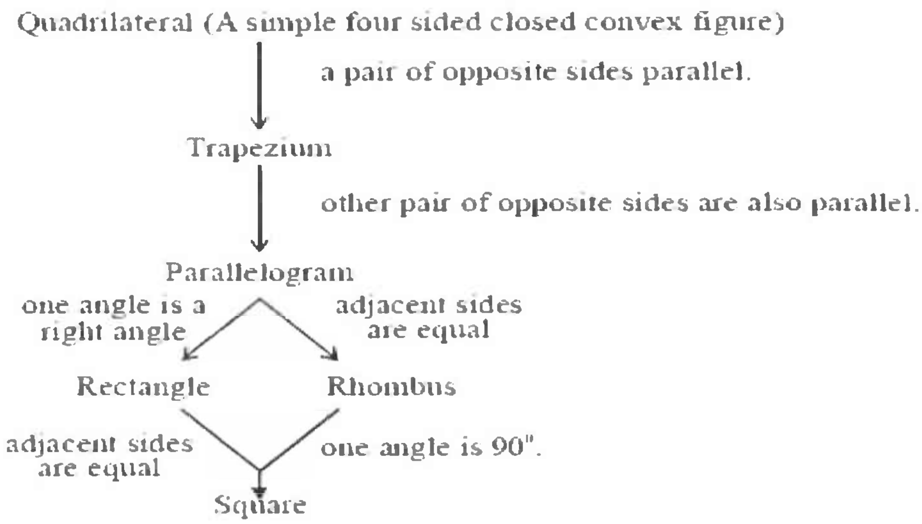
Selecting the content for a lesson involves a number of considerations. The selection of the content should be related to the overall programme of study for pupils. The first and foremost task of the teacher is to separate a topic into distinct elements or aspects like concepts, generalisations facts & prescriptions and to design a sequence or progression in a hierarchical way through which these elements that makes coherent and intellectual sense that effectively facilitates learning and their retention. He has to discriminate between essential and unimportant matters within the subject, to know where to place emphasis and where to anticipate difficulties, their occurrence occur, their precise nature, and know how to help students to avoid or overcome them after selecting the content.

Concept Analysis

Concept Map of the Concept "Matrix"



Concept Map of the Concept "Quadrilateral"



Here we give concept analysis of a few concepts.

Example 1

Concept Name : Perpendicular Bisector

Concept Definition : Perpendicular Bisector of a line segment is a line that divides the line segment at right angles into two equal parts.

Essential Attributes

1. The line intersects the given line segment at right angles.
2. The line divides the given line segment into two equal parts.

Non-Essential Attributes

1. The length of the line segment.
2. Configuration of the line segment.

Examples :

1.

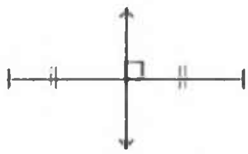


Fig. 6.22

2.

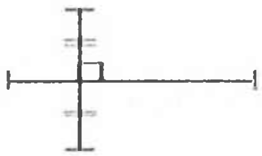


Fig. 6.23

3.

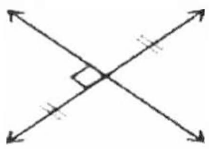


Fig. 6.24

4. Alphabet T

5. Vertical bar in a two pane window.

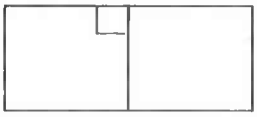


Fig. 6.25

6. Simple balance at rest.

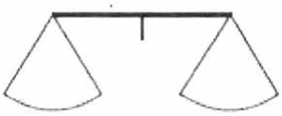


Fig. 6.26

7. CD is perpendicular bisector of AB in an Isosceles Triangle in the figure.

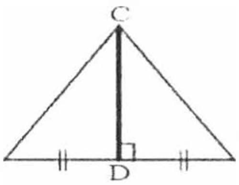


Fig. 6.27

8. Perpendicular bisector of a chord passes through the center of the circle.

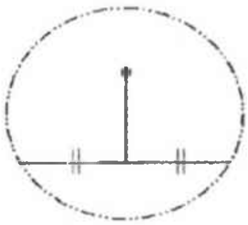


Fig. 6.28

Non-Examples

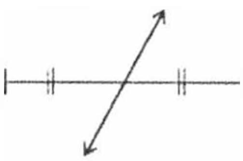
1. 

Fig. 6.29

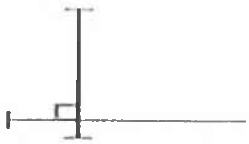
2. 

Fig. 6.30

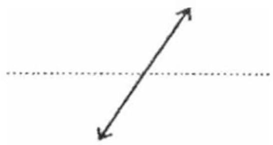
3. 

Fig. 6.31

4. Vertical post of a football goal.

Fig. 6.32

5. Vertical post in a door.

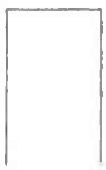


Fig. 6.33

6. Radius of a circle is perpendicular to the tangent.

Fig. 6.34

Super ordinate Concept : **Line**

Co-ordinate Concept : **Transversal**

Note : Perpendicular bisectors shown in the figures are line segments contained in the respective lines.

Example 2

Concept Name : Identity Matrix

Concept Definition

Identity matrix is a square matrix with principle diagonal elements unity (1) and all other elements zero.

Essential attributes

(i) It is a square matrix.
(ii) Each of the main diagonal elements equals to 1.
(iii) Each non diagonal elements equal to zero.

Non essential attributes

order of the square matrix.

Examples

(i) $$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

(ii) $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(iii) $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Non examples

(i) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}$ It is not a square matrix (Matrix but not an identity matrix).

(ii) $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ Non-diagonal elements are not zero (Square matrix but not identity matrix).

(iii) $\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$ Main diagonal elements are not unity (Diagonal matrix but not an identity matrix).

(iv) (a) $\begin{bmatrix} 0 & 5 \\ 2 & 0 \end{bmatrix}$ Main diagonal elements are not unity and all other elements are non zero.

(b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ Main diagonal elements are not unity & non diagonal elements are non zero.

Super ordinate concept : Square matrix

Example 3

Concept Name: Latus Rectum

Concept Definition: A **latus rectum** of a parabola is a **line segment** which is perpendicular to the axis of the parabola, that passes through the focus and whose end points lie on the parabola.

Essential Attributes

i) It passes through the focus.
ii) It is perpendicular to the axis of the parabola.
iii) Its end points lie on the parabola.

Non-Essential Attributes: Orientation and opening of the parabola.

Examples

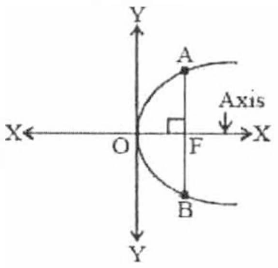


Fig. 6.35

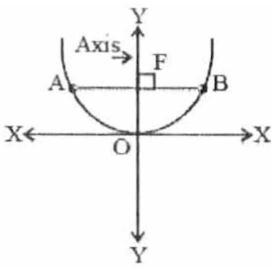


Fig. 6.36

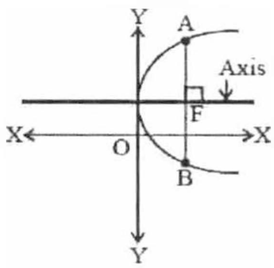


Fig. 6.37

Non-examples:

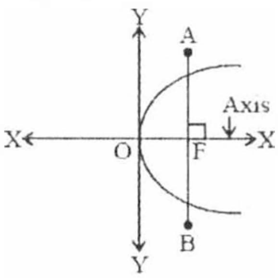


Fig. 6.38

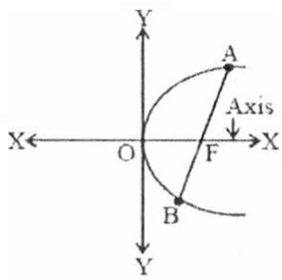


Fig. 6.39

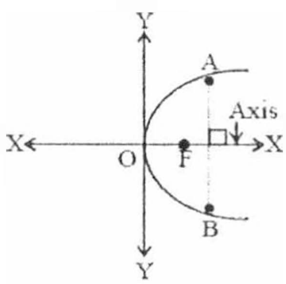


Fig. 6.40

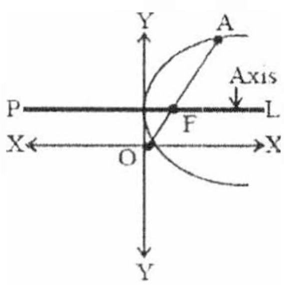


Fig. 6.41

Superordinate Concept : Line Segment.

Example 4

Concept Name : Symmetric Matrix

Concept Definition : A **symmetric matrix** is a square matrix such that the **element in the *(i, j)* element is equal to *(j, i)*th element for every *i* and *j*.**

Essential Attributes : (i, j)th element must be equal to (j, i)th element.

Non-essential Attributes : Order of the square matrix.

Example

1. $\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$

2. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -3 \\ 1 & -3 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 0 & 2 & 3 \\ 2 & 0 & -1 \\ 3 & -1 & 0 \end{bmatrix}$

Non-example : $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 5 \\ 3 & 1 & 0 \end{bmatrix}$

Super ordinate concept: Square Matrix

Example 5

Concept Name : Equivalence Relation

Concept Definition : An **Equivalence Relation** on a set is a **binary relation** which is Reflexive, Symmetric and Transitive.

Essential Attributes : Reflexive Property, Symmetric Property and Transitive Property.

Non-Essential Attributes : Type of set is non-essential and type of relation is non essential.

Examples :

1. The relation "equal to =" in Q set R is an equivalence relation.
2. The binary relation "is congruent to" in the set of all triangles.
3. The binary relation "is similar to" in the set of all triangles.
4. The binary relation "has same area as" in the set of all triangles.
5. The binary relation "has same perimeter as" in the set of all triangles.
6. A binary relation "is parallel to" in the set of all lines in a plane.

