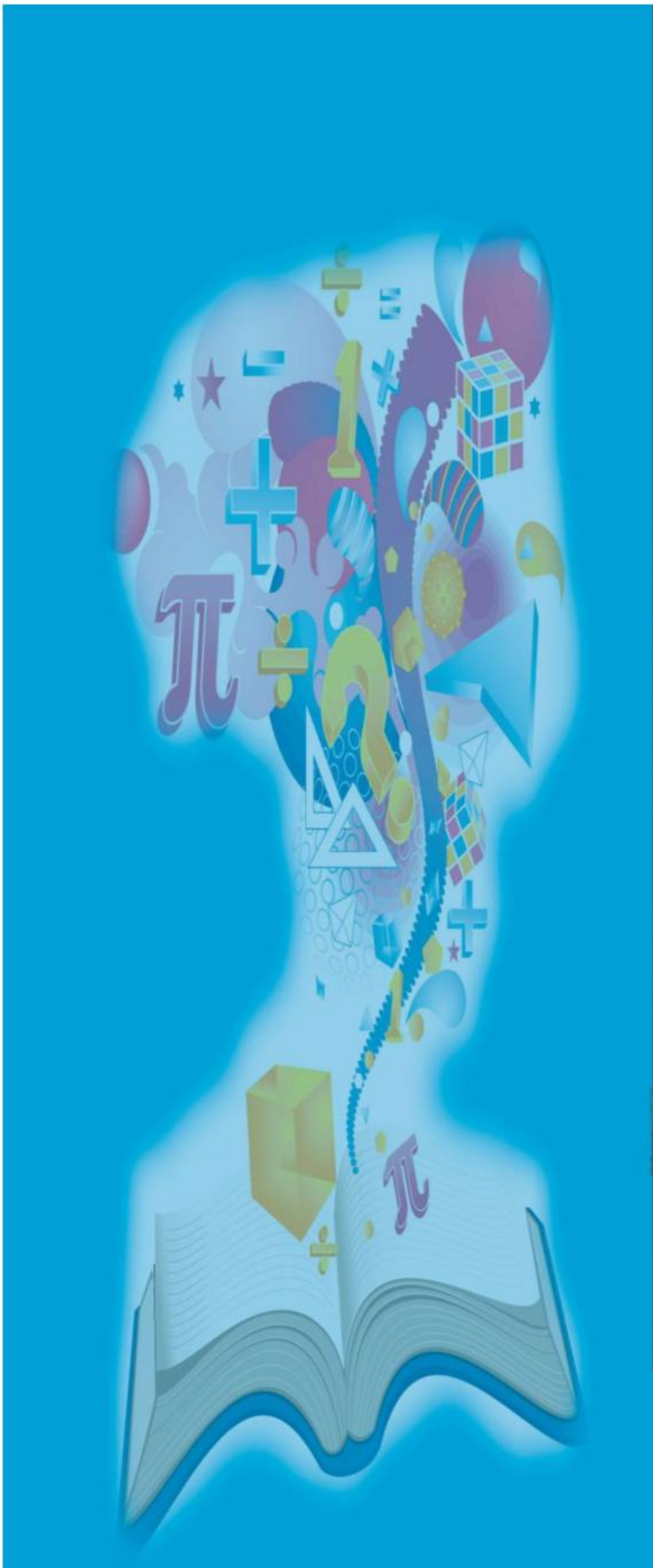

ENRICHMENT MATERIAL

FOR CONTENT-CUM-PEDAGOGY PROGRAMME FOR KRP'S ON MATHEMATICS AT SECONDARY LEVEL

FOR TELANGANA AND ANDHRA PRADESH

Programme Coordinator

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PREFACE

School Mathematics Education always faces lot of difficulties due to the abstract nature of the subject. Children develop fear in learning mathematics due to their experience in understanding fundamentals as well as lack of motivation. Teacher's role is also very important in creating good learning situation in classroom. Also they have to develop different teaching strategies to different groups of students to make their understanding in mathematics very comfortable. Also there is need to connect their daily life situations to concepts in mathematics which gives a satisfaction, joy and reduces the fear of subject.

With this background Governments of Telangana and Andhra Pradesh states requested to take up this training program and accordingly it is formulated.

In the beginning two days in-house meeting was conducted with all the faculty of mathematics department on 16^{th} and 17^{th} August 2018 in RIE Mysuru to identify the difficult areas and develop some modules for training.

Next, a five day's training program was conducted in RIE Mysuru from 24^{th} to 28^{th} September 2018. Totally 34 KRPs from Telangana and Andhra participated in this program. The training program was designed in interactive as well as activity based approaches with participants.

All the resource persons handled the sessions in an effective manner. Participants' involvement was very good in all the sessions. Most of the difficult contents identified were discussed thoroughly. The training was appreciated and well received by all the participants.

Finally, based on the training and interaction some modules were finalized for the resource material after a careful observation.

I would like to convey my whole hearted gratitude to the resource persons Prof. N. M. Rao, Prof. B. S. P Raju, Sri. T. P. Prakashan, Sri. T. K. Vijay Kumar and Dr. Sharad Sure for their constant supervision and involvement in the program.

I am grateful to our beloved principal Prof Y. Sreekanth for his continuous encouragement and inspiration throughout the program. Also I am thankful to Prof A.

Sukumar, Head DESM, Prof C. H. Venkatesha Murthy, Head DEE and Prof M. U. Paily, in-charge CAL for their support and help.

I am very thankful to Dr. T V Somashekar, Sri. B Madhu, Sri. Ajay Kumar, Ms. Pallavi and Sri. Shrinath H. for their support throughout the training program. I am also thankful to the staff of DESM, DEE and CAL for their secretarial help and cooperation.

Last but not the least I am thankful to all the participants for their valuable participation and continuous interaction. I hope that this material will help them to improve the quality of classroom transactions and facilitate joyful learning in teaching mathematics.

V S Prasad

Program co-ordinator

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Regional Institute of Education, Mysuru-570 006
Department of Extension Education

Title of the Programme: **Content-cum- Pedagogy Enrichment programme for KRP's on Mathematics at Secondary Level**

Programme Coordinator: **Prof V S Prasad** **Date: 24th to 28th September 2018**

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1. ARITHMETIC PROGRESSION

- An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.
- The fixed number is called *'Common Difference'* and it may be positive, negative or zero.

Examples of Arithmetic Progression :

1. 200, 250, 300, 350, ...
2. a, a+d, a+2d, a+3d, . . .

To know about A.P, we need to know the first term '**a**' and the common difference '**d**'

For Example :

1. If first term **'a'** is 6 and the common difference **'d'** is 3 then the A P is 6, 9, 12, 15, . . .
2. If a = 6 ; d = -3 then, AP is 6, 3, 0, -3, . . .

On the other hand if you are given a list of numbers, we can check easily that whether it is an AP or not and if it is AP then we can find the first term and the common difference **'d'** also.

- The n^{th} term of the A P with first term 'a' and common difference 'd' is given by,

$$\mathbf{a_n = a + (n - 1)d}$$

- The sum of the first n terms of an arithmetic progression is given by

$$\mathbf{S_n = [2a + (n -1)d]\ n/2}$$

Problem 1: Determine the AP whose 3^{rd} term is 5 and 7^{th} term is 9
Solution : We have $a_5 = a + (3 -1)d = a + 2d = 5$
$a_7 = a + (7 -1)d = a + 6d = 9$
a = 3 ; d =1
The A P is 3, 4, 5, 6, 7 . . .

Problem 2 : How many 2 digit numbers are divisible by 7 ?
Answer : n = 13

Problem 3 : Find the 31^{st} term of an AP whose 11^{th} term is 38 and 16^{th} term is 73
Answer : $a_{31} = 178$

Problem 4 : How many 3 digit numbers are divisible by 7 ?
Answer : 128

Problem 5:
A single square is made from 4 match sticks. To make two squares in a row it takes 7 match sticks, while three squares in a row takes 10 matches. If there are 109 match sticks. Calculate the number of squares in the row.
Answer : 36

2. RATIONAL NUMBERS

INTRODUCTION:

As we know that a linear equation of the form ax + b = 0 with a≠0, b≠0, has no solutions in integers. This shows that there is a need for the extension of integers which gives these solutions. Such system is called rational number system denoting by Q. So a rational number is a number of form p / q where p, q ϵ Z and q≠0.
Hence for the equation ax + b = 0, the solution is x=-b/a which is rational number. Hence it has solution in the rational number system.

EQUALITY: Let p/q and r/s be two rational numbers such that p/q=r/s. will it implies p=r and q=s?. no, need not be. For example

$$2 / 3 = 6 / 9$$

But, 2 ≠ 6 and 3 ≠ 9

Hence there is need to define the equality of rational numbers. It is given as follows:

$$p / q = r / s \text{ if and only if } ps = rq$$

OPERATIONS:

1. Q is closed under addition, subtraction and multiplication.

2. Division is not closed in q as 0 ϵ q and 1 ϵ q, but '1/0' is not defined in q.

3. Properties like p/0, 0/q, 0/0.

If q ≠ o then note that 0/q is taken to be as zero which has no problem. But the cases p / 0 and 0 / 0 are two extreme cases, needed to be clarified.

For p/0:
By taking very small numbers in the denominator let us observe the following

a) If q = 0.1 then 1 / q = 1 / 0.1 = 10

b) If q = 0.01 then 1 /q + 1 / 0.01 = 100

c) If q = 0.0001 then 1 /q = 1 / 0.0001 = 10000

So as denominator smaller and smaller than rational number 1/q became bigger and bigger which may tend to ∞.
Similarly if we take the denominator as negative then 1/q tends to -∞.
So there is no accuracy in getting the exact value of 1/q. thus we conclude that when q=0,the rational numbers 1/q is not defined.

For 0 / 0:

If you assign any values to 0/0, it holds, for example 0/0=3,0/0=4,……

So it is considered that 0/0 is an indeterminate form. Similarly some other indeterminate form are given by 0^0, 0^∞, $0.\infty$, ∞-∞, ∞^∞. . .etc

*associative law holds in Q under addition and multiplication
*distributive law holds in Q

RECIPROCAL OF A NUMBER:
Let p / q be a rational number. We say that r / s is called the reciprocal or multiplicative inverse of p / q if (p/q) x (r/s) = 1

REPRESENTATION OF RATIONAL NUMBER IN THE NUMBER LINE:

1. **Representation of fraction p / q with p < q**

Join q in the y-axis to 1 in the x-axis through a line segment. Now draw a parallel line to this passing from p in the y-axis cutting the x-axis. The point of intersection of this line with x-axis represents p/q on the number line (x-axis).

For example take 2/3:

Join 3 in the y-axis to 1 in the x-axis through a line segment. Now draw a parallel line to this passing from 2 in the y-axis cutting the x-axis. The point of intersection of this line with x-axis represents 2/3 on the number line (x-axis).

2.**Representation of fraction p / q where p > q**

Join q in the y-axis to 1 in the x-axis through a line segment. Now draw a parallel line to this passing from p in the y-axis cutting the x-axis. The point of intersection of this line with x-axis represents p/q on the number line (x-axis).

For example take 3/2:

Join 2 in the y-axis to 1 in the x-axis through a line segment. Now draw a parallel line to this passing from 3 in the y-axis cutting the x-axis. The point of intersection of this line with x-axis represents 3/2 on the number line (x-axis).

3.**Representation of mixed fraction**

Convert it into improper fraction and apply the above procedure

For example: Consider 1 $\frac{1}{2}$. Convert this into $\frac{3}{2}$ and apply the above procedure.

RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS:

Simple technique to find more rational numbers between two rational numbers :

Let p / q and r / s be two rational numbers. Then by adding their numerators and denominators directly, we get another number which is clearly lies between them.

$$p / q < (p + r) / (q + s) < r / s$$

by continuing in this way we have

$$p / q < (p + p + r) / (q + q + r) < (p + r) / (q + s) < (p+ r+ r) / (q + s +s) < r / s \text{ and so on.}$$

Hence in this way we can find any number of rational numbers between given two rational numbers.

Example 1:

1/3 and ½

look at

$$1/3 < 2/5 < 1/2$$
$$\text{i.e} \quad 1/3 < 3/8 < 2/5 < 3/7$$
$$\text{i.e} \quad 1/3 < 4/11 < 3/8 < 5/13 < 2/5 < 5/12 < 3/7 \ldots$$

and so on

Example 2: 1/4 and 1/2

Look at

$$1/4 < (1+1) / (4+2) < 1/2$$
$$\text{i.e.} \quad 1/4 < 2/6 < 1/2$$
$$1/4 < 3/10 < 2/6 < 3/8 < 1/2$$

and so on.

Exercise.

Find ten rational numbers between 3/5 and ¾

sol.

$3/5 < 3/4$

step 1: $3/5 < 6/9 < 3/4$

step 2: $3/5 < 9/14 < 6/9 < 9/13 < 3/4$

step 3: $3/5 < 12/19 < 9/14 < 15/21 < 6/9 < 15/22 < 3/4$

step 4: $3/5 < 15/24 < 12/19 < 21/33 < 9/14 < 24/35 < 15/21 < 6/9 < 21/31 < 15/22 < 18/26 < 3/4$

step 5:

$3/5 < 15/24 < 12/19 < 21/33 < 9/14 < 24/35 < 15/21 < 21/30 < 6/9 < 21/31 < 15/22 < 18/26 < 3/4$

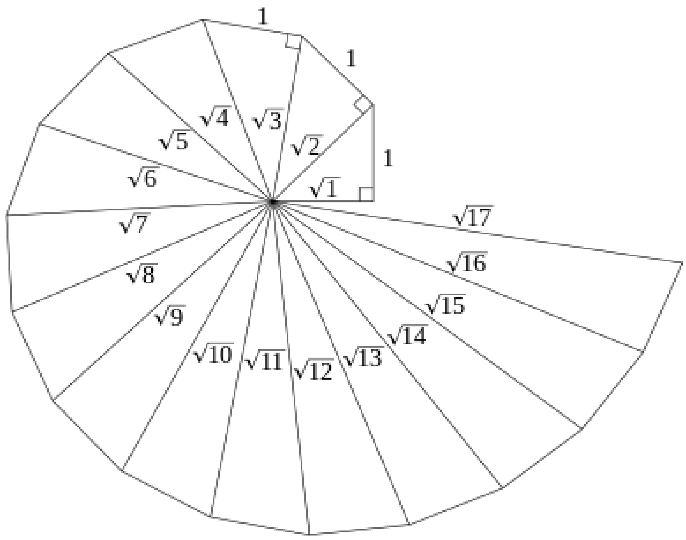
Thus by applying simple basic addition on numerator and denominator. we can find any number of rational numbers between two rational numbers.

Remark:

*There are infinitely many rationals between two rationals
*There are infinitely many numbers which are not rationals between two rationals

IRRATIONAL NUMBERS:

Locating square root numbers on number lines.
constructing the square root spiral



<u>VISUALIZING 1/√2 ON NUMBER LINE:</u>

We know that 1/√2 is irrational may be it is easier to handle if the denominator is a rational number. so we want to write 1/√2 as an equivalent expression in which the denominator is a rational number.
Multiplying 1/√2 by √2 / √2 we get $1/\sqrt{2} = (1/\sqrt{2}) \times (\sqrt{2}/\sqrt{2}) = \sqrt{2}/2$.
in this form it is easy to locate 1/√2 on the number line. it is half way between 0 and √2.

Showing 0.999… = 1:

$$
\begin{aligned}
0.999\ldots &= x \\
9.999\ldots &= 10x \\
9+0.999\ldots &= 10x \\
9+x &= 10x \\
9 &= 9x \\
x &= 1
\end{aligned}
$$

therefore 0.99999………… = 1.

3. STATISTICS

We know the importance of collection and representation of data. We also know how to represent data: in both tabular form and graphical form. Here we discuss how a single value can summarily describe a data. Such a value is called a central value or a measure of **central tendency**. We describe through a teacher-students' conversation in a classroom situation.

Consider the following situation: Suppose an employer hires 6 salesmen. Salesmen individually made profits (in thousand rupees) as follows;

16, 18, 11, 15, 13 and 17

The employer only has the knowledge of total profit. If he considers each one have done equally best, what would he think each one's contribution is?

S: Employer thinks that each one has contributed 15 thousand rupees.

T: Good! But how do you say it is 15 thousand?

S: Because the total profit is 90 thousand rupees and that there are 6 salesmen, in case of equal contribution it has to be 90/6 = 15 thousand rupees.

T: Good! So we observe that the above data of profits made by six salesmen could summarily be denoted by **15** thousand per salesman.

So it is always convenient to denote a data (whether grouped or ungrouped) by means of central tendencies. Many a times more than mere convenience it becomes a necessity to know the central tendencies. For instance,

Consider a man who possesses a piece of land measuring 3 acres. How can we judge his economic status relative to his locality? May be his economic condition is adjudged fairly good in village A while it may not be the case in village B. So it purely depends on the central values of the corresponding village.

Thus the measures of central tendencies play a very significant role in statistics and its applications to economics and other areas.

Mainly we use three measures of tendencies: **Mean, Median** and **Mode.**

We shall learn the meaning of Mean, its formulation and computation for both grouped and ungrouped data.

T: In vague words mean/ arithmetic mean may be considered as a value $\overline{x}$ (in the range of the variable) for which the sum of deviations of each value in the data from $\overline{x}$ is zero.

We can clearly see that in the first example for $\overline{x} = 15$, we have:

$$16 - 15 = 1$$
$$18 - 15 = 3$$
$$11 - 15 = -4$$
$$15 - 15 = 0$$
$$13 - 15 = -2$$
$$17 - 15 = 2$$

Adding all these we obtain 0.

Now let us consider another example:
Suppose you are a batsman and your team needs 55 runs from 4 overs. Ideally how many runs do you plan to score every over to win this game?

S: I would plan to score 13.75 runs per over, that is about 14 runs per over.

T: Yes! That is what is called Required runrate (RR) in cricket which is indeed the mean required runs in an over.

Looking into these examples, Can you formulate the mean for ungrouped data?

S: Yes! Mean would be the ratio of sum of all values in the data to the number of values recorded.

T: Good! Mean of a given ungrouped data is given by:

$\overline{x} = (x_1 + x_2 + \ldots + x_N)/N$
Where, N denotes the number of values in the given ungrouped data.

Teacher now provides an exercise problem of finding mean of an ungrouped data.
If students have obtained following marks in a class test, find the class average:
66, 69, 73, 58, 78, 81, 53, 58

Students find the mean,
$\overline{x} = 67$

S: The class average in the test is 67.

T: Good! Now it is clear how to find the mean of an ungrouped data. But we see that when the values are very large number the computation of mean consumes a lot of time. So, in such situations we can compute mean by what is called the **assumed mean method.**

We know that mean is a value $\overline{x}$ (in the range of the variable) for which the sum of deviations of each value in the data from $\overline{x}$ is zero. This gives us an idea that when a data involves very large numbers, we can work out the mean by assuming a value A to be the mean and then adding to this the arithmetic mean of deviations from A.

That is, you assume a particular figure in the data as the arithmetic mean on the basis of logic/experience. Then you may take deviations of the said assumed mean from each of the observation. You can, then, take the summation of these deviations and divide it by the number of observations in the data.

The actual arithmetic mean is estimated by taking the sum of the assumed mean and the ratio of sum of deviations to number of observations.

Symbolically let,
A = assumed mean
X = individual observations
N = total numbers of observations
d = deviation of assumed mean from individual observation,
i.e. d = X – A

Then mean is given by,

$$\overline{x} = A + (\Sigma d)/N.$$

Teacher provides an exercise problem of finding the mean using assumed mean method.

T: But sometimes we see that the deviations from assumed mean are themselves large. In such case we can further simplify our computation process by dividing every deviation by a fixed value. This method is called **Step-deviation method.**

Step-deviation method:
Let A be the assumed mean. Determine the deviations, d = X – A for each value X. If deviation d is large then determine d' for each value by, d' = d / c for a fixed number c. Then the mean is given by,

$$\overline{x} = A + (\Sigma d')c / N,$$

where N is the number of observations.

Teacher provides an exercise problem of finding the mean using the step-deviation method.

T: Good! We now learned how to compute the mean of a given ungrouped data. Now let us consider grouped data.

We know that whether it is a discrete variable or a continuous variable, in a grouped data the observed value times the corresponding frequency, denoted by 'fx' is the significant value in understanding the data. So, keeping the above three methods for ungrouped data, can you analogously formulate the mean for a grouped data with discrete variable?

T: Good! So now you see how naturally we obtain these formulae.

Now, finally let us consider the situation of grouped data with continuous variable:

After understanding the mean of grouped data with discrete variable, it is very easy to determine the mean of a grouped data with continuous variable. It is simply formulated by substituting the role of 'x' by the midponits of the class intervals, 'm'. so we have:

Direct method:

$$\bar{x} = (\Sigma fm) / (\Sigma f),$$

where f stands for frquency of class-interval whose mid-point is m

Assumed mean method:

$$\bar{x} = A + (\Sigma fd) / (\Sigma f),$$

Where A is the assumed mean (chosen logically among the mid-points) and d = m - A

Step-deviation method:

$$\bar{x} = A + (\Sigma fd') \, . \, c / (\Sigma f)$$

Where d' = d / c, 'c' a fixed value.

Teacher provides some exercise problems to the students on computing mean of grouped data (both discrete and continuous variables).

T: Students before we end our discussion on Arithmetic mean let me ask you some questions which we will be addressing in our next class:

1. Is mean a reliable measure of central tendency always?
2. Do we come across situations where mean fails to summarize the given data?

In addressing these questions we find the need for other measures of central tendencies.

MEDIAN

1. Ungrouped data:

   The central most value obtained when the observations are arranged in ascending or descending order is called the median.
   Let $x_1, x_2, \ldots, x_n$ be the observations,
   If n is odd, then median = $(n + 1/2)^{th}$ value.
   If n is even, then median = mean of $(n/2)^{th}$ and $(n/2 + 1)^{th}$ observations.

2. Median for discrete grouped data:

Frequency distribution table:

Observations	**frequency**	**Cumulative frequency**
X_1	f_1	f_1
X_2	f_2	$f_1 + f_2$
X_3 . . X_n	f_3 . . f_n	$f_3 + f_2 + f_3$ . . $f_1 + f_2 + . . . + f_n$

Steps to calculate median:

1. Find the cumulative frequencies.
2. Calculate $n = \sum f_i$
3. Find n/2
4. Identify the cumulative frequency that is near to n/2 and greater than n/2
5. The observation corresponding to that cumulative frequency gives us the median of the given data.

Briefly we describe the other two central tendencies

3. Median for continuous grouped data:

Class interval	**frequency**	**Cumulative Frequency**
a_1 - a_2	f_1	cf_1
a_2 - a_3 L - U . a_n - a_{n+1}	f_2 . . f_n	cf_2 . . cf_n

Steps to calculate median:

1. Find cumulative frequencies
2. Find $n = \sum f_i$
3. Calculate n/2.
4. Identify the median class:
   The class interval that contains the $(n/2)^{th}$ observed value is called "MEDIAN CLASS".
   Identify the interval where $(n/2)^{th}$ observation lie where n/2 is ≤ cumulative frequency.
5. Identify the lower and upper limit of median class.

i.e., $(n/2)^{th}$ observation lies between the lower limit L and upper limit U of the median class.

6. The formula for calculating median is given by,

a) $L + ((n/2) - cf) \times (h/f))$

Here, cf = cumulative frequency of the preceding median class. It is obtained by adding cumulative frequencies from above.