Non-Examples

1. A binary relation "is a subset of" in a set of sets.
2. "is less than" in the set of all rational numbers.
3. "is a factor of" in the set of all natural numbers.
4. "is perpendicular to" in the set of all lines in a plane.
5. "> (greater than)" in the set of all integers.

Conceptual Hierarchy

1. Super ordinate Concepts – Binary Relation.
2. Coordinate Concepts – Anti-symmetric relation, inverse relation.

Write the concept analysis of the following mathematical objects:

(a) median (b) adjacent angles (c) function

(d) power set (e) singular matrix (f) right angled triangle

(g) linear pair of angles

TEACHING MATHEMATICAL CONCEPTS

The study of mathematics deals with certain objects such as Natural numbers, Circles, Triangles, Functions and Proof.

In learning about these mathematical objects, we are concerned with what these objects are. For example

1. What an angle is how to call whether or not something is a rectangle, what the definition of a parallelogram ?

2. What are the relations among mathematical objects ?

When we teach students what an object is, and how to identify it, we are teaching a concept of that object.

Concepts are the most basic learnable objects and first things learned by young children.

By means of concepts, other concepts and other kinds of subject matter are learned.

A concept is the meaning of a term used to designate the concept.

According to Hunt, Marine and Stone (1966), "A concept is a decision rule which, when applied to the description of an object, specifies whether or not a name can be applied. Thus a student who knows the definition of a circle as the locus of points in a plane from a given point in the plane has a rule that can be used to tell whether any given object is to be called a circle.

Moves in Teaching a Concept

Some concepts are taught, for other the term designating the concepts are used.

For example, a teacher who has deliberately taught a concept of a finite set might not teach a concept of an infinite set but would simply use the term.

1. **Defining**

   Because most concepts in mathematics are precise, definitional moves can be used.

   Definitions are often written in the form <u>(1)</u> is a <u>(2)</u> such that <u>(3)</u>

The first space is filled by the term being defined, the second space is filled by a term denoting a superset in which the set of objects denoted by the term defined are included and third space is filled by one or more conditions that differentiate the set of objects denoted by the term defined from all the other subsets of the superset.

2. **Stating a sufficient condition or sufficient condition move**

It is the form in which a characteristic or a property of an object is stated that identifies it as a sufficient condition.

A rhombus is an equilateral parallelogram. Being an equilateral parallelogram is sufficient for being rhombus.

The sufficient condition is clearer in the statements:

"If a quadrilateral is an equilateral parallelogram, it is a rhombus".

Other forms are "If a parallelogram is a square, it is a rhombus"

A triangle is a right-angled triangle provided that it has one right angle.

The logic of the move of sufficient condition enables a student to find examples of objects denoted by a concept, assuming such an example exists.

3. **Giving one or more examples**

Examples are objects denoted by the concept i.e. members of the set determined by the concept.

Examples clarify concepts because they are definite, specific and if well chosen familiar.

Teachers frequently elicit examples of concepts from students to decide whether the students have acquired the concepts.

Examples cannot be given for every concept. For example, even prime number greater that 2, greatest integer, and for self-contradictory concepts like square, circle, six-sided pentagon.

4. **Giving an example accompanied by a reason why it is an example.**

Accompanying an example with a reason that it is an example is an effective move because the reason is a sufficient condition.

This move is helpful to slow learners, because the logical connection is made explicit by supplying a reason.

5. **Comparing and contrasting objects denoted by the concept.**

By comparing objects of the concept being taught with objects with which students are familiar, a bond of association can be established between familiar and less familiar.

In teaching a concept of parallelogram, the teacher may compare it with non-parallelogram (trapezium).

Comparison points out similarities. But since objects compared are not identical, a contrast identifies some of the differences, if not all.

If a teacher has taught a concept of equal set and then teaches a concept of equivalent set, the next step may be to contrast these two concepts in order that the students do not miss the distinction between them.

6. **Giving a counter example**

A counter example is an example that disproves a false definition of a concept.

Two kinds of counter examples are possible for an incorrect definition.

i. Give a member (an example) of the set determined by the term defined that is not a member of the set determined by the defining expression.

ii. Give a member (an example) of the set determined by the defining expression that is not a member of the set determined by the term defined.

Though this kind of move is effective in sustaining and ultimately facilitates comprehension of the desired concept, students may feel that the teacher was badgering and embarrassing them. Teachers have to exercise good judgement when deciding how frequently to use counter example moves.

7. **Stating a necessary condition**

If two sides are parallel, a quadrilateral is a parallelogram. This statement indicates the absence of a necessary condition for a quadrilateral to be a parallelogram.

One form of the definition of a parallelogram to satisfy the necessary condition is,

If both pairs of opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.

Another form in which a necessary condition is stated uses *only if*.

Example, A quadrilateral is a parallelogram *only if* both pairs of sides are parallel.

A necessary condition move enables a student to identify examples of objects *not* denoted by a concept.

8. **Stating a necessary and sufficient condition**

This move is used, if condition by which objects can be denoted by a concept is both necessary and sufficient condition. One form for this is the explicit use of terms necessary and sufficient, as

It is both *necessary* and *sufficient* that a parallelogram be equilateral and equiangular for it to be a rhombus. Another form is the use of *if* and *only if*. Thus the statement is equivalent to,

A parallelogram is a rhombus *if* and *only if* it is equilateral.

The definition in terms of necessary and sufficient condition proceeds by subsuming the set of objects to be defined from all other subsets of the superset. Thus, a definition of a rhombus might be

A parallelogram having pair of adjacent, congruent is a rhombus.

The definition implies that there are two conditions necessary for an object to be a rhombus.
(i) being a parallelogram and (ii) having a pair of adjacent congruent sides. The combination of these two necessary condition is sufficient. But for some students, the necessary and sufficient conditions in the above statement may not be clear. For them, the teacher can make use of *if* and *only if* form.

A sufficient condition move enables a student to identify examples and a necessary condition move enables students to identify non-examples of a concept. A combination of these enables students to discriminate both examples and non-examples of a concept.

An object not in the set determined by a concept is a non-example of the concept.

9. **Giving non-examples**

Like the move of giving examples, giving non-examples helps to clarify a concept. Definition of a concept followed by examples and non-examples of the concept is a common move for a teacher.

10. **Giving a non-example accompanied by a reason why it is a non-example**

This move is similar to that of giving an example together with a reason that is an example. The reason that accompanies the non-example is the failure to satisfy a necessary condition.

Its logic is that of conditional reasoning.

"If a quadrilateral is not a parallelogram, it is not a rhombus. This quadrilateral is not a parallelogram. Therefore it is not a rhombus".

Strategies of Teaching a concept

A *strategy* is defined as a temporal sequence of moves.

So, theoretically, there are thousands of strategies for teaching a concept, of which some are logically impossible.

Examples of some strategies of teaching a concept

1. Definition----- Example-----Example with a reason---- Non-example with a reason

2. Example-----Non-example-----Comparison and contrast-----Characteristics-----Definition-----Example with a reason----Non-example with a reason

In such strategies, the definition identifies the necessary and sufficient conditions, examples clarify them and reasons reinforce necessary and sufficient.

Uses of Concepts

1. Knowledge of a concept helps in classifying given objects into examples and non-examples of the concept
Since we can classify, we can discriminate. For example, a student who has concept of rhombus can pick out rhombus from other quadrilaterals.

2. Knowledge of concepts helps in communication. Communication breaks when people do not have the knowledge of certain concepts.

   A definition of a term tells you both how to use the term and also how to avoid using it.
   Example, A rhombus is an equilateral parallelogram.

   This definition tells that a rhombus means, "an equilateral parallelogram" and if the students do not have the concept of an equilateral parallelogram, the teacher can think of the definition. An equilateral parallelogram is a four-sided figure whose sides are line segments having the same length.

3. Concepts helps in Generalization.

4. Concepts help in discovery of new knowledge.

Elements of lesson Plan

There is not a unique way to preparing of a lesson plan. One could think of a variety of formats to suit different situations. It is not very important that a particular form for drawing up a lesson plan should be ritualistically followed. What is important is to recognize that a lesson plan has a distinct purpose in relation to good teaching that is to follow it. A plan must help to clarify to the teacher the specific learning outcomes in pupils, in relation to the topic and indicate how these are proposed to be realized and evaluated.

While there need be no rigidity about the form or pattern of a lesson plan, it may be suggested that the following essential elements find a place in every good lesson plan.

1. Statement of instructional objectives and learning outcomes in relation to the topic. It is desirable to state the objectives in terms of changes in pupils behaviour so that the evidences for the changes might also be sought in pupils behaviours.

2. Selection and sequential organization of learning activities in term of the objectives.

3. Selection of appropriate aids and materials and the resources to be used.

4. Selection of appropriate devices to evaluate the learning outcomes at different stages.

(a) Instructional Objectives

Cognitive Level objectives: Knowledge level (Remembering level)

An objective requiring students to remember a specified response, but not a multistep sequence of responses to a specified stimulus is at the knowledge level.

Knowledge of symbols, terms, definitions, relationships, formulae, conventions, axioms,

Examples

(1) state the definition of a triangle

(2) state the Pythagoras theorem

(3) state the formula for finding the area of a rectangle

(4) recognize the symbol for summation

(5) state the angle sum property of a triangle

(6) states the SAS-axiom

(7) state the laws of surds

(Comprehension) Understanding Level

An objective requiring students to remember a sequence of steps in a procedure and then be able to apply to arrive at a response to specified stimuli i.e. this category includes the recognition or recall of specific facts and terminology and the ability to perform a given algorithm in a familiar context. It also includes the ability to translate a verbal description to a pictorial representation, the ability to read and interpret problems, discriminates, comparing and contrasting, detecting errors, giving illustrations etc.,

Examples

(1) gives examples of an inverse proportion

(2) cites examples from the environment which are in the shape of a square

(3) given the dimensions of a rectangle, compute its area

(4) gives the reasons for the assertions in the proof of the theorem

(5) explain how the formula for the area of a right triangle can be derived from the formula for the area of rectangle.