L= Lower limit of the median class.

h=class size(upper limit – lower limit)

f= frequency of the median class.

(OR)

b) $U-((n/2)-cf) \times (h/f))$

Here, cf = cumulative frequency of the succeeding median class. It is obtained by adding cf's from below.

U= Upper limit of the median class.

MODE

Mode is the most repeated observed frequency.

Frequency distribution table:

a_1-a_2	f_1	
a_2 - a_3	f_2	
L – U	.	
. . a_n- a_{n+1}	. . f_n	

Steps to calculate mode:

1. Identify the modal class, i.e., the class which contains the highest frequency.
2. The formula for calculating mode is given by,

$L + ((f_1-f_0) / ((f_1-f_0) + (f_1-f_2)) \text{ X } h$

Here, f_1 is the frequency of modal class.

f_0 is the frequency of preceding modal class.

f_2 is the frequency of succeeding modal class.

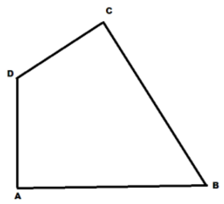
L is the lower limit of modal class.

h is the size of the class.

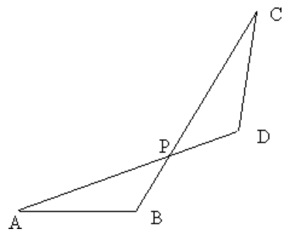
4. EXPLORING QUADRILATERALS

Let us explore the facts that we know about quadrilaterals together.
Before getting into the topic let us recall few terminologies from our previous knowledge.
As we all know the definition of a **polygon** is “ a simple closed curve made up of only line segments”.
Oh just wait for a minute. What do we mean by a simple closed curve then? (think of it!)
Let us have the better understanding of the definition by giving examples which are not polygons

3.



(2)

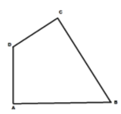
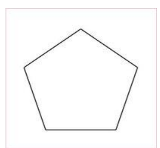
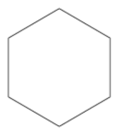


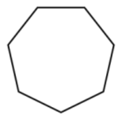
(3)

Look at the figure (1). Is that a polygon? If not which criterion it is failed to satisfy in order to be a polygon?
We can examine the same thing for other figures also.

Classification of polygons:

As we know by the definition of polygons, a polygon is made up of straight lines.
It is convenient to classify them according to number of sides they have.
Following table gives the classification of polygons according to the number of sides they have.

Number of sides	Classification	A simple figure
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	

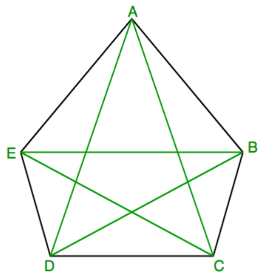
7	Heptagon	
.		
.		
.		
N	n-gon	

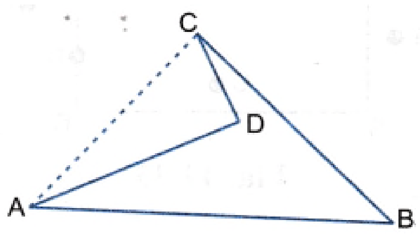
Here comes our Quadrilateral…!

Diagonals: In a polygon a diagonal is a segment connecting two non-consecutive vertices.

Examples:

(1) **(2)**





In figure one, BD, BE, AC, AD & EC all are diagonals.
In figure two can AC be a diagonal?

In figure 1) all the diagonals lie inside the polygon ABCDE. But in figure 2) AC lies outside the polygon ABDC.

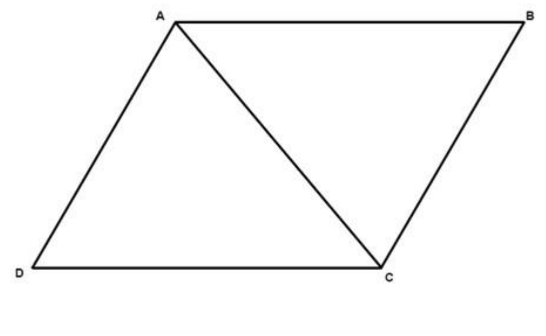
This observation gives us one more criterion on basis of which we can classify polygons.
We do classify polygons into two types
1) Convex polygons
2) Concave polygons

1) **Convex polygon** is a polygon in which all its diagonals lie inside it. The polygon in figure 1) is one such example.
2) **Concave polygon** is a polygon in which not all its diagonals lie inside it. The polygon in figure 2) is an example of concave polygon.

Quadrilaterals:

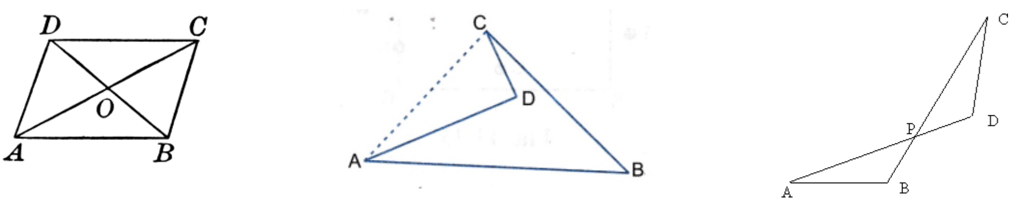
As mentioned above a quadrilateral is a polygon with four sides. Given a quadrilateral we can find 4 lines.

Then if we draw a diagonal in a quadrilateral ABCD, it is divided into two triangles. By doing so can we find the sum of all the angles of a quadrilateral?



Since sums of angles of two triangles triangle ABC and triangle ACD constitute the sum of the angles of quadrilateral. The sum of all the angles in a quadrilateral ABCD is 360^0. Because by the angle sum property of a triangle we know that the sum of the angles of both the triangles triangle ABC and triangle ACD is 180^0.

Note: The points at which two lines meet in a polygon is called as a vertex.
Let us consider following quadrilaterals



(1) (2) (3)

Let us look at the quadrilateral in figure 1). In that quadrilateral all its diagonals lies inside the quadrilateral ABCD.

Consider the quadrilateral ABCD in figure 2). In this, the diagonal AC lie outside the quadrilateral.

In figure 3) both the diagonals lie outside the quadrilateral and also the lines AD and BC are crossed each other at the point other than vertex.

Based on these criterion we can classify quadrilaterals as follows.

1) A simple convex quadrilateral:

It is a convex quadrilateral in which the sides only meeting at vertices. That is, a convex quadrilateral in which no two sides cross each other is called a simple convex quadrilateral.

2) A simple concave quadrilateral:

It is a concave quadrilateral in which the sides only meeting at vertex.

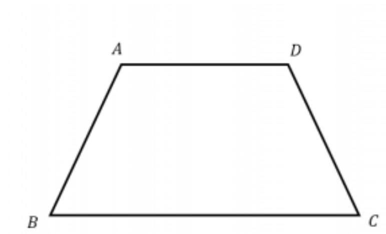
3) A crossed quadrilateral:

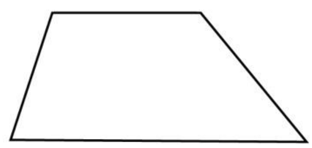
It is a quadrilateral with two of its sides crossing each other at a point other than vertices.

On the basis of nature of sides and nature of angles a simple convex quadrilaterals can be grouped as follows:

1) Trapezium:

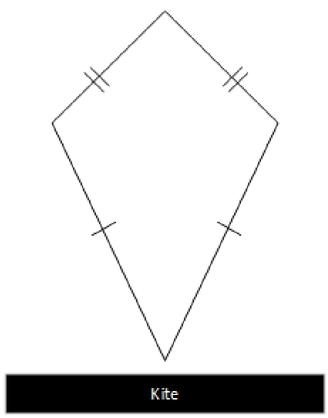
A trapezium is a quadrilateral with at least one pair of parallel sides.

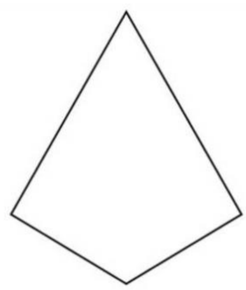




2) Kite:

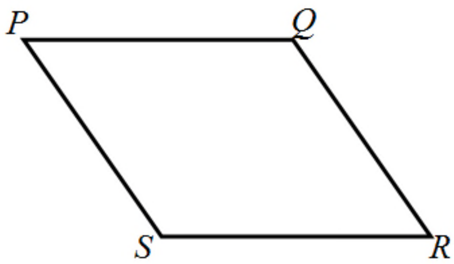
A quadrilateral whose four sides can be grouped into pairs of adjacent sides with equal length.

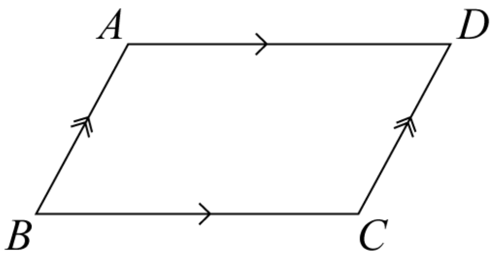




3. Parallelogram:

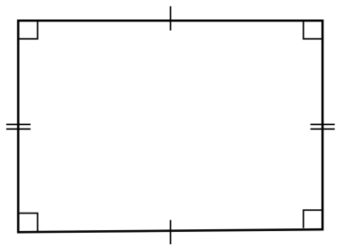
A quadrilateral with two pairs of parallel opposite sides is called as a parallelogram.

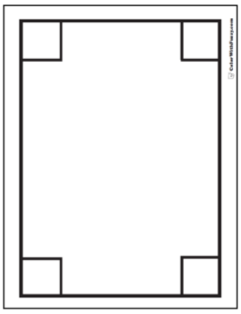




3) Rectangle:

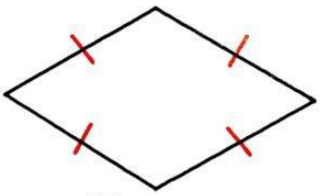
Rectangle is a quadrilateral with four right angles.

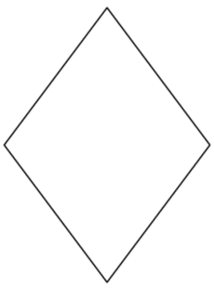




4) Rhombus:

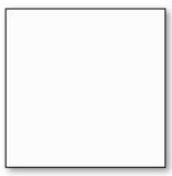
Rhombus is a quadrilateral with four sides having same length.





5) Square:

A square is a rectangle in which all the four sides have same length.



Is it possible to identify the hierarchical structure among quadrilaterals?
For example a parallelogram is a trapezium as it has a pair of parallel sides.
Is a rhombus a parallelogram also?

5. POLYNOMIALS

Definition and illustrations.

A polynomial is what we call any function that is defined by an equation of the form
$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0.$
where $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers or complex numbers which are called as coefficients of the polynomial.
Here x is called the variable. If we want to say specifically then p(x) is called a polynomial in the variable x.

We can have plenty of examples. Let us see few of them.
$p(x) = 2x^4 + 3x^3 + 7$, $p(x) = 2$, $p(x) = x^{10} + 7x^5 + 10x$ and so on.

Degree and leading term of a polynomial

Degree of a polynomial is defined to be highest power of the variable with non zero co-efficient present the polynomial is called it's degree.
For example the degree of the polynomial $5x^4 + 3x^2 + 8$ is 4,

Similarly degree of the polynomial $p(x) = 9$ is 0.

Then what we can say about the degree of the polynomial $p(x) = 0$, called the **zero polynomial**?
We do not assign any degree of the zero polynomial, as all its coefficients are zero.
The term with highest power is called the **leading entry of the polynomial** and the coefficient of the leading entry is called the **leading coefficient.**
For example in the polynomial $p(x) = 6x^2 + 2x + 1$, the leading entry is x^2 and leading coefficient is 6.

Classification of polynomials.

One characterization that we can consider in order to classify polynomials is degree of a polynomial.
Yes depending on the degree, a polynomial can be named as follows.

- A polynomial of degree **zero** is called a **constant polynomial.**
- A polynomial of degree **one** is called a **linear polynomial.**
- A polynomial of degree **two** is called a **quadratic polynomial.**
- A polynomial of degree **three** is called a **cubic polynomial.**
- A polynomial of degree **four i**s called a **bi-quadratic polynomial.**
- A polynomial of degree **five** is called a **quintic polynomial.** and so on..

Zero or root of a polynomial

For a given polynomial p(x) a number say r may be real or imaginary is said to be a zero of the polynomial p(x) if p(r) = 0.

For example consider $p(x) = x^2 + 2x + 1$. Then 1 and -1 both are roots.

Then given any polynomial does it have a zero?
YES. Given any polynomial of positive degree it has a root. This result we do call as **the "Fundamental theorem of Algebra".**
Then the next question comes is, how many zeros a polynomial can have?
YES. For this also we have an answer with us. **Given a polynomial of degree n, it has at most n roots.**

Relation between the roots and coefficient of a polynomial.

Let us consider a polynomial $p(x) = a_nx^n + a_{n-1}x^{n-1} +...+ a_1x + a_0$ with a_n not equal to 0. That is a p(x) is a polynomial of degree n. Then we know that p(x) has exactly n roots. Let $r_1, r_2,...,r_n$ be those n roots. Then we can write p(x) as
$p(x)=a_0 (x - r_1) (x - r_2) . . . (x - r_n)$
Equating the two expressions for p(x) we obtain
$a_nx^n + a_{n-1}x^{n-1} + . . . + a_1x + a_0 = a_n (x - r_1) (x - r_2) . . . (x - r_n)$

Dividing both the side by a_n we get,
$x^n + (a_1 / a_n) x^{n-1} + . . .+ (a_0 / a_n) = (x - r_1) (x - r_2). . .(x - r_n)$
$x^n + (a_1 / a_n) x^{n-1}+ . . .+ (a_0 / a_n) = x^n- (r_1 + r_2 +...+ r_n) x^{n-1} + (r_1r_2 + r_2r_3 +. . .+ r_{n-1}r_n + r_nr_1) x^{n-1}- ...+ (-1)^n (r_1 r_2 . . . r_n).$

Equating the corresponding coefficients of the powers of x we get,
$(r_1 + r_2 +. . .+ r_n) = -(a_1 / a_n),$
$(r_1r_2 + r_2r_3 +. . .+ r_{n-1}r_n + r_nr_1) = (a_2 / a_n),$
.
.
.
$r_1 r_2. . .r_n = (-1)^n (a_0 / a_n).$

Let us have some **illustrative examples.**
Consider the polynomial $p(x) = 2x^2 + 5x + 3$. Then the roots of this polynomials are -3/2 and -1.
Now we have -(3 /2) + (-1) = - (5 / 2), Which is same as $-(a_1 / a_2) = -(5 / 2)$ and
-(3 / 2) (-1) = 3 / 2, which is same as $(-1)^2 (a_0 / a_2) = 3 / 2$.
Similarly we can verify this for a polynomial of any degree.

6. GEOGEBRA ACTIVITIES

Geogebra is an interactive mathematical software. The word Geogebra is a combination of names of two branches of mathematics geometry and algebra. As the name shows this software combines both the branches and works as a path between various other branches of mathematics.

This software can be used from primary class to university level. This is available on multiple platforms.In geogebra constructions can be made with points, vectors, segments, lines, polygons etc. The major advantage of this software is ,all the drawn figures can be changed easily and dynamically. There are various input commands like square roots and inverse trigonometric functions.

The project of developing an interactive mathematics software was started in 2001 at University of Salzburg by Markus Hohenwarter. Now the project florishes at University of Linz together with the help of open-source developers who work without considering any profit.

Usage of geogebra need not be restricted to mathematics.Various branches like statistics,physics and even social science uses geogebra.State Schools of Kerala, geogebra is included in ICT syllabus.Maths text books of Kerala make cross references of geogebra applets. Teachers and students use geogebra as a teaching-learning tool.

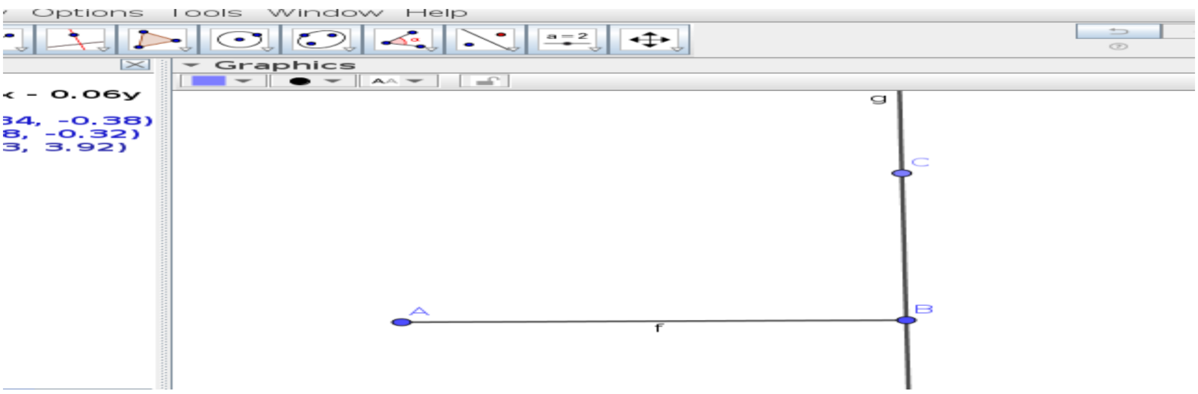
GeoGebra depends on software licensed under the GNU General Pub- lic License (GPL), the LGPL, the Apache license and others. The software is licensed under the ”GeoGebra Non-Commercial License Agreement”. The International GeoGebra Institute (IGI) works with more than 140 (in 2014 March) user groups at universities and non-profit organizations around the
world. We can download geogebra from the website www.geogebra.org free of cost.

Here we will present some exemplar activities using geogebra.

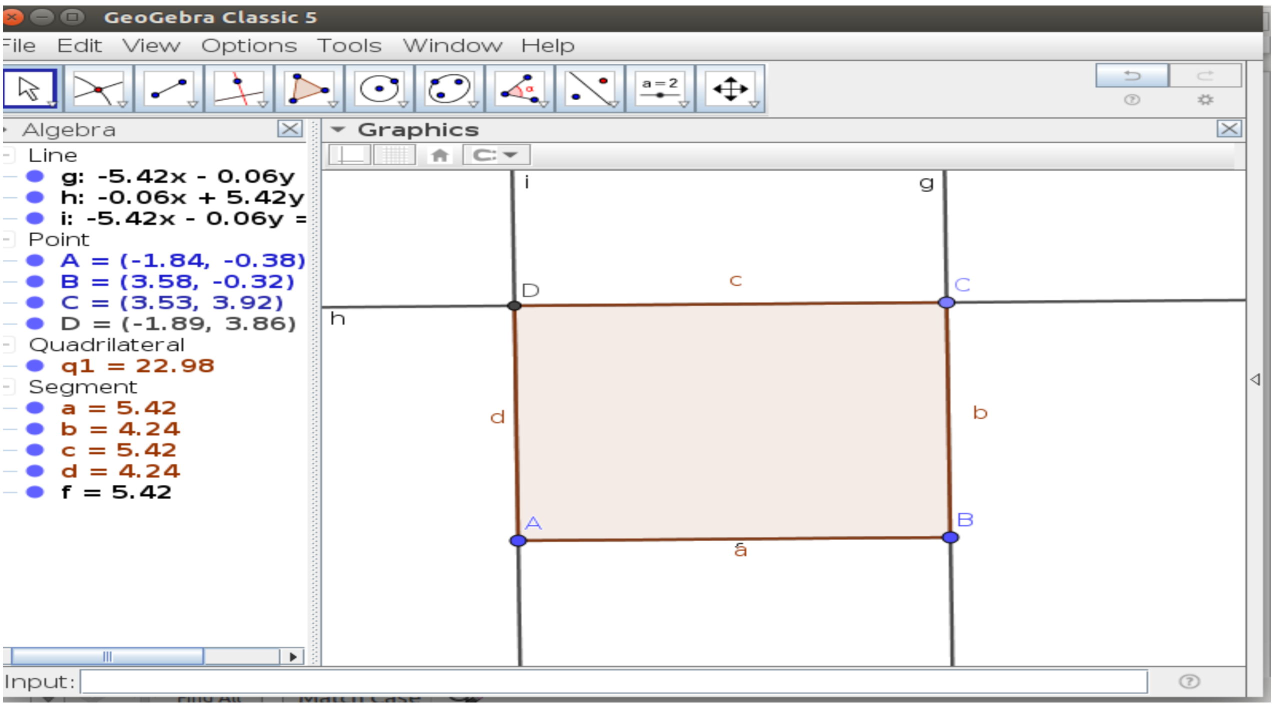
Activity 1: Rectangle construction

Question : Construct a rectangle ABCD

1. Create segment AB
2. Create perpendicular to segment AB through B.
3. Insert a poit C on the perpendicular line



4. Construct parallel line to segment AB through the point C.
5. Create a perpendicularline to the segment AB through A.
6. Construct intersection point D.



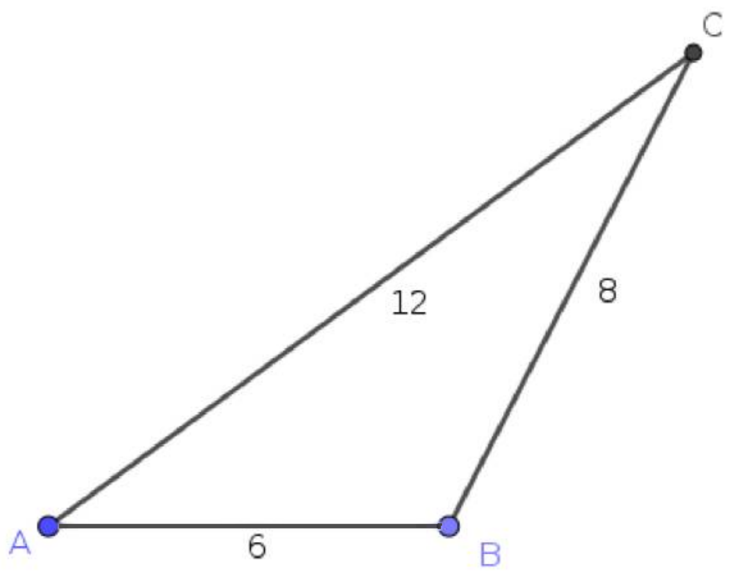
7. Create polygon ABCD

Activity 2 : Constructing a triangle with given measures

Question : Construct traingle ABC with sides AB=6 units, BC= 8 units, CA= 12 units