(6) classify similar and non similar surds

(7) identify surds from a given set of irrational numbers

(8) rationalize the denominator of a given surd

(9) computes the product of two given matrices which are conformable for multiplication

In the understanding level objectives, one needs to know how to execute the steps in arriving at the response to the stimulus. He recalls the procedure and applies over it.

For Example

If the stimuli is 2+3 for a secondary school student, then the response from the student is immediate and it is a recall from the addition facts rather than figuring it out.

Hence this is a knowledge level whereas if the stimulus is 86+57 then it requires the student to recall the procedure of addition of two addends, and in place of addends he has to substitute 86 and 57 and the procedure is to be carried out to get a response.

Hence it is an understanding level objective.

Application Level: An application objective requires students to use deductive reasoning to decide how to utilize, if at all, a particular concept relationship or process to solve problems (Kelley, 1988, pp.169-254). Here "solving problems" is used broadly referring to situations in which students determine strategies for addressing questions and tasks. Solving problems does not include solving exercises, which are very routine, and no novel thinking is required. It also includes gathering and analyzing information, separating a problem into its constituent parts, identifying relevant information for solving a problem, recognizing analogues problems, recognizing patterns, relationships, generating new patterns or relations, forming hypotheses, inferring, predicting and doing other things in new situation. This is the highest level of cognitive behaviour. Suggest new or alternate or modified method of attack or a method of solving problems.

Examples: (1) Given the expression $\sqrt[n]{\frac{a}{b}}$ where n, a and b are positive integers, the student should be able to determine whether the given expression is irrational.

(2) When confronted with a real life problem, determine whether or not computing the surface area will help solve that problem.

(3) Given a polynomial equation and a root to this equation, the student should be able to show that the root is rational or irrational.

(b) Pre Requisite Knowledge

One needs knowledge to learn. It is not possible to absorb new knowledge without having some structure developed from previous knowledge to build on. New knowledge cannot be learnt in isolation. So in a classroom, the teacher has to hook the past learning experiences of the students and brought forward to facilitate present learning, so that a higher degree of learning can be achieved with much ease and also that shortens the length of the time that it takes to acquire the new learning otherwise. In the context of the lesson plan, the pre-requisite knowledge is that knowledge which the student is expected to possess that will facilitate to learn the new knowledge to be imparted through the lesson. Pre-requisite knowledge does not include all the knowledge he possesses before learning a particular concept or a generalization. It includes only that knowledge which is required and which is relevant to the present situation.

For example: To learn to prove the basic proportionality theorem the pre-requisite knowledge required by the student is

(1) Formula for the area of a triangle i.e., ½ x base x height.
(2) To draw an attitude of an acute angled triangle.
(3) Ability to recognize the two triangles having the same base and lying between the two parallels in different orientation and recall the statement that they have the same area.

Note that, it is not necessary that he should know the derivation of the formula ½ x base x height, or proof of the theorem "that the triangles having the same base lying between the same parallels are equal in area".

(c) Teaching aids

Example (1)

Objective: To obtain the formula for the surface area of a sphere.

Material Required A ball, cardboard/wooden strips, thick sheet of paper, ruler, cutter, string, measuring tape and adhesive.

Method of Construction

1. Take a spherical ball and find its diameter by placing it between two vertical boards (or wooden strips) [see Fig. 1]. Denote the diameter as *d*.

2. Mark the topmost part of the ball and fix a pin [see Fig. 2].

3. Taking support of pin, wrap the ball (spirally) with string completely, so that on the ball no space is left uncovered.

4. Mark the starting and finishing points on the string. Unwind the string from the surface of the ball. Measure the length between the starting and finishing point on the string and denoted by l.

5. On the thick sheet of paper, draw 4 circles of radius 'r' (which is equal to the radius of the ball).

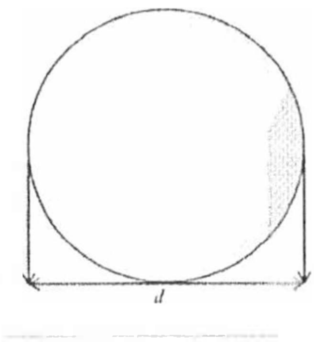


Fig. 1

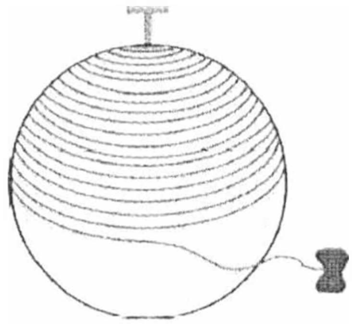


Fig. 2

6. Start filling the circles [see fig. 6.44] one by one with string that you have wound around the ball.

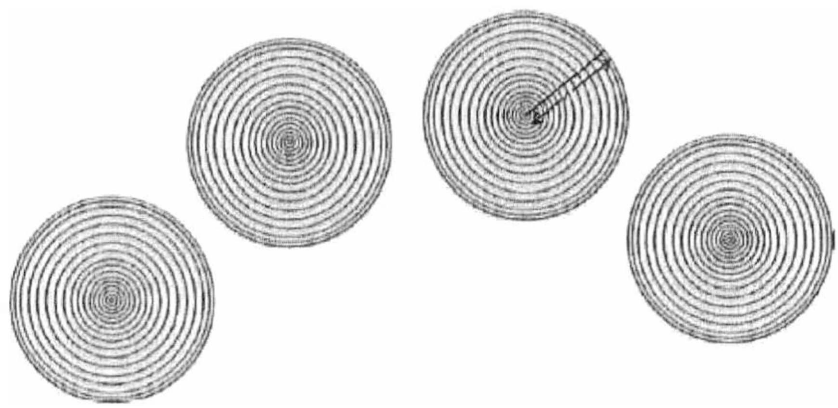


Fig. 3

Demonstration

Let the length of string which covers a circle (radius r) be denoted by a.

The string, which had completely covered the surface area of ball, has been used completely to fill the region of four circles (all of the same radius as of ball or sphere).

This suggests:

Length of string needed to cover sphere of radius r = 4 × length of string needed to cover one circle i.e., $l = 4a$
or, surface area of sphere = 4 × area of a circle of radius r
So, surface area of a sphere = $4\pi r2$

Example (2)

Objective To find experimental probability of unit's digits of telephone numbers listed on a page selected at random of a telephone directory.

Material Required Telephone directory, note book, pen, ruler.

Method of Construction

1. Take a telephone directory and select a page at random.
2. Count the number of telephone numbers on the selected page. Let it be 'N'.
3. Unit place of a telephone number can be occupied by any one of the digits 0, 1, ..., 9.
4. Prepare a frequency distribution table for the digits, at unit's place using tally marks.
5. Write the frequency of each of the digits 0, 1, 2, ...8, 9 from the table.
6. Find the probability of each digit using the formula for experimental probability.

Demonstration

1. Prepare a frequency distribution table (using tally marks) for digits 0, 1, ...,8,9 as shown below:

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	n_0	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_9

2. Note down frequency of each digit (0, 1, 2, 3,...,9) from the table.
Digits 0, 1, 2, 3, ..., 9 are occurring respectively $n0$, $n1$, $n2$, $n3$, ..., $n9$ times.

3. Calculate probability of each digit considering it as an event 'E' using the formula

$$P(E)=\frac{\text{Number of trials in which the event occured}}{\text{Total number of trials}}$$

4. Therefore, respective experimental probability of occurrence of 0, 1, 2, ..., 9 is given by

$$P(0)=\frac{n_0}{N},\ P(1)=\frac{n_1}{N},\ P(2)=\frac{n_2}{N},\ldots,\ P(9)=\frac{n_9}{N}.$$

(d) Learning Experiences

Learning experiences are the activities undertaken by the students planned deliberately by the teacher with a particular purpose to bring about desirable changes in their behaviour. The selection of learning activities offers much scope and choice for teachers. The decision about which activity or combination of activities to use within a lesson depends on the teachers beliefs about the relative effectiveness of the different activities for the type of learning intended.

Criteria for framing good learning experiences:

1. Learning experiences should be based on the specific objectives to be attained
2. Learning experiences should be framed in relation to the prescribed syllabus
3. Learning experiences should meet the needs of the particular age group of pupils
4. Learning experiences should provoke good deal of sustained interest in the pupil so as to ensure full participation of the pupil in the experience.
5. Learning experiences should be prepared by taking account of their abilities, interests and motivation.
6. Learning experiences should provoke the desired reaction in the students.
7. Learning experiences should be closely related to the environment of the students.

Planning Learning Experiences

When thinking about learning activities to be used, teacher also need to think of the lesson as a coherent whole, such that the total package of experience provides for pupils, achieves teachers intended learning outcomes. Thus, not only must the activities promote the appropriate intellectual experience for this learning to occur, but they must also enable pupils to readily engage and remain engaged in this experience. The degree of teacher success will depend on how best the teacher has planned the experiences. So it is very essential that the teacher knows how to plan learning experiences.

The following points should always be borne in mind while providing the learning experiences.

1. Learning experiences should:

   - be arranged from simple to complex
   - to provide the experiences so that conceptualization occurs first, followed by knowledge and comprehension level learning and finally culminating into application level learning.
   - be organized in such a way so as to provide the desired reaction.
   - be provide, time and again to reinforce the desired behaviour.
   - seek pupils participation to the maximum possible extent.
   - be directly linked with the desired goal.
   - be arranged according to the facilities available in the immediate environment of the school.
     (Students should be engaged in learning activities for conceptualizing a concept or relationship. Then the name for a concept or relationship should be introduced. Before conceptualizing the concept or relationship, memorizing words to attach a concept or relationship is meaningless for most of the students).

2. The initial phase of the lesson requires the activities to be designed to set the scene and elicit interest and introduces the topic. The second phase which is the major part of the lesson should involve main learning experiences that engage the pupils thinking process and holds their attention, while he/she asks questions, discusses a new skill or concept. The final phase in which the teacher assesses and review the pupils understanding of the learning outcomes.

3. Though a variety of activities is important, each activity must be appropriate to the learning at hand. A variety of activities provides pupils with an opportunity to learn in different ways, and thereby to build up and develop the skills to do so effectively. At the same time, it does not mean that every lesson must involve a variety of activities. It depends on the complexity of the subject matter the ability and interests of the students.