1. Choose segment with given length tool .
2. Select point A.

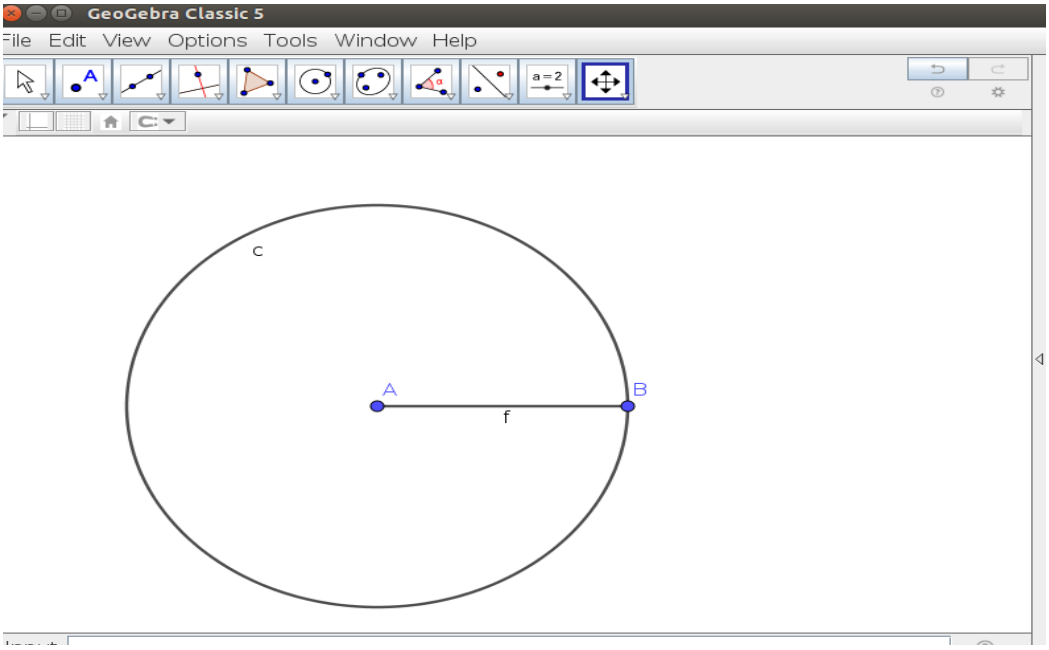
3. Enter 6 in pop up for length of segment.
4. Select circle with radius tool.
5. Click on A
6. Enter radius as 12
7. Click on B.
8. Enter radius as 8.
9. Choose point of intersection tool.
10. Click on point of intersection of circles c and d to get point C.
11. 11. Join AC and BC with segment tool.
12. 12. Hide circles c and d



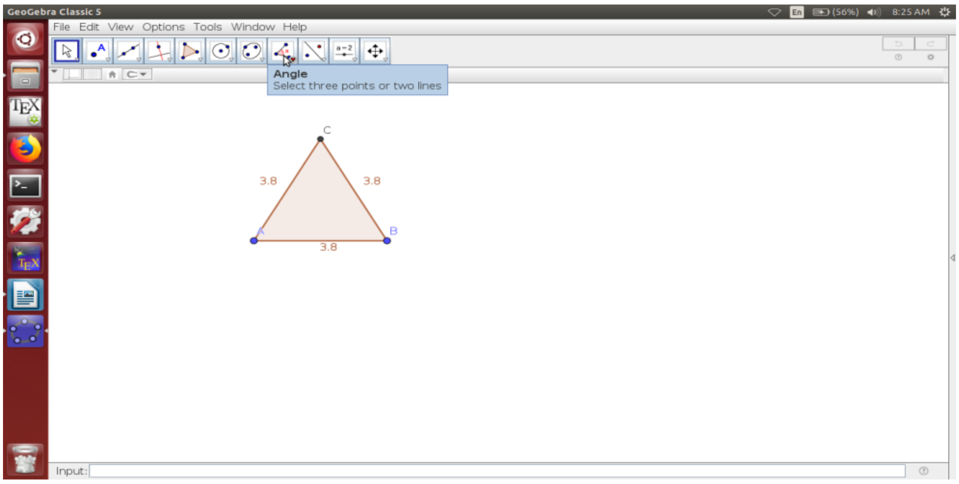
Activity 3 : Constructing an equilateral triangle.

Question: Construct an equilateral triangle

1. Create segment AB.
2. Construct circle with center A through B.



3. Construct circle with center B through A
4. Intersect both circle to get C.
5. Create polygon ABC in clockwise direction.
6. Hide circles.



7. Show the interior angles by choosing angle tool and clicking somewhere inside the triangle.

Activity 4 : Linear Pair

Question : Construct an applet to introduce linear pair

Step 1: Draw a line AB using ***segment tool***

Step 2 : Mark a point C on the line using ***point tool***

Step 3: Draw a circle with centre C,using ***circle with center*** ***through point*** tool

Step 4: Mark a point D on the circle using ***point*** tool

Step 5: Draw the line CD .

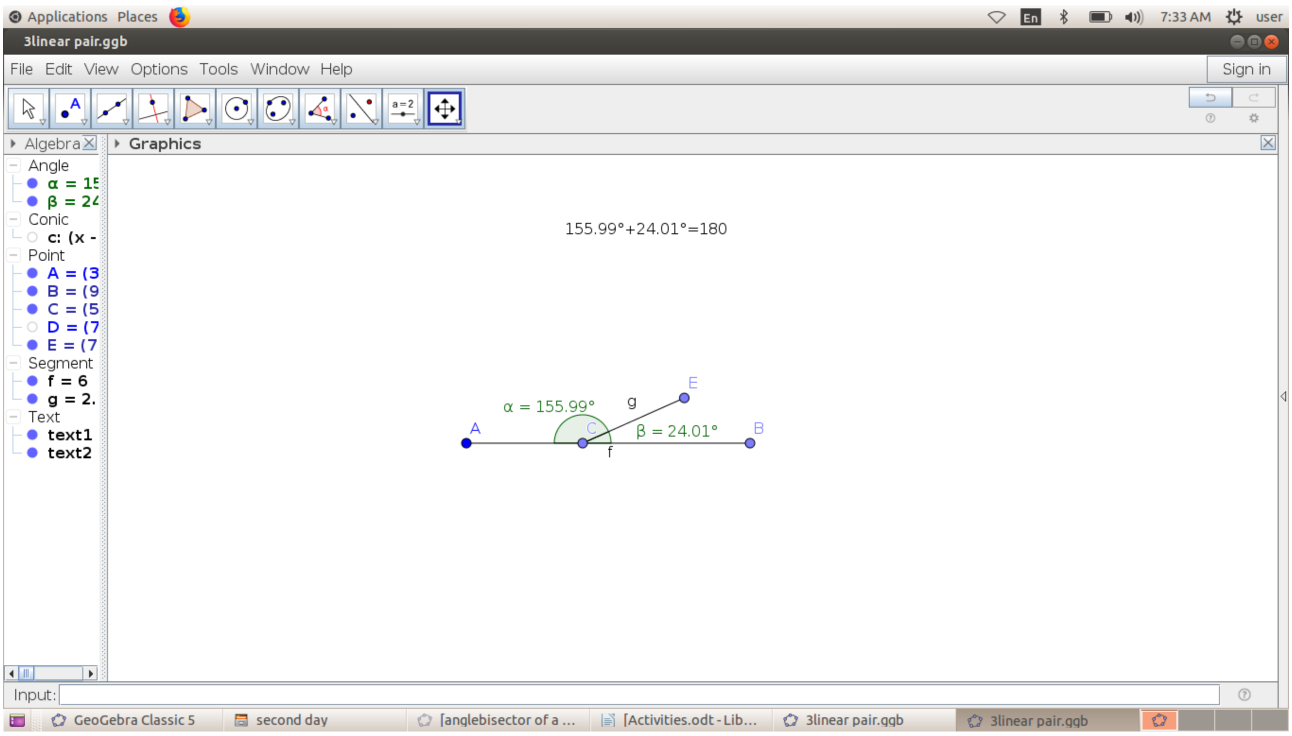
Step 6: Hide Circle.

Step 7: Mark the measure of <BCD using ***angle*** tool

Step 8:Mark the measure of <ACD

Step 9: Move point D using ***move*** tool

Step 10 : Observe the Sum of the two angles



Activity 5 Vertically opposite angle

Question : Construct an applet to observe the properties of vertically opposite angles when two line sgments intersect

Step 1: Draw a circle with center A

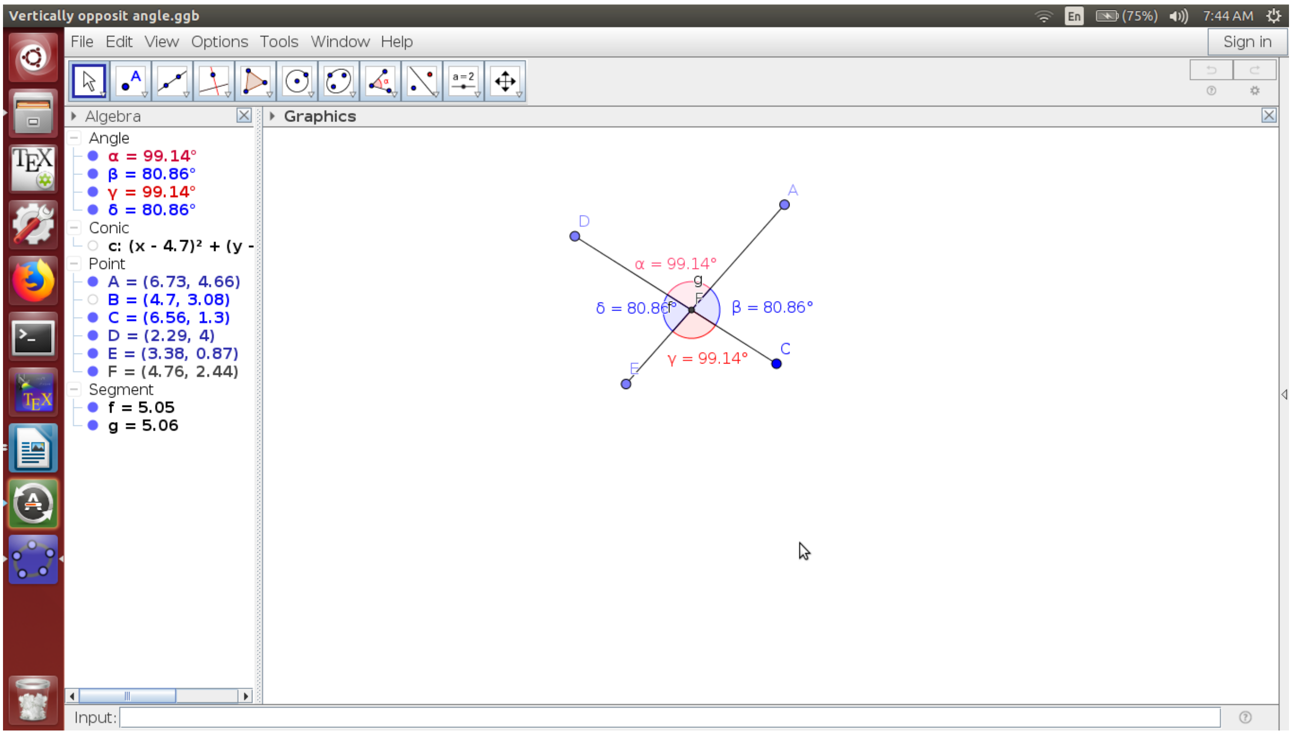
Step 2: Mark four points A,B,C,D on circle

Step 3: Join opposit points ,Ac and BD

Step 4: Hide circle

Step 5: measure the four angle measures using ***angle*** tool

Step6: Observe the property of opposite angles



7. PROBLEM SOLVING

Problems on Numbers

1. Angles of a polygon are in the arithmetic sequence 120,125,130,... What is the sum of all angles of the polygon.

2. A body is moving along a straight line in same direction.Speed of the body is increasing uniformly at the rate of 4m/s in every second. Speed of the body in 4^{th} second is 19m/s and in 9^{th} second is 39m/s .

(a) Write the sequence of velocity of the body at the ends of 1s,2s,3s,...

(b) Write the sequence of distance travelled by the body in each second

(c) What is the speed of the body at the 10^{th} second and what is the distance travelled by the body during 10seconds ?

3. Form an arithmetic sequence with common difference 4 and sum of first 8 terms is equal to sum of first 12 terms.

In an arithmetic sequence -60,-54,-48,...sum of first n terms is equal to sum of first m terms .What is the maximum value of m and n?

4. Consider two types of triangular patterns formed by using the terms of sequence 5,9,13,...as given below:

5	5
9 13 17	9 13
21 25 29 33 37	17 21 25
....	....
....	....