Example for Learning activities for the initial phase of the lesson i.e. introduction or motivation.

T : Hello : My dear students tell me what is 2^3?
S1: Oh! $2^3=8$
T : How did you get that?
S1 : I multiplied 2 itself as a factor 3 times

T : Good. Then what is 2^4 -------- S_3. How do you get the value.
S3 : Sir, it is 16., we obtain by multiplying 2 by itself as a factor 4 times.
T : O.K. can you tell me S^{15} what is $2^{3.6}$?
S4 : No, How can you find?
How can we multiply 2 by itself 3.6 times.
T : you may not be able to multiply 3.6 times.
can you estimate the value of $2^{3..6}$.
can you tell between which two values $2^{3.6}$ exists ?
S : since $2^3=8$; $2^4=16$
Then probably a number that lies between 8 and 16 must correspond to $2^{3..6}$
T : So, let us see. How to find $2^{3.6}$?

Example for Learning activity for the objective:

To Distinguish between examples and non examples of circles and explaining the defining attributes of a circle (Cognitive: conceptualization).

T : I would like someone to tell us about the racing game we played with the throw ball yesterday. Okay. Mr. Sam.
S : Two of us raced at a time to see who could get the ball first.
T : Draw a picture showing how we first line up and where the ball was.
S : Draws the picture

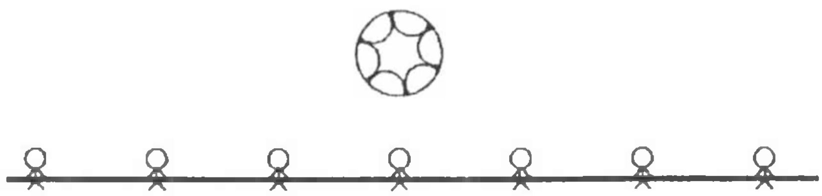


Fig. 6.45

T : Now draw, a picture showing how we lined up after changing the rules.
S : Draws the picture

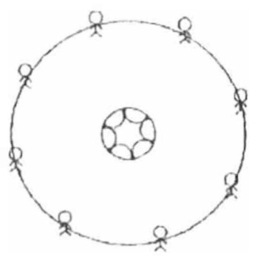


Fig. 6.46

T : Why did we change the game?
S4 : Because it was not fair before; some people were closer to the ball than others.
T : Why was the second way fairer?
S5 : Because everybody was just as close.
T : What shape did you make after we fixed the game?
S10: A circle
T : Why was that better than being on a straight line?
S15 : Because we were around the ball, so that no one was closer or farther away from the ball.
T : So, what is a circle then?
S5 : A circle is a kind of thing that is all around a ball.
T : Yes, good.

When you are standing on a line some of you are closer and some are farther from the ball. When you are around the ball and with the same distance from the ball, you make a shape of a circle.
So a circle is a set of points (like your positions) in a plane which are at equidistance from a fixed point.

(e) Assignments

At the end of the lesson plan, after review or summary of the lesson, the assignment follows. The teacher has to carefully plan the assignment to be given to students. It may consist of exercises or problems to be done outside the classroom, i.e., in the hostels or in their houses leisurely. Decisions about the length, type, nature of the assignments should be made while planning the lesson. Assignments that are not clear, confusing or unrealistic will only inhibit learning.

The purpose of assignments is to provide activities in mathematics. Problems and exercises provide a context for applying generalizations and for practicing algorithms basic to the development of mathematical skills.

The assignment should be assigned as early as possible after the review is over and when the teacher has the full attention of the students. The teacher should tell the students clearly what is to be done, and how to do it.

With a wide range of abilities, reading, speeds, interests and aptitudes operative in any class, a differentiated assignment can be given. Differentiation does not need to be blatantly obvious. Minimum assignment for all with the suggestions that pupils who find the going rather easy move ahead to problems of a more complex nature.

Assignments act as a barometer for students achievement for the teachers. The teachers can use the student's work in doing assignments as a diagnostic tool. The type of students errors can provide the teacher with valuable information for future lessons, re-teaching and constructing follow up activities.

Student's work in doing assignment can be a starting point or can be used as a platform to initiate new learning. Assignments can also be provided with some answers so that student's can gain confidence after achieving parts of the assignments, which in turn motivates to complete the assignments.

Structure of a Lesson Plan

A comprehensive plan might indicate at the top the essential identification data like the name of the teacher, school, standard, section, subject, topic, period and time, date etc. The instructional objectives could then be selected and clearly written. What are often stated as general aims could be taken for granted and so need not be repeated lesson after lesson. For example. Students will have the knowledge of, ..., understands ...., applies .... etc., could be assumed and need not be specifically stated unless some important aspect of this is to be specifically emphasized in the lesson.

The instructional objectives should be stated in terms of specific objectives, stated in terms of pupil's observable behaviours.

The instructional objectives should be followed by the previous knowledge ie, background of experiences and understandings, that the student be expected to have, which will facilitate to link the new knowledge the concepts, the generalizations which the teacher is supposed to impart on that day.

Then under the head of teaching points, a mention is made about the concept names, the statements of the generalization etc., that the teacher is going to teach.

The learning aids, materials etc., to be used could also be indicated immediately after teaching points.

Since all teaching and testing have to be objective based and learner centered, due emphasis should be given on the student behaviour in the process of learning as well as in the product to be formally tested, the plan should clearly show such learning outcomes in terms of pupils observable behaviours.

These then should be the starting points for indicating the corresponding content, learning activities, evaluation devices and items etc. Such things could be given in a structured way. **There is no rigidity about the number or different order of columns,** but a good comprehensive plan adopting, this approach should essentially indicate the important expected learning outcomes, sequential learning activities and an evaluation procedure. The content need to be spelt out in detail and every question and also every small detail be given under learning activities so that the teacher (atleast a new teacher) finds it at ease in actual classroom. As far as possible the learning activities could be given from the pupils point of view indicating the teachers role by implication. Questions for testing the product outcomes could be given to see whether the stated objectives have been attained by the pupils.

In addition the plan should include a review or a summary followed by an assignment.

All these then could then be given in a structured way in about three columns with one-to-one horizontal relationship.

(a) Example of Lesson plan

Topic : Algebra — Class : XII
Unit : Relations — Time : 45 minutes

Instructional Objectives:

At the end of this lesson, a student will be able to

1. define an equivalence relation
2. state the characteristics of an equivalence relation
3. identify equivalence relation from the given relations
4. cite examples of equivalence relation
5. relate equivalence relation with other type of relations.

Teaching Point

Equivalence relation on a set is a relation on the set which satisfies Reflexive Property, Symmetric Property and Transitive Property.

Previous Knowledge

Reflexive, Symmetric and Transitive Relations.

<table>
<tr><th>Expected Learning Outcomes</th><th>Sequential Learning Activities with inbuilt Evaluation</th><th>Blackboard Work</th></tr>
<tr><td></td><td>Introduction</td><td></td></tr>
<tr><td></td><td>T: Good Morning students (seeks the attention of the class). We have seen some properties of relation and based on these properties, we have distinguished between types of relations.</td><td></td></tr>
<tr><td></td><td>T: What are (or) Mention the properties of a relation? ... S_1</td><td></td></tr>
<tr><td></td><td>S_1: Reflexive, Symmetric, Transitive, Anti-symmetric.</td><td></td></tr>
<tr><td>Recalls and lists</td><td>T: Good. Mention the types of relations? ...S_2</td><td></td></tr>
<tr><td></td><td>S_2: Reflexive, symmetric, transitive, anti-symmetric.</td><td></td></tr>
<tr><td></td><td>T: I have given you some homework to write the examples of reflexive, symmetric and transitive relations. Give me an example of reflexive relation? ....S_3</td><td></td></tr>
<tr><td>Gives an example of a reflexive relation.</td><td>S_3: "is equal to" in a set of R.
T: Does this binary relation satisfy any other properties? ...S_4</td><td></td></tr>
<tr><td></td><td>S_4: Symmetric and transitive.</td><td></td></tr>
<tr><td></td><td>T: Good, in the same manner check the other relations. (Gives some time to the students). What about the other binary relation "is perpendicular to". ....S_5</td><td></td></tr>
<tr><td></td><td>S_5: It satisfies only symmetric property.</td><td></td></tr>
<tr><td></td><td>T: Repeats the same for few more binary relations ("is less than", "is parallel to").

Development of the Concept

T: Look at the binary relations (showing to chalkboard). Are there any commodities between the binary relations "is equal to", "is parallel to" etc. from that of other binary relation? ....S_6</td><td>
<table>
<tr><th>Binary Relation</th><th>R</th><th>S</th><th>T</th></tr>
<tr><td>"is equal to"</td><td>√</td><td>√</td><td>√</td></tr>
<tr><td>"is ⊥ to"</td><td>x</td><td>√</td><td>x</td></tr>
<tr><td>"is less than"</td><td>x</td><td>x</td><td>√</td></tr>
<tr><td>is parallel to</td><td>√</td><td>√</td><td>√</td></tr>
</table>
R- Reflexive
S- Symmetric
T- Transitive</td></tr>
</table>

Expected Learning Outcomes	Sequential Learning Activities with inbuilt Evaluation	Blackboard Work
Compares and contrasts the binary relations which satisfy all the properties from that of other binary relations.	S_6: "is equal to", "is parallel to", etc. are satisfying all the three properties when compared to other binary relations.	Teacher rounds of those binary relations which satisfy all the three properties.
	T: Good. The binary relations which satisfy all the three properties are called equivalence relations (writes on the board).	Equivalence Relation: Equivalence Relation is a binary relation which satisfies reflexive property, symmetric property and transitive property.
	T: What is required for a binary relation to be an equivalence relation? ....S_7	
States the sufficient condition.	S_7: It should satisfy reflexive, symmetric and transitive properties.	
	T: Good. (Gives a binary relation "is less than and equal to" and asks). Is this an equivalence relation? ....S_8	1. "is less than or equal to" in a set of Real numbers. 2. "is to the left of" in a set of points on a line. 3."is collinear with" in a set of points in a line.
	S_8: "is ≤" is not an equivalence relation.	
	T: Why do you think so? ....S_9	
Gives reason for the binary relation as the non-example of equivalence relation.	S_9: It does not satisfy symmetric property.	
	T: Good. Give me an example of an equivalence relation? ....S_{10}	
	S_{10}: "5 divides x-y" in a set of all integers.	
	T: Why is it an equivalence relation? ....S_{11}	
Give reasons for a binary relation to be an equivalence relation.	S_{11}: Since it satisfies all the three properties.	
	T: Right. We have seen that there are 3 conditions for a binary relation to be an equivalence relation.	1. Reflexive. 2. Symmetric and 3. Transitive properties.

Review and Evaluation:

What is an equivalence relation?

What are the characteristics of an equivalence relation?

An equivalence relation is a kind of .... relation?

What similarities and differences do you find between equivalence relation and other relations?

Today we learnt about equivalence relation.

Strategy :

Compare and contrast → Definition → Sufficient condition → Non-example → Necessary condition → Example → Sufficient condition.

(b) Example of Lesson Plan

Topic : Linear equations in two variables
Unit : Algebra

Class : IX
Time : 45 minutes
Date : 21.10.10.

Instructional Objectives : At the end of the class students will be able to

1. states the definition of linear equations in two variables.
2. state the characteristics for an equation to be linear equation in two variables.
3. cites example for the linear equations in two variables.
4. identifies linear equations in two variable.
5. formulates the linear equations in two variables.

Previous Knowledge

1. Definition of linear equation in one variable
2. Formulate linear equation in one variable

Teaching points :

1. Linear equations in two variables is an equation that contains two different variables each of degree one.
2. General form of linear equation in two variable, is $ax+ by +c =0$, where a, b and c are real number, with a and b both not equal to zero.

Expected learning outcomes	Sequential Learning Activities with inbuilt evaluation	Evaluation/Blackboard work
	T : Good morning	
	S_3 : Very good morning sir	
	T : In your earlier classes you have learnt linear equations in one variable. Give a few example for linear equations in one variable.	
recalls examples for linear equations in one variable.	S_{10} : $x + 5 = 0$ $y - 2 = 0$ $2z = 6$	$x + 5 = 0$ $y - 2 = 3$ $2z = 6$
	T : Why do these are called linear Equations ?	
States the necesary condition for equations to be linear.	S_{15} : These are linear equations because, the degree of the variable is 1.	
	T : How many variables are there in each of the equations	
Identifies the number of variables in the equations.	S_4 : One	
	T : Ram and Rahim together have 5 more books than Ram. Find the number of books Ram and Rahim have?	
	S_9 : Let x be the number of books Ram have. So Rahim will have $x + 5$	
Solves the problem by formulating the linear equation in one variable.	$\therefore$ $x + x + 5 = 25$ $2x + 5 = 25$ $2x = 25 - 5 = 20$ ie $x = 10$.	
	So Ram have 10 books Rahim have $10 + 5 = 15$ books.	
	T : Good. See here on the black board a set of equations given in one column as set A and in other column as set B.	
	All the equation in set A has some thing common which is not found in each of the equations of set B observe and tell me what is that?	

Set A	Set B
1) $x + 2y = 3$	$x - y + z = 8$
2) $2r + 5s = 0$	$n = 5$
3) $u = 2v + 4$	$u + v + t = 0$
4) $x - 2z = 6$	$2x + y + u + 3 = 5$
5) $3t = 5u - 8$	$x + 8u = 3z$

Table - 1

Compare each of the equations in set A & contrasts with equations in B and state the findings.	S_7 : equations in set A contains two variables where as in set B the equation have the number of variable either 1 or more than 2.
	T : Yes good in set B, each of the equations contain two variables.
	T : Now observe the equations in table - 2. see the each of the equations in set A has some thing common and which is not found in every equation of the set B. Try to find out that.
Compares and contrasts and finds the commonalities in set A.	S_{10} : All the variable is each of the equation given in set A are of degree 1, while it is not so in the equations given in set B.
	T : Good. Now any one of you Come to black board and write Equations which contain two Variables and each of degree 1.
Writes the equations	S_2 : (Writes) $x + y = 5$ $2t \div 5 = 3u$
	T : Good. What do you call an equation containing variables of degree 1
Recalls from the previous knowledge	S_3 : It is a linear equation.
	T : what is a linear equation in two variables.
Tries to define	S_5 : A linear equation in two variables is an equation that contains two variables and of degree 1.
	T : Then, is $x + 2y^2 = 5$ a linear equation in two variables?
Recognizes the lack of necessary condition	S_1 : No, S_{12} Because one of the variable is of degree 2.
	T : Can any one restates the definition given by S_5.
Modifies the definition given by S_5.	S_8 : A linear equation in two variables is an equation that

Set A	Set B
1) $x + 2 = 0$	$x^2 - 5 = 3$
2) $x + y = 5$	$x^2 + y^2 = 4$
3) $u = 2v + 4$	$r + 2s^2 = 0$
4) $s - 2t + 5$ $u = 6$	$r^2 + 2t - u^3 = 9$

Table - 2

	contains two variables and each of degree 1.	
	T : Yes good. So a linear equation in two variables is an equation having two variables and each of degree 1.	
	T : S_7 give two examples of linear equation in two variables	
Gives examples.	S_7 : 1) $15u - t = 0$ 2) $x - 8y = 5$	
	T : Good. (Write an equation on the black board)	
	Is this a linear equation in two variables ?	$x = 8$
	S_5 : No	
	T : Why?	
Identifies the lack of necessary condition.	S_5 : Because it does not contain two variables.	
	T : (Writes) Is this linear equation in two variables ?	$t + 5u = 8$
	S_8 : Yes.	
	T : Why?	
Identifies the sufficient condition for an equation to be linear in two variable.	S8 : Because it contain two variables and also each of the variable is of degree 1.	
	T : (Writes) Is this a linear equation in two variables ?	$t^2 = 5$
	S_5 : No	
	T : Why.	
Identifies lack of necessary Condition.	S_5 : It is not linear. T : Yes, for an equation to be Linear in two variables it should contain two variables and also each variable should be of degree 1.	
	T : (writes a set of equations on the black board) Write down the equations which are linear in two variables and that are not.	1) $x + y = 8$ 2) $2x^2 - y = 4$ 3) $2t + u = 10$ 4) $x^2 - u^2 = t^2$ 5) $s^3 - y^3 = 8$
List the linear equations in two variables & others	S_6 : Writes	6) $x + y = 2t$

linear in two variables are equation . 1, 3, 7, 10

others 2, 4, 5, 6, 8, 9 one not the linear equations in two variables.

7) $s + 2u = -8$

8) $r + s^2 = t^2$

9) $a + b = c$

10) $5a + 6b = 9$

T : Mr. kalyani has a few hen and a few pigs in hen form. Total legs of the hens and pigs is 50 and the heads is 17.

Can you express these in the form of equations.

S : (Silence)

T : Is one variable enough to represent the situation.

S : Looks no

T : Then how many variables are necessary?

Identifies the variables

S : May be we require two variables one for pigs and one for hens. So let the number of hens be x and the number of pigs be y.

T : Then how many heads each one of the hen and the pigs has.

S : Each one has one head.

T : How many heads x hens and y pigs have

Represents in the form of an equation.

S : $x \times 1 + y + 1 = x + y$

But it is given equal to 17

$\therefore x + y = 17$

T : Now can you represent the legs of the hen & pigs in equation form.

S : $2x + 4y = 50$

T : Each student in IX class has 4 text books & 6 Note books and each student in V class has 2 text books 2 note books. Total number of Text books are 100 and the note books are 140 can you represent these in the form of equations.

Represents the data in the form of equation.

S : Let x be the number of IX class students

& Let y be the no. of V class students

then

$4x + 2y = 100$

$6x + 2y = 140$

Review : In a right angled triangle. Write the equation representing the sum of the other two angles (besides the right angle) what are the variables.

Assigument : Prabhakar has some Rs 5/- notes and a few Rs 10/- notes. Total value of the money is 125. Can you express this as a linear equation in two variables.

(c) Example of Lesson plan

Topic : Arithmetic — Class : VIII
Unit : Profit and Loss — Time : 45 minutes

Instructional Objectives: To enable the student

1. to acquire the knowledge of terms and procedures used in determining profit or loss in an enterprise.

   Specifications :
   a) Recognizes the terms profit, loss, selling price, cost price etc.
   b) Recalls the meaning of these terms.

2. to understand the terms and procedures in the context of profit or loss.

   Specifications :
   a) Illustrates the meaning of the terms.
   b) Translates verbal statements into mathematical relations.
   c) Compares similar situations.
   d) Analyses the data.

e) Uses appropriate relations to determine the profit/loss in given situations.

f) Verifies the results obtained.

3. to apply the knowledge of profit and loss to new situations.

Specifications :
a) Derives new relations between cost price, selling price and profit/loss
b) Draws inferences from the relations about profit/loss.
c) Solves problems involving the profit and loss concepts.

Concepts :

a) Of terms – profit, loss, cost price and selling price, profit (or loss) percent.

b) Of relations -

i) Profit = selling price – cost price.

ii) Loss = cost price – selling price

iii) Cost price = selling price – profit or selling price + loss.

iv) Selling price = cost price + profit or cost price – loss.

v) Profit (or loss) percent = $\dfrac{\text{Total profit (or loss)}}{\text{Cost price}} \times 100$

c) Of symbols – Cost price is denoted by C.P.; Selling Price is denoted by S.P.

Previous knowledge :

1. Arithmetical Calculation (with numbers),
2. Percent,
3. Discount
4. Graph Reading.

Instructional Aids : Charts of graphs showing the relation between C.P. and S.P. when there is (a) Profit (b) Loss.