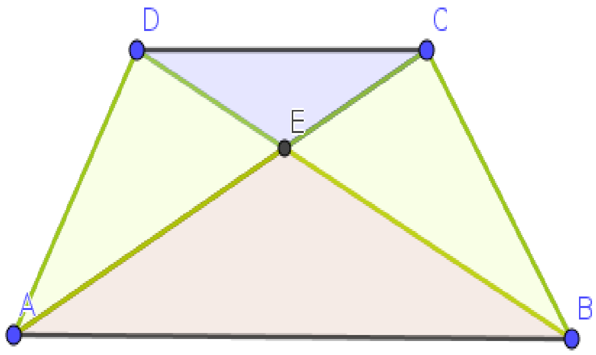
Find 3^{rd} term in 16^{th} raw of each pattern.

5. Form an arithmetic sequence of natural numbers of common difference 8 and have no any perfect square term.

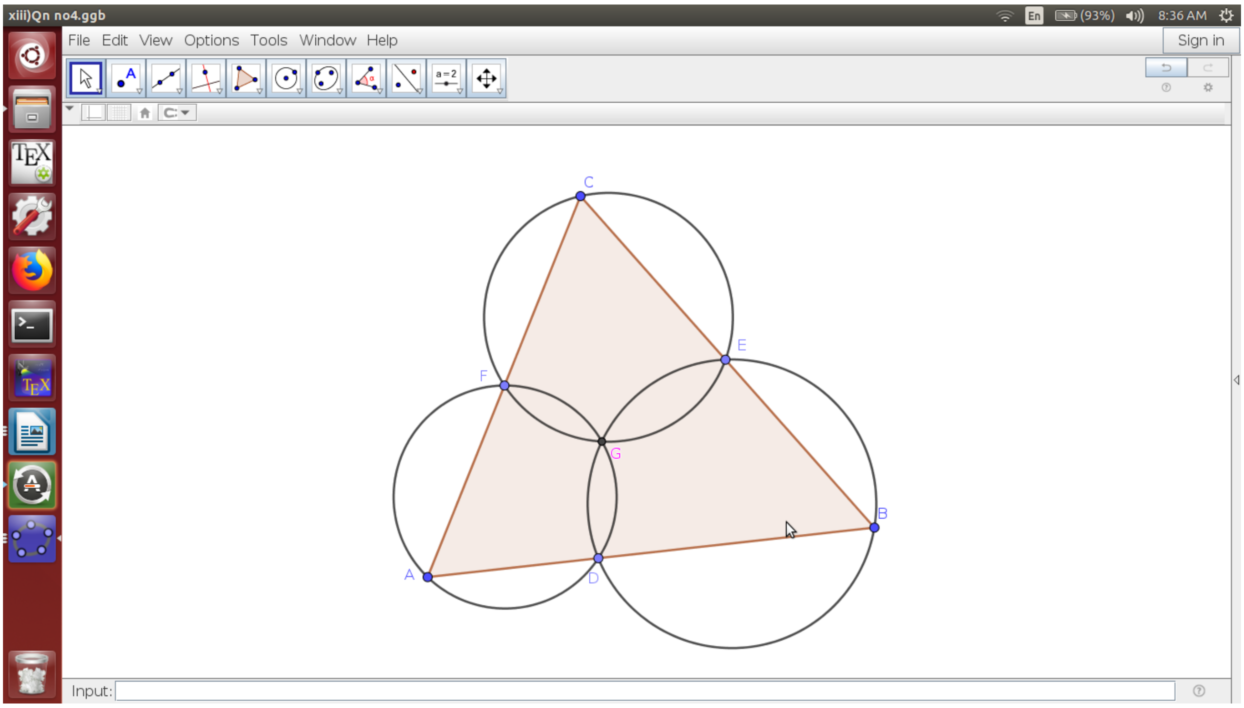
6.A coir of length 20m is bent in the form of a rectanglewith a long wall as one side. Prove that maximum area of the rectangle is 50 sq cm.

Problems on Geometry

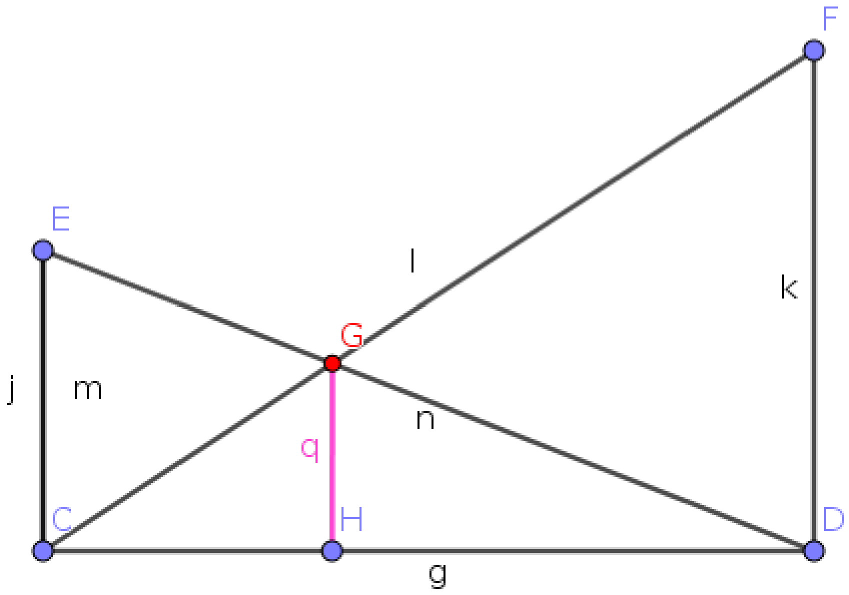
1. What is the Perimeter of a triangle whose sides are 1 units and 99 units ?
2. ABCD is a trapezium.Diagonals are meeting at E. Area of coloured triangles are 36 and 25 sq cm. What is the area of the trapezium.



3. Prove that the circles with two sides of a triangle as diameters are pasing through a point on third side.
4. ABC is a triangle.D,E,F are 3 points on the sidesAB,BC,AC. Prove that the circumcircles of triangles AFD,DEB and CEF are concurrent.



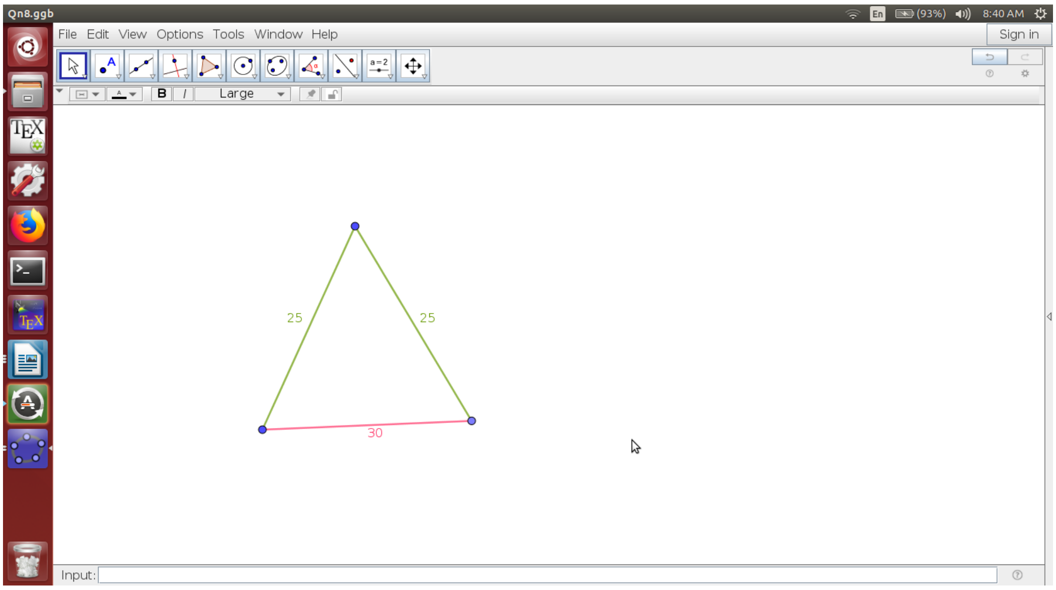
5. CE and DF are two poles mounted vertically on a level ground from the top of the pole CE a string is connected to the foot of the pole DF and another string is connected from the top of DF to C. They are intersecting at G.Whatever maybe the distance between the poles, prove that height of G from the ground is same.



6. A coir of length 20m is bent in the form of a rectangle with a long wall as one side .Prove that maximum area of the rectangle is 50sqcm.

7. Area of a right triangle is 54sqcm and length of its hypotenuse is 15cm .Calculate its radius.

8. what is the area of largest semicircle that can be inscribed in a triangle of sides 25,25,30.



8. CRITICAL MATHEMATICS EDUCATION

Introduction

- Mathematical education has evolved into a domain on its own
- Draws frameworks from Philosophy, Sociology, and Psychology fields
- Mathematics education has asked many key questions on to the existing practices of teaching and learning
- Some of these questions have been very critical for the development of the field.
- Today we explore one set of such key questions that has shed light on some significant aspect of the teaching and learning of mathematics – critical mathematics education

Agenda

- What does 'critical mathematics education' mean? What is a critical attitude of mind?
- What different sorts of mathematics are there?
- What are the aims of teaching and learning mathematics?
- What factors influence students' learning and how do we know that?
- How should learning mathematics empower learners?
- How to implement critical mathematics education curriculum?

Critical?

- What is this critical attitude of mind?
- General understanding of being critical - being inclined to judge severely and find fault
- 'Being critical' is about engaging in a critique, making careful judgements, using all available evidence, reasoning and balanced arguments to evaluate claims and to reach conclusions.
- Not taking traditional explanations and views for granted but questioning them to see if they stand up
- 'Critical mathematics education' is about this critical attitude of mind applied to mathematics and its teaching.

Critical questions about the nature of mathematics

- Is there an agreement about nature of mathematics?

- Absolutist conception of mathematics
    - Timeless, although we may discover new theories and truths to add
    - Superhuman and untouched by social and historical developments
    - Pure, abstract, isolated and wholly logical knowledge
    - Value-free and culture-free

Critical questions about the nature of mathematics

- Is there an agreement about nature of mathematics?
    - Fallibilist conception of mathematics
        - The argument is not that some or all of mathematical knowledge may be wrong
        - But that mathematics is a historically developing area in which ideas of truth and proof as well as mathematical theories, rules and results themselves are modified, changed and redefined over time
            - 1 + 1 = 2 is true, but only for as long as we keep our definitions and rules fixed
            - Law of commutativity for quaternions (a complex number of the form w + xi + yj + zk, where w, x, y, z are real numbers and i, j, k are imaginary units that satisfy certain conditions.)
        - Fallibilism does not assert that the old mathematics is false; it is just that in the new system the old truth is no longer true

Fallibilist views of nature of mathematics

- Fallibilist views reject the notion that there is a unique, fixed and permanent hierarchical structure comprising mathematical knowledge. Instead fallibilist views see mathematics as made up of many overlapping structures.
- A feature of the fallibilist view is that mathematics is seen to be made up of (and in) different social practices, each with its history, persons, institutions, symbolic forms, purposes and power relations.

Is there an agreement about nature of mathematics?

- Mathematics can no longer be taken to be above dispute
- Mathematics is created by humans in their various cultures

Ethnomathematics

- Ethnomathematics—the culturally embedded mathematics outside the bounds of academia

- From marble exchange values in British school playgrounds,
- Symmetrical sand drawings in Mozambique,
- Islamic tiling patterns,
- Kabbalistic numerology in medieval Europe
- knotted Quipu strings in Central and South America

Ethnomathematics

- An ethnomathematical or cultural view of mathematics argues that mathematics is an intrinsic part of most people's cultural activities, and that academic mathematicians have appropriated, decontextualised, elaborated and concentrated that mathematics, until it seems to have a life of its own, thus denying its ethnomathematical origins.

- From the ethnomathematical perspective, this is a historical and philosophical falsification.