Sequential Learning Activities (SLA) (1)	Expected Behavioural Outcomes (2)	Evaluation (3)
Introduction Preparation : Phase 1 1. The teacher (a) narrates some instances from business involving selling, (b) draws the pupil's attention to the newspaper announcements about prices of articles. Why does one do business ? Are articles sold always for the people paid by the seller ? Give some examples/instances where the articles are sold for profit/loss.	Recognise that business is a means of livelihood. Recall some instances where articles are sold for profit or loss.	If a shopkeeper buys 10 kg butter from a villager for Rs.280/- and sells it for Rs.320/- does he gain or loss ?
2. The salesman buys articles from some source. What costs him is called the Cost Price. What is the cost price when a seller buys a) Books paying 80 ps. Each ? b) Rice paying Rs.250/- per quintal ? The cost price shall be denoted by C.P. Use the notation to the cost price in the above examples.	Recognise the cost prices in each case.	Fill in; The cost price of a student note book of 100 pages is ______ if each book costs Rs.1.20 for the seller. Find the cost price of Toordal per kg. if the seller buys it from the grower paying Rs.570/- per quintal. Find the selling price in each case below: i) A radio costs Rs.250/- to a buyer in a radio shop.
3 Selling price is the price of the article fixed by the seller to sell the article. If the seller sells apple for Rs.8/- a kg. What is the selling price of the apple ?	Recognise the selling price of the article.	(ii) A fruit seller sells Mosambi at Rs.5/- a kg. (iii) A vegetable vendor, heralds that beans per kg. is Rs.2/-. Complete: If a pencil costs 40 ps. in a shop, then ______40 ps.(P.S., C.P., S.P.)
Just as C.P. denotes the cost price, the selling price of an article shall be denoted by S.P. Using the notation, write the selling price of apple in the above example.	Uses appropriate notation to denote the selling price.	

Development Phase II

4. a) Consider a situation when some articles become scarce. That means the articles is in great demand. If a stockiest has it, does he sell it (a) for the cost price ? (b) for more than cost price? C) for less than cost price?
   b) If the article is available in plenty, it is not in demand, then the stockiest may sell it for less than the cost price.
   When does a merchant?
   a) Gain, b) loss?
   What is the relation between S.P. and C.P., when there is a) profit b) loss.
   Identify the situations below, resulting in profit or loss, by writing P or L accordingly.

   i) Fish was brought at Rs.15/- per kg. and sold at Rs.18/- per kg.
   ii) Peas was sold for Rs.4 a kg, while the same costing the seller Rs.3 per kg.
   iii) Lemons bought at Rs.8 per hundred was sold at Rs.6 per hundred.
   iv) Gold costing Rs.1800/- per ten grams was sold for Rs.1950/- per ten grams.
   v) Paddy was sold for Rs.80/- per quintal while its cost price was Rs.62/- per quintal.
   vi) Potato per kg is sold at Rs.2.40 per kg. while the cost price is Rs.2.10 per kg.

Identifies the situation when the merchant sells it for more than the cost price.

Infers that when a) there is profit, S.P. is more than C.P. b) there is loss, S.P. is less that C.P.

Identifies the relation between C.P. and S.P. in case of
a) profit b) loss.

Rice was purchased by a merchant at Rs.200/- per quintal during harvest and he can sell it at Rs.320/- per quintal during i) the harvest ii) off season (Select appropriately).

A fruit seller's cost price for orange for 1000 pieces was Rs.80/- when he finds that the orange has flooded the market and the orange he has will perish soon, his selling price a) exceeds the cost price, b) cannot exceed the cost price c) falls short of the cost price. (Select the most probable).

Find the (amount of) profit or loss in each of the cases (i) to (vi).

5. The amount of profit (or loss) is the difference between the selling price and the cost price. Express this as a mathematical relation.

Infers the relation between S.P. and C.P.
Formulates the relation for profit or loss as
a) Profit = S.P. – C.P.
b) Loss = C.P. – S.P.

1. X sold eggs at 40 ps. each, buying it from a farm at 32 ps. per piece. Find the profit/loss per piece.

6. It is convenient to express profit or loss as if the cost price is 100 rupees. When it is so expressed it is called profit percent or loss percent. If a watch costing Rs.150/- nets a profit of Rs.24/-, what is the profit percent ?

Expresses profit as profit percent using the formula. Profit (or loss) percent =

$$\frac{\text{Net profit (or loss)}}{\text{Cost Price}} \times 100$$

2. Y sold beans for Rs.2/- a kg. making a profit of 20 ps. What was the cost price ?
3. Z bought tomatoes for a cost price of 75 ps. per kg. and had a loss of 10 ps. per kg. What was the selling price ?

Problem: A table bought for Rs.80/- was sold for Rs.85/- Find the profit or loss percent. In this problem, what is the a) cost price b) sale price ? Which is more ? Is it profit or loss ?
What is the net profit ?
What is the formula for profit percent ?
What is the profit percent ?
Problem: Profit of 8% was made by selling a two-in-one transistor for Rs.2160/-. What is the cost price ?
When the profit is 7% what does it mean?
How do you calculate profit or loss percent ?

Pupils recognize the cost price and sale price. Pupils recognize that S.P. is greater than C.P. and is a case of profit. Compute the profit using Profit = S.P. – C.P.
Recalls the formula:

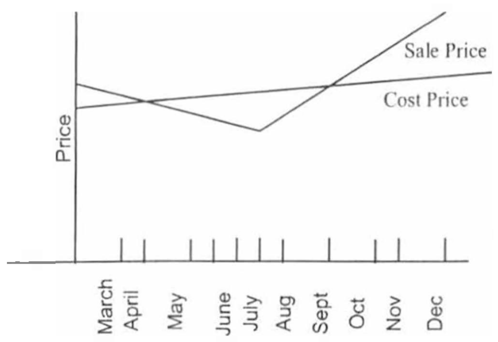
$$\text{Profit percent} = \frac{\text{Profit}}{\text{Cost Price}} \times 100$$

Calculates the profit percent.
Recognises the C.P. was100 rupees.
Recalls that a) S.P. exceeding C.P. means profit and b). C.P. exceeding S.P. means loss.
Recalls the formula: Profit or loss percent =

$$\frac{\text{Profit or loss}}{\text{Cost Price}} \times 100$$

Fill in –
When profit percent is equal to actual profit, the cost price equals ______

The graph below shows the C.P. and S.P. of an article over 10 months. Find the periods of profit and loss.



During which month(s) there is
a) no loss, no profit b) loss ?
c) profit ?

Reads the graph. Interprets the portions of graph. Determines when there was a) profit b) loss c) no profit and no loss.
Reasons that a) an upward graph indicates increased profit b) a downward graph indicates decreased profit c) there is loss when the sale price graph is below the cost price graph and d) there is profit, otherwise.

Why do you say that
1. July-September is the period of loss?
2. The rest 10 months is the period of profit ? When profit decreases, what is the relation between the ordinates of the sale price graph? For profit, what is the relation between the ordinates of sale price and cost price at a given time. Answer the above, in case of loss.

Review Questions :

1. Explain the meaning of the term – Cost Price, Selling Price, Profit, Loss, Profit Percent, Loss Percent.

2. When will profit/loss occur ? How do you calculate profit or loss percent ?

3. What is profit (or loss) percent. How do you calculate profit or loss percent ?

4. A man lost 5% by selling a radio for Rs.250/- For what price should he have sold in order to get a profit of 5%

5. A photographer allows a discount of 10% on the advertised price of a Camera. What price must be marked on the Camera which costs him Rs.600/- to make a profit of 20% ?

Assignment:

1. A man buys 300 articles for a total cost of Rs.630/-. He fixed the selling price of each 32% above the cost price and sold 230 articles. He then reduced the selling price by 20% and sold the rest of the articles. Calculate the profit percentage of his original outlay.

2. Collect newspaper cuttings showing the cost prices of a variety of articles.

3. Note the cost price and selling price of some articles whose price fluctuates, over a period of 10 days. Draw the graphs of cost price and selling price over the period. Determine the periods of profit and loss.

4. Note the prices of different cereals and pulses announced over AIR for a week's period. By comparing these with the cost prices, find out the profit/loss each day.

5. Prepare charts displaying how profit or loss occurs, to a rural man who can read sparingly.

Moves in the Expository Strategy of Teaching a Generalization [The Remainder Theorem]

1. Introduction Move

T : Good morning students. In the previous class, we learnt the division of polynomials by polynomials. I had given you some problems on the same to be worked out at home. Has everyone done it?
Good.

Focusing :

T Today we shall learn a method for finding the remainder without actually performing the process of division, when a polynomial with any degree greater than one is divided by a binomial of the form (x – a).

The Objective Move:

T : In the process, we would learn a theorem called "The Remainder Theorem".

The Motivation Move:

In the previous class, we performed many divisions. We say that bigger the polynomial, longer is the process of division.
By learning the Remainder theorem, the division of polynomial of degree > 1 by a binomial of the form (x – a) can be done by a short cut method, without going through the lengthy process of division.

Reviewing the Pre-requisite Knowledge

The pre-requisite knowledge to learn the remainder theorem are:
Meaning of a polynomial
Degree of a polynomial
Subtraction of a polynomial
Division of a polynomial
which could be tested during the development of the teaching-learning process.

2. Assertion Move

T : The remainder Theorem which we are going to learn is stated as (on the blackboard)
Let p(x) be any polynomial of degree ≥ 1 and 'a' any real number. If p(x) is divided by (x – a) then the remainder is p(a).

3. Instantiation Move

T : In the second problem of the yesterday's home work, the dividend is the polynomial.
$x^3 + 4x^2 + 5x + 11$
The divisor was $x - 2$.
You see that both the polynomials satisfy the condition specified in the theorem.
What are they?.....S_1

S_1 : The dividend is a polynomial of degree ≥ 1.
The divisor is of the form $(x - a)$.

T : Good. S_1...What is the remainder of this division ?

S_1 : 21

T : Have you all got the same answer ?

S : Yes.