Ethnomathematics

- Mathematics is cultural knowledge that derives from humans engaging in the six universal activities of counting, locating, measuring, designing, playing and explaining in a sustained and conscious manner (Bishop 1988).

- The traditional view of mathematicians is that it is their specialist knowledge that is applied to real world and other problems, and in watered-down form used in informal cultural contexts (i.e. ethnomathematics).

Critical question

- What kind of mathematical knowledge we need to expose students to? An absolutist mathematics? A fallibilist mathematics? Or even ethnomathematics?
- If so how, a question we take it up for discussion later.

Critical questions about the aims of teaching mathematics

- Why do we or should we teach mathematics?
- What are the purposes, goals, justifications and reasons for teaching
- it?
- How can current mathematical teaching plans and practices be justified?
- The aims of mathematics to be considered in relation to society, because aims reflect the intentions of individuals or groups.
- Different individuals would respond differently to the earlier questions

Mathematical aims	*View of mathematics*	*Typical group members*	*Name for the Group*
Acquiring basic mathematical skills and numeracy and social training in obedience (authoritarian, basic skills centred)	Absolutist set of decontextualised but utilitarian truths and rules	Radical 'New Right' conservative politicians and petty bourgeois	Industrial Trainers
Learning basic skills and learning to solve practical problems with mathematics and information technology (industry and work centred)	Unquestioned absolutist body of of applicable knowledge	Meritocratic industry-centred industrialists, managers, etc., New Labour	Technological Pragmatists
Understanding and capability in advanced mathematics, with some appreciation of mathematics (pure mathematics centred)	Absolutist body of structured pure knowledge	Conservative mathematicians preserving rigour of proof and purity of mathematics	Old Humanist Mathematicians
Gaining confidence, creativity and self expression through maths (child-centred progressivist)	Absolutist body of pure knowledge to be engaged with personally	Professionals, liberal educators, welfare state supporters	Progressive Educators
Empowerment of learners as critical and mathematically literate citizens in society (empowerment and social justice concerns)	Fallible knowledge socially constructed in diverse practices	Democratic socialists and radical reformers concerned with social justice and inequality	Public Educators

Few conceptions of learning mathematics

- Learning Mathematics is learning new skills – computation and
- Learning Mathematics is all about 'doing' Mathematics
- Learning Mathematics is learning to solve problems
- What is a critical mathematics education perspective?
- Students should be able to think mathematically, use it in their lives to empower themselves both personally and as citizens, and appreciate its role in history, culture, and the contemporary world.

What factors influence student learning?

- Old argument – underperformance need to be understood in terms of the biological structures, their learning obstacles had nothing to do with the school structure

- New argument - social aspects play a fundamental part in a person's intellectual and emotional development. Learning obstacles are established beforehand and are not to be located in the school structure. **Learning obstacles**

- Physical learning obstacles
- The actual distribution of wealth and poverty includes a distribution of learning possibilities and learning obstacles.
- Learning obstacles can take the form of a **ruined foreground**, and that ruining the foreground of a certain group of children is a socio-political act.

Background and foreground

- Meaning and background - 'cultural background' should not remain the only key notion when meaningfulness in mathematics education is discussed.
- **Foreground** of a person is the opportunities, which the social,political and cultural situation provides for this person. However, not the opportunities as they might exist in any socially well-defined or 'objective' form, but the opportunities as perceived by a person.

Foreground

- Why does foreground so important in learning?
- Learning as an intentional act as intentionality or intentions-in-action is a defining element of an action
- The intentions of a person are not simply grounded in his or her background, but emerge also from the way the person sees his or her possibilities.
- Actions become not simply caused by the past, but represent forms of grasping the future.
- Their foregrounds might prevent them from putting effort into the designated activities.

Foreground

- Achievement (low or high) as being related to the opportunities that the school structure and the sociopolitical context in general make open for the students to perceive as their opportunities.
- ***When a society has ruined the future of some group of children, then it has also obstructed the incitements of learning*. A ruinedfuture can be the most brutal form of learning obstacle.**
- In order to establish meaning in education, students should be involved in meaning production, and each student's foreground is an essential resource for this production.
- **Meaning not only represents the past. It also represents the present and the future.**

Learning obstacle

- 'Schooling' can be seen not only as a support for entering the network society, but it can also become a gatekeeper, and an 'excluder' from the network society.
- It is important that mathematics education provides opportunities and when these appear to be real opportunities, seen from the students' perspectives, they can become active in their processes of learning.
- A learning obstacle becomes acted out as a learning resistance.

Practical approaches to critical mathematics education

- Use examples, discussion points and projects within the mathematical curriculum with relevance for pupils:
  - immediate personal relevance by getting learners engaged in activity;
  - evident utilitarian and examination relevance;
  - Links with their out-of-school interests;
  - connections with local, regional, national and global issues that affect learners or humanity more widely.
- Include activities which require cooperation, discussion, creativity, judgement, and for which there is no ‘right’ answer; pursuing cross-curricular links and avoiding demarcating subjects strictly;

Practical approaches to critical mathematics education

- Critical citizenship through mathematics

  Local Environment
  - Local survey of shops: types including charity shops and closed shops; changes over the years.
  - Parks and playgrounds: types, size, safety.
  - Survey of homeless: local and national statistics.
  - Analysis of accident black spot data.
  - Should there be more children’s parks, schools and shops on new housing estates? Are shops, post offices and postboxes placed in the best positions?
  - What happens to local roads when a new housing development is built?
  - Just how much does it cost to build a house in the area? How much is it sold for?
  - Develop projects of local interest—publish the results in the local press to show that school projects are valued by the community.

Practical approaches to critical mathematics education

- Critical citizenship through mathematics

  Environment
  - Pollution, destruction of the rainforest, comparative surface areas; local and national figures for recycling—is it actually worth driving to the recycling center?
  - Paper use in the school, taluk, and district.

- Petrol usage—how much pollution is caused by different cars? Who causes most pollution? Who drives the most polluting cars? Which country pollutes more? Which countries suffer most?
- Discuss and challenge controversial statistics about the environment, etc. Is the world getting hotter?

Practical approaches to critical mathematics education

- Critical citizenship through mathematics

  Local 'folk' mathematics

  - What mathematics does the school caretaker use? What about a carpenter? Bricklayer? Nurse? Doctor? Find out where mathematics is used in different jobs by inviting people in and taking pupils on trips.

Practical approaches to critical mathematics education

- Critical citizenship through mathematics

  Health education

  - Analysis of mortality, life expectancy by state, district and other demographics.
  - Find out the profits of the three main tobacco companies in the world.
  - Given that it kills you, why do people smoke?
  - How much does the tobacco industry get in profits for each person who dies
  - from smoking-induced lung cancer?

Practical approaches to critical mathematics education

- Critical citizenship through mathematics

  Political data

  - Statistics on employment, education, mental health among various social groups
  - Gender and exam success: using ASER and DISE data, school, district and state data
  - Gender stereotypes in adverts and newspapers.
  - Comparative spending in India and other countries on education, health, defense, etc.
  - Analysis of unemployment figures by region, qualification, social class, gender.

Mediating between concrete and abstract - RME approach

9. LEARNING MATHEMATICS

Supporting learning of mathematics

- Making learning meaningful
- Making experiences meaningful
- Making the experiences concrete

Concretizing the experiences

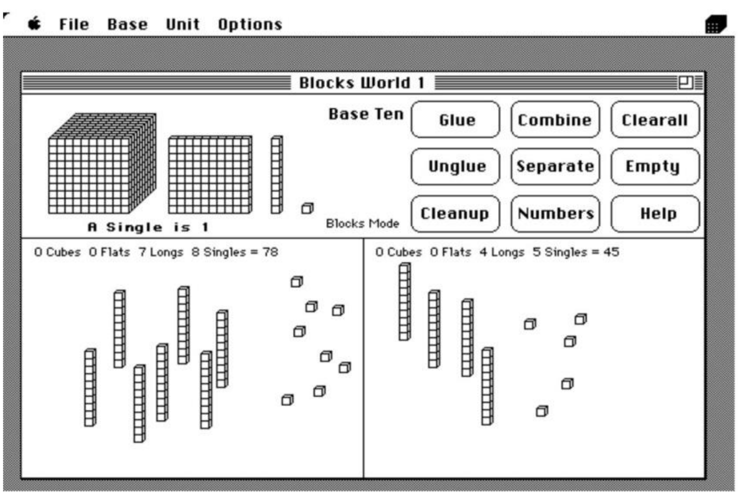
- Popular approaches:
  Use of teaching aids
  Use of concrete materials

- What has been the effect?
  Researches show that concretizing the experience doesn't necessarily helpful

- An example: Dienes blocks

Student learning of number structure with
Diene's blocks

- Diene's blocks
  Numbers are represented by singles, longs, flats and cubes

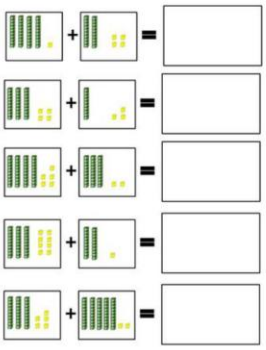


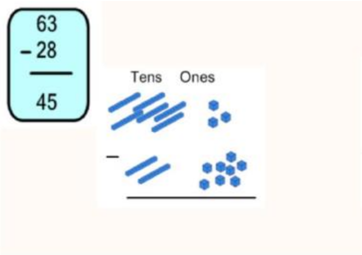
How does it work?

- Exchange groups of ten blocks for a higher-order blocks if there are more than ten of a kind
- Notate the number of blocks in a strict order, corresponding with an increase in value

Concretizing learning experiences by Dienes Blocks

- Dienes blocks are used to show
  - number structure
  - To carryout number operations (addition/subtraction/multiplication/division)
  - Transition from concrete to semi concrete and then to abstract



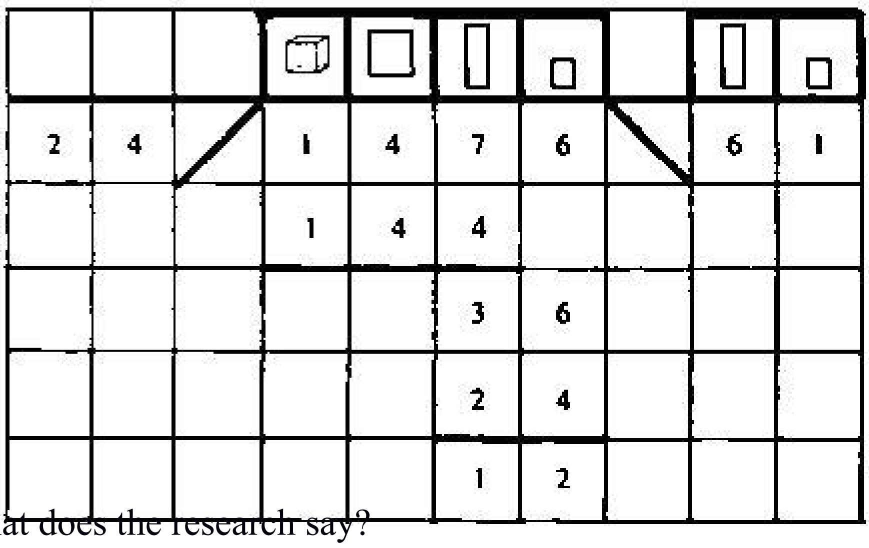


Long division with Diene’s block model

- Long division is built on the idea of division as fair sharing. One starts with small numbers. A problem like 81÷6, will be thought of as dividing 81 blocks over 6 persons. First 6 of the 8 ten-rods are distributed, then the remaining tens are exchanged for ones, and the resulting 21 ones are divided, each gets 3 and the remainder is 3.
- Next, to make the shift from dealing with blocks to the paper and pencil algorithm, a procedure like the one outlined above is soon replaced by procedures executed with imaginary blocks, using a standard form that resembles the written algorithm

Long division with Diene’s block model

- Take for instance 1476÷24. Here the students would have to exchange a one thousand block and four squares of one hundred to get 147 rods that will have to be divided over 24 people



What does the research say?