T : Okay. Now, we will try to find the remainder by the remainder theorem. As per the remainder theorem, the remainder is p(2). Let us evaluate p(2) [on the blackboard]
$P(2) = 2^4 + 2(2)^3 + 3(2^2) + 2 - 1$
$= 16 + 16 - 12 + 2 - 1$
$= 21$
Thus in this case we see that p(2) is equal to the remainder, you got by the division method.
Now, let us consider another problem – the 5th one.
The polynomial to be divided is $y^3 + y^2 - 2y + 1$, the polynomial (binomial) with which you are going to divide is $y - 3$.
S_2, you have done the problem. What is the remainder you have got?

S_2 : 31

T : Okay. Let us try to find the remainder, using the remainder theorem.
The divisor polynomial is $y - 3$.
The real number a in this case is 3. Therefore, the remainder is given by p(3).
$p(3) = 3^3 - 3^2 - 2.3 + 1$
$= 27 - 9 - 6 + 1 = 31$
So even in this case we have found that the remainder found by the division method and the one found by using the remainder theorem are equal.

Application Move :

Now take down this problem.
Divide the polynomial $y^3 - 2y^2 + 3y - 18 = 0$ by $(y - 3)$ and find the remainder. Verify its value using the remainder theorem.
.......[Pauses for the students to work out].
Have you all solved the problem ?
S_1 work it out on the board.

$$
\begin{array}{r|llllll}
\multicolumn{1}{r}{} & y^2 & + & 4 & +\,6 & & \\ \cline{2-7}
y-3 & y^3 & - & 2y^2 & +\,3y & - & 18 \\
 & -(y^3 & - & 3y^2) & & & \\ \cline{2-7}
 & & & y^2 & +\,3y & - & 18 \\
 & & & -(y^2 & -\,3y) & & \\ \cline{4-7}
 & & & & 6y & - & 18 \\
 & & & & -(6y & - & 18) \\ \cline{5-7}
\multicolumn{1}{r}{} & & & & & & 0
\end{array}
$$

By the remainder theorem
$p(3) = 3^3 - 2(3)^2 + 3(3) - 18$
$= 27 - 18 + 9 - 18$
$= 36 - 36$
$= 0$

T : So you see in this case that the remainder is zero and you have also verified one thing through it. What is it ? .....S_7.

S_7 : ....

T : If the remainder is zero, what can you say about $(y - 3)$...S_7?

S_7 : $(y - 3)$ is a factor of the given polynomial.

T : Very good.

So by using the remainder theorem, you can also verify if the given binomial is a factor of the polynomial.
Take down this problem. Divide $y^3 + y^2 - 2y + 1 = 0$ by $(y - a)$ and find the remainder. Verify if you get the same remainder through the remainder theorem.

Interpretation Move :

The generalization can be paraphrased as :

T : When a polynomial of degree ≥ 1 is divided by a binomial containing difference between a term whose power and coefficient is one and a real number, the remainder is the value of the polynomial function for the real number.

Analysis Move :

T : So in the above theorem, you see that there are specific characteristics for the polynomial. What is the characteristic of the divided....S_1.

S_1 : It is a polynomial of degree ≥ 1.
What is the characteristic of the divisor....S_2.

S_2 : It is a binomial of the form $(x - a)$, where x is any variable and a is a real number.

T : Given the characteristic polynomials, how do you find the remainder using the remainder theorem?.....S_3

S_3 : The remainder is found by substituting the real number in the binomial, i.e. a in the given polynomial which is the dividend.

Counter Example Move :

The wrong interpretation of the binomial is the divisor. They might misconceive it as $x + a$ or any other binomial, in which case remainder theorem is not applicable. It could also be a wrong sign of the real number which they substitute the polynomial they find the remainder using the theorem.

Justification Move :

T : Though we have seen that the theorem holds good for the problems we considered, for it is to be accepted universally for all divisions of the kind mentioned, we should prove the theorem.
So, let p(x) be the polynomial, q(x) be the quotient, r(x) be the remainder.
What are the possibilities for r(x)....S_1?

S_1 : It can be zero.

T : Good. If the binomial is a factor r(x) will be zero. What else...S_2?

S_2 : It can be any number.

T : Yes, i.e. you see that the degree of r(x) is then less than the degree of $(x - a)$ in the divisor i.e. zero which means that r(x) is a CONSTANT. So let us call it r.
In division, if D is the dividend, d is the divisor and r the remainder and q is the quotient, what is the relationship between them?
...
If you are given all the four values, how will you find D?...S_2.

S_2 : $D = d \,.\, q + r$.

T : Very good.
So in this case we can say that for all values of x,
$p(x) = (x - a) \,.\, q(x + r$.
What is it that we find using the theorem....S_1.

S_1 : p(a).

T : Good.
$\therefore\; p(a) = 0 \,.\, q(a) + r = 0 + r = r$.
i.e. $p(a) = r$ and this proves the theorem.

Application (in a generalization)

T : We know that if $p(a) = 0$, then $(x - a)$ is a factor of $p(x)$.
We would use this in a theorem which we would learn in the next class i.e. the Factor Theorem.

A **problem situation** for an expository strategy using the above moves would be :

T : You all know the method to divide a polynomial by a polynomial. If I tell you that the dividend which is a polynomial of degree ≥ 1 and the divisor is a binomial of the form $(x - a)$.
Can you find the remainder without actually dividing them?

USE OF ICT IN TEACHING LEARNING PROCESS

ICT has the potential to make a significant contribution to pupil's learning in mathematics by helping them to:

(i) practice and consolidate learning skills by using software to revise or practice skills and to give rapid assessment feed back.

(ii) Develop skills of mathematical modeling through the exploration, interpretation and explanation of data by choosing appropriate graphical representations for displaying information from a dataset; by experimenting with forms of equations in trying to produce graphs which are good fits for data-plots; by using a motion sensor to produce distance time graphs corresponding to pupils own movements.

(iii) Experiment with, make hypothesis form, and discuss or explain relationships and behaviour in shape and space and their links with algebra, by using software to

   (a) automate geometric constructions,
   (b) carryout specified geometric transformations
   (c) perform operations on co-ordinates or draw loci

(iv) Develop logical thinking and modify strategies and assumptions through immediate feed back by planning a procedure as a sequence of instructions in a programming language or a sequence of geometrical constructions in geometry software or a set of manipulations in a spreadsheet.

(v) make connections within and across areas of mathematics for example:- To relate a symbolic function, a set of values computed from it, and a graph generated by it to a mathematical or physical situation, such as the pressure and volume of a gas, which it models.

(vi) work with realistic and large, sets of data. For example: carrying out experiments using large random samples generated through simulation.

(vii) Explore, describe and explain patterns and relationships in sequences and tables of numbers; by entering a formula in algebraic notation to generate values in an attempt to match a given set of numbers.

(viii) Learn, and memorise, by manipulating graphic images. For example, the way the graph of a function such as $y = x2$ is transformed by the addition of, or multiplication by a constant.

ICT also has the potential to offer valuable support to the mathematics teacher by:

(i) helping them to prepare teaching materials

For example: Downloading materials for classroom use from the internet, such as mathematics problems for pupils to solve with accompanying teachers notes, software for computers and reviews of published resources.

(ii) providing a flexible and time saving resource that can be used in different ways and at different times without repetition the teachers input by enlarging fonts, adding diagrams or illustrations, adapting parameters used in problems.

(iii) providing a means by which subject and pedagogic knowledge can be improved and kept up-to-date by accessing the virtual teacher centre to obtain practical advice, to exchange ideas with peers and 'experts' outside school.

(iv) Aiding record keeping and reporting by storing and regularly updating formative records which can form the basis of a subsequent report.

(from PAMELA COWAN, Teaching Mathematics, A hand book for primary & secondary school batches).

List of Persons & Resource Persons attended for the Second Activity from 29th November to 3rd December 2010

Sl.No.	Name and Address	
1.	Ms.R Prema TGT (Retired) No.423, A & B Block, Navilu Road Kuvempunagar, Mysore-570 023, Ph.2541393	Resource Person
2.	Ms.Asha B N Selection Grade Lecturer Onkarmal Somani College of Education Saraswathipuram, Mysore-570 009, Ph.2524829	Resource Person
3.	Mr.M Adi Narayana Reddy PGT (Mathematics), Jawahar Navodaya Vidyalaya Krishnapuram (PO), Marripadu (M), Nellore (Dist.) Andhra Pradesh-524 230, Ph.08620-228722	Resource Person
4.	Mr.Kalagathoori Balaji TGT (Mathematics) KV No.1, Tirupati, Ahdhra Pradesh-517 507 Ph.9703417268	Resource Person
5.	Mr.V B V V S S S H S R Sastry Junior Lecturer in Mathematics A P S W R Junior College, Balawanta Reddy Complex New Nagole, Hyderabad-500 008, Ph.24546399	Resource Person
6.	Ms. H R Kavitha Assistant Teacher, G H School, Lakshmipuram, Mysore	Resource Person
7.	Mr.Sharad Sure Lecturer in Education, Amrita School of Education Bogadi, Mysore-570 026, Ph.0821-2340911	Resource Person
8.	Ms.N Suma Rao Assistant Mistress, Kautilya Vidyalaya, No.9/1, J Block, 3rd Stage, Kanakadasanagar Dattagalli, Mysore-570 023, Ph.2460267	Resource Person

10.	Mr.Manje Gowda A Assistant Master Govt. Girls Junior College, K R Nagar, Mysore	Resource Person
11.	Ms.Geetha B Assistant Teacher, Govt. P U College Gavadagere, Hunsur Taluk, Mysore	Resource Person
12.	Mr.Somaraju Assistant Teacher, Govt.High School Musuvina Koppalu, Dyvapattana (via), T Narasipura Taluk Mysore Dist., Ph.9739639380	Resource Person
13.	Ms.Priyadarshini R Raikar Govt. High School, Vyasarajapur, T Narasipura Taluk, Mysore Dist. Ph.9242080938	Resource Person

List of Persons attended the first activity of the programme from 27th to 29th September 2010

Sl.No.	Name and Address
1.	Mr.C G Lokesh TGT, GHPS, Shantinagar, Chikamagalur Taluk Chikamagalur Dist.-577 101
2.	Mr.Khadar Patel Assistant Teacher, Govt. Boys P U College Shorapur Taluk, Yadagiri Dist.
3.	Mr.Gowdru K G Assistant Teacher Govt. Upgrade Higher Primary School Kamalapura, Harihara Taluk, Davanagere Dist.
4.	Mr.Shashidhara S K Assistant Master, Govt. High School Shivapur, Tumarikoppa Post, Kalghatagi Taluk, Dharwad Dist.-581 204
5.	Mr.Mallikarjun Kannale Assistant Master, Govt. High School Saigaon, Bhalki Taluk, Bidar Dist.0585 416
6.	Mr.D G Umesha Assistant Master, Govt. High School Ichchangi, Savanur Taluk Haveri Dist.-581 110
7.	Mr.Lakshmana B N Assistant Master, Govt. High School Huthur, Kolar Taluk, Kolar Dist.-563 101
8.	Mr.Jithendra Babu M N Assistant Teacher, GHP School Janekunte, Bellary Taluk Bellary Dist.-583 115
9.	Mr.R F Magi Assistant Master, Govt. High School Bevindkoppa, Bailhongal Taluk Belgaum Dist-591 102
10.	Mr.Seetaram Patgar Assistant Teacher, Govt. High School Hulidevarabana, Sagar Taluk, Shimoga District
11.	Mr.Mallikarjun K Assistant Master, Govt. High School for Boys Jalahalli, Devadurga Taluk Raichur District-584 116

12.	Mr.Rajendra Bhat K Assistant Teacher, ABMVS High School Inna, Karkala Taluk Karkala-576 146
13.	Mr.Vinayak G Avadhani Assistant Teacher, Govt. High School, Hodke Shiroor, Honnavar Taluk North Canara Dist., Uttara Kannada
14.	Ms.Sunanda H P Assistant Teacher Grade II D.K.Z.P.G.P. School, Manchi, Kukkaje Bantwal, Dakshina Kannada-574 323
15.	Mr.Kadetotad B M Assistant Master, Girls English School, Deshpandenagar, Hubli Dharwad District-580 029
16.	Mr.Siddalingeshwar Poolbhavi Assistant Teacher, S.S.B.P.P Govt. High School Chikkadanknakal, Gangavathi Koppal

Annexure II

DETAILED PROGRAMME PROPOSALS FOR 2010-11

Format for Developing Proposals for DAB/ IAB

ABSTRACT

1.	Title of the Programme	:	**Development of Training Package for KRPs to train teachers in Classroom Transactions in Mathematics for classes VI to IX**
2.	Type : Research / Development/ Training/ Extension/ Development - cum - Training, etc.		Development
3.	Category New/ Ongoing/ Carried over (If new, give justification in brief, if ongoing/ carried over mention the progress)	:	New In view of the recent trends and changes in the technological society, mathematics is found an important tool and this necessitates to equip the mathematics teachers with the necessary skills so that they can be able to produce quality future work force.

4. Specific Objectives :

1. To identify the content areas that are difficult for teachers for transacting them in the classrooms.
2. To develop the training package for KRPs in transacting the identified content areas.

5. Methodology :

(If research programme, please also indicate sample, research question / hypotheses and tools)

1. Workshop involving teachers to identify content areas that are difficult for transaction.
2. Workshop involving KRPs and RPs to develop the materials in the identified areas.
3. Workshop for Review and Finalisation of the materials.

6.	Total Budget	:	Rs.2,33,050/-
7.	Plans for Utilization and Dissemination of the end product(s)	:	The KRPs will use the training package in training the teachers at secondary level.

REGIONAL INSTITUTE OF EDUCATION (NCERT): MYSORE - 570 006
(National Council of Educational Research and Training, New Delhi)

PROFORMA FOR PROGRAMME PROPOSAL FOR 2010-11

1.	Name of the NCERT Constituent/Department	:	RIE, Mysore
2.	**Title of the Programme:**	:	**Development of Training Package for KRPs to train teachers in Classroom Transactions in Mathematics for classes VI to IX**
3.	**(a) Type of the Programme** (Research/ Development / Training / Extension/ Development cum research)	:	Development
	(b) Category of the Programme (New/ Ongoing/Carried over)	:	New
	(c) If the Programme is on going or carried over, mention the PAC Code No. and year of approval	:	
4.	**Total Duration of the Programme as phased in Col. 10 (Months)**	:	July 2010 - January 2011
	(a) Date on which Programme commenced / to be commenced	:	15.7.2010
	(b) Target date of completion	:	30.1.2011
5.	**Target Groups:**		
	a) If Programme is meant for a group with special needs (Special Groups, SC, ST, Minority, Girls, etc).		
	b) Stage of Education to which the Programme is meant (Pre-Primary/ Primary/ Upper Primary/ Secondary / Senior Secondary / Tertiary/ Any other)	:	Upper primary and Secondary
	c) If Programme is State/Region/ Agency specific, please specify.		Karnataka
6.	**Beneficiaries (Please tick)**	:	Teachers of Karnataka

7. **Need and Justification**
(If an ongoing / carried over programme, please also state briefly the progress achieved and the work likely to be completed by the end of the current financial year).

In view of the recent trends and changes in the technological society, mathematics is found an important tool and this necessitates to equip the mathematics teachers with the necessary skills so that they can be able to produce quality future work force.

8. **(a) Specific Objectives:**
1. To identify the content areas that are difficult for teachers for transacting them in the classrooms.
2. To develop the training package for KRPs in transacting the identified content areas.

(b) Methodology
(If research programme, please also indicate sample, research questions/ hypotheses and tools)
1. Workshop involving teachers to identify content areas that are difficult for transaction.
2. Workshop involving KRPs and RPs to develop the materials in the identified areas.
3. Workshop for Review and Finalisation of the materials.

9. Collaborating Agencies(if any)	Name of Agency	Nature of Collaboration
(a) NCERT Constituents		
(b) Outside Agency	SCERT	

10. **Phasing of the Programme with precise information on Activities** (including in-house activities involving expenditure or otherwise clearly indicating the methodology to be followed).

Sl. No.	Activities proposed to be organized	Proposed dates From To	Estimated Expenditure in Rs.
1	3 Days workshop to identify the difficult areas	July 12 – July 14	58,700/-
2	5 days workshop for development of the material	Sept. 13 – Sept. 17	1,21,800/-
3	5 Days workshop for review and finalization	Dec 6 – Dec. 10	52,550/-
		Total	**2,33,050/-**

Amount Required in the Proposed year **: Rs. 2,33,050/- (Rupees two lakhs thirty three thousand and fifty only)**

11. Details of each Budget Activity under Item No. 10 (in the following format).

11.1	**Activity No.**	:	11.1
	Title	:	Workshop to identify the difficult content areas
	Proposed Dates	:	12.7.2010 to 14.7.2010

Sl. No.	Item of Expenditure	Estimated Expenditure in Rs.	Remarks, if any
1	TA for 20 teachers (2000 × 20)	40,000/-	
2.	DA (food charges) for 20 teachers (Rs.200 × 20 × 3 days)	12,000/-	
3.	Accommodation for 20 teachers (Rs.25 × 20 × 3 days)	1,500/-	
4.	Working lunch and Refreshment (5 × Rs.80 × 3)	1,200/-	
5.	Stationery	3,000/-	
6.	Miscellaneous	2,000/-	
	Total	**59,700/-**	

11.2	**Activity No.**	:	11.2
	Title	:	Preparation of the Training Package
	Proposed Dates	:	September 13 – September 17

Sl. No.	Item of Expenditure	Estimated Expenditure in Rs.	Remarks, if any
1.	TA for 20 KRPs (Rs.2000 × 20)	40,000/-	
2.	DA (food charges) for 20 KRPs (20 × Rs.200 × 5)	20,000/-	
3.	Accommodation for 20 KRPs (20 × Rs.25 × 5)	2,500/-	
4.	TA for 5 RPs (Rs.4000 × 5)	20,000/-	
5.	DA (food charges) for 5 RPs (Rs.250 × 5 × 5)	6,250/-	
6.	Accommodation for 5 RPs (Rs.40 × 5 × 5)	1,000/-	
7.	Honorarium for 8 RPs (Rs.500 × 8 × 5)	20,000/-	
8.	Local Conveyance for 3 RPs	2,250/-	

	(Rs.150 × 3 × 5)		
9.	Working lunch and Refreshment For 7 persons (Rs.80 × 5 ×7)	2,800/-	
10.	Stationery	4,000/-	
11.	Miscellaneous	3,000/-	
	Total	**1,21,800/-**	

11.3	**Activity No.**	:	11.3
	Title	:	Review and Finalisation of the Package
	Proposed Dates	:	Dec 6 – Dec 10 2010

Sl. No.	Item of Expenditure	Estimated Expenditure in Rs.	Remarks, if any
1.	TA for 10 KRPs (Rs.2000 × 10)	20,000/-	
2.	DA (food charges) for 10 KRPs (Rs.200 × 10 × 5 days)	10,000/-	
3.	Accommodation for 10 KRPs (Rs.25 × 10 × 5)	1,250/-	
4.	Local conveyance for 2 local RPs (Rs.150 × 2 × 5)	1,500/-	
5.	Honorarium for 2 local RPs (Rs.500 × 2 × 5)	5,000/-	
6.	Working Lunch and Refreshment For 7 persons (Rs.80 × 7 × 5)	2,800/-	
7.	Stationery	5,000/-	
8.	Typing, Printing and Xeroxing	5,000/-	
9.	Miscellaneous	2,000/-	
	Total	**52,550/-**	

12. Scheme of Evaluation :

13. Dissemination of the findings :
The package is given to State authorities.

14. Plans for follow up/Feed back on utilisation of the outcome

15. a) Name and Designation of the Programme Coordinator:
Dr.B.S.P. Raju, Professor in Mathematics

b) Name(s) and Designation(s) of other faculty member(s) involved

Five resource persons will be identified from NCERT and other RIEs

And RIEM Mathematics faculty.

Signature of the Head, Department	B. Syam prasad Raju **Signature of the Programme Coordinator**
Date :	**Date :** 15/1/2010.