- Instructors expect the blocks to help students see the number structure in concrete form
- Students do not gain much insights (Labinowitcz, 1985; Resnick and Omanson, 1987)
- Students lack the capabilities of applying the concepts and procedures (Schoenfeld, 1987)
- Why does this happen?
- Is it counter intuitive?

Why does this happen?

- Though the position system is represented, it is not an exact representation – as in the positional notation change of order of digits change the number but that is not the case with the Diene’s blocks
- Hence more rules are added

  like how they have to be ordered

  What is the sequence of an operation like a subtraction – one always has to start from the unit’s place

Why does this happen?

- This is done in order to be consistent with the algorithms they are learning later on but contrary to a common sense approach
- This means that the main concern is not the positional notation system but the algorithm

Why does this happen?

- Although the model is concrete, the mathematics embedded in the model is not concrete for students
- Students see concrete objects but not concrete mathematics
- Teachers make sense of the mathematics embedded in the concrete materials because, for them the concepts are already attained. In this case, the number structures are already in place in their mind. Hence they see the blocks being revealing the number structure
- The model is taken from the abstract mathematical knowledge

What do the researches indicate?

- Need for domain specific, situated, informal knowledge and strategies
- Rather than teaching the concepts and procedures to apply them in problems – i.e., to delay the application of concepts and principles
- Problem may be in neglecting the informal knowledge and strategies

What is the alternative approach?

- Realistic Mathematics Education (RME) – an approach followed by Netherlands, based on the research by Freudenthal Institute

Long division in RME

- In the realistic approach “contextual problem" are used as a starting point, preferably problems that allow for a variety of informal solution procedures; that is to say, applied problems precede instruction on the algorithm
- Children of about 8 or 9 years old were asked to solve the following problem:

Tonight 81 parents will be visiting our school. Six parents can be sated at each table. How many tables do we need?

Context

- Contextual problems describe situations where a problem is posed. More often this will be an everyday life situation, but not necessarily so; for the more advanced students, mathematics itself will a context.

Student strategies

- The students produced all kinds of solutions:
- Some used repetitive addition: 6 + 6 + 6 + ... , or stepwise multiplication, probably based on addition, I x 6, 2 x 6, 3 x 6, ,
- Some only wrote down the resulting sequence 6, 12, 18, ... ;
- Some used 10 x 6 as a starting point, in order to continue by multiplication
- One student knew 6 x 6 = 36 by heart, which was doubled to get 12x6=72, one 6 was added, and finally one more 6.

Student strategies

- Teacher simulated to compare their solutions. Most found the first jump to 10 x 6 a handy short-cut. When a similar problem (concerning the same night at school) was administered afterwards, it appeared that a substantial number of the students imitated the 'ten times' short-cut spontaneously. The problem read like this:

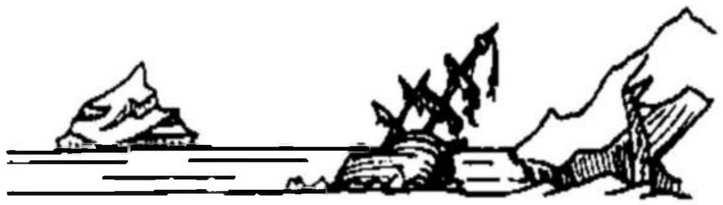
- One pot serves seven cups of coffee. each parent gets one cup. How many pots of coffee must brewed for the 81 parents?

Problems and Solutions

(a) 34 / 467 \ 13.7
34–
127
102–
250
238–
12

(b) 34 / 467.0 \ 10+3+0.7=13.7
340– [10 × 34]
127
102– [3 × 34]
25.0
23.8– [0.7 × 34]
1.2

FIG. 13.4. (a) Standard procedure and (b) Interpretation.



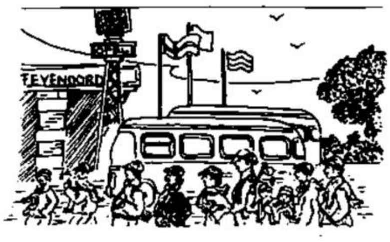
The captain of the stranded ship is told that there are 4000 biscuits left. The crew consists of 64 members. Each man gets 3 biscuits a day, which means 192 biscuits a day for the whole crew. How long will this supply last?

FIG. 13.5. Overwintering in Nova Zembla.

4000		4000		4000	
<u>192</u>	−1 day	<u>192</u>	−1 day	<u>1920</u>	−10 days
3808		3808		2080	
<u>192</u>	−1 day	<u>384</u>	−2 days	<u>1920</u>	−10 days
3616		3424		160	
<u>192</u>	−1 day	<u>768</u>	−4 days		
3424		2656			
<u>192</u>	−1 day	<u>1536</u>	−8 days		
etc.		etc.			

FIG. 13.6. Repetitive subtraction of smaller or larger quantities.

Problems and Solutions



1296 supporters want to visit the away soccer game of Feijenoord. The treasurer learns that one bus can carry 38 passengers and that a reduction will be given for every ten buses.

FIG. 13.7. Feijenoord.

8 / 1296	\	36 / 1296	\	36 / 1296	\
380 -	10x	380 -	10x	1140 -	30x
916		916		156	
380 -	10x	760 -	20x	152 -	4x
536		156		4	
380 -	10x	76 -	2x		
156		80			
38 -	1x	76 -	2x		
118		4			
38 -	1x				
80					
38 -	1x				
42					
38 -	1x				
4					

FIG. 13. 8. Various levels of curtailment.

Realistic Mathematics Education (RME)

- Mathematics as a human activity
    - Mathematics has to be taught in order to be useful
    - Could not be accomplished by teaching a 'useful mathematics'
    - Mathematics should be taught as mathematizing
    - Doing mathematics is more important than learning mathematics as a ready-made product

RME

- Mathematisation is not just a translation into ready-made symbol system
- A way of symbolizing might emerge in the process of organizing the subject matter
- Making more mathematical
    - Generality: generalizing
    - Certainty: reflecting, justifying, proving
    - Exactness: modeling, symbolizing, defining
    - Brevity: symbolizing and schematizing
- Mathematics education for young children should aim at mathematizing everyday reality as there is no mathematical matter that is experientially real to them

RME

- Ideas to deal with learning mathematics:

- Guided reinvention – route to learning along which a student is able to find the intended mathematics by himself/herself has to be mapped out

    Start with a thought experiment – use history of math as a heuristic device

    Emphasize on the learning process rather than on inventing as such

    Selection of a contextual problems that allow for a wide variety of solution procedures

    Planning of possible learning trajectories – various problems to be posed, anticipated mental activities of the students, action that should be taken to facilitate reinvention

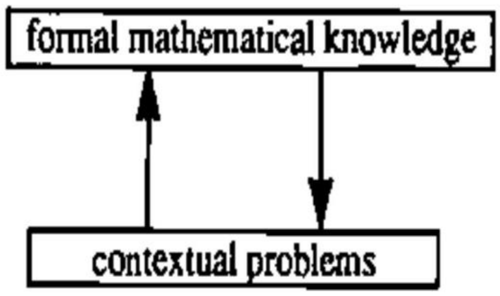


FIG. 13. 9. Application of formal mathematics.

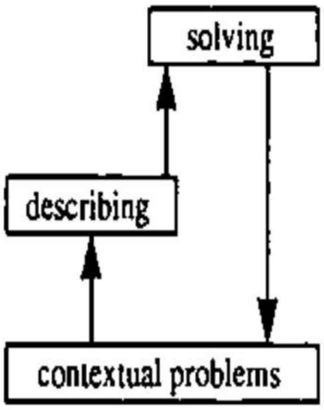


FIG. 13.11. Realistic problem solving.

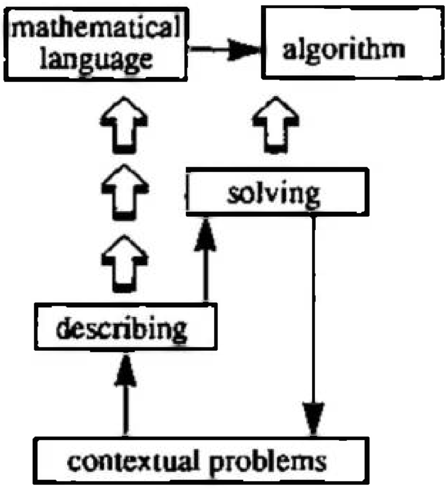


FIG. 13.12. Vertical mathematising.

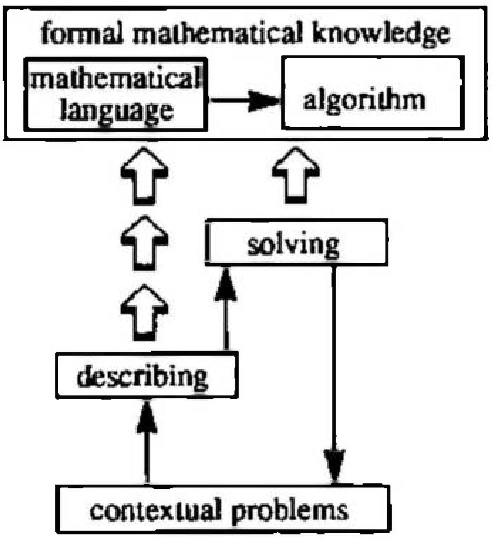


FIG. 13.13. Reinvention.

More problems

1. 26 passengers have to be transported by cars.
   Each car can carry 4 passengers.
   How many cars will be needed? [7]
2. A rope of 26 metres is cut into pieces of 4 metres.
   How many pieces does one get? [6]
3. If 26 bananas are to be fairly divided among 4 people,
   how many bananas will each get? [6½]
4. A 26 km walk is divided into 4 equal stretches.
   How long is each of them? [6.5]
5. A rectangular pattern of 26 trees with 4 trees per row,
   how many rows will there be? [?!!]
6. A rectangular terrace with a size of 26 square metres
   has a width of 4 metres. How long is this terrace? [6.5]

Self-developed models

- Models are less prominent
- Models differ in role and character
- Intermediate model approach envisage models as two way intermediary between situated knowledge and formal knowledge
- RME models - Situation models and mathematical models
- Situation models - A context specific *model of* situation
- Mathematical models - a *model for* mathematical reasoning on a formal level

11. Activity Based Teaching in Mathematics

Mathematics has become important because "it works". By working we mean that mathematics has become an integral part of the world in which we live. Mathematics helps us to understand our environment. It is not an oversight and exaggeration that mathematics controls the nature. Our primary purpose should be to understand nature in order that we may learn better to shape the bountiful awards of nature and co-exist on this earth with all living things. We have all seen the results of man's attempt to master his environment, construction of dams across rivers, construction of bridges, tunnels, etc. Mathematics is an invaluable tool to understand the nature.

Mathematics is the language of science and directly related to this is the fact that mathematics is the means for communication between scientist. If we emphasize the language role that mathematics is fulfilling we not only add interest and importance to subject but we also point out important consideration for learning of the subject. If a pupil is conversant with symbolism of maths and what it represents, if s/he has good head, start on learning how to explore the variety of concepts that attracts her/him. Maths is still an abstract system of ideas and must be seen as such by our students if we are to present an accurate picture to them. Thus as the student use maths to solve problems, we shall have to be prepared to indicate clearly, how the mathematics things and physical things are needed for being one and the same. This becomes particularly important in the study of geometry as it is easy to confuse and visual representation of mathematical concepts with concepts themselves. Helping students begin to formulate concepts of nature of mathematics is equally important. Mathematics perhaps the finest creation of mind of man as such it stands as an example of what heights man may reach when he relies upon her/his power and reasoning, Certainty and permancy are not the characteristic of many fields of endeavour in present century society. But they are accessible to ideas in mathematics, through the use of logic. This alone would serve to establish an unusual position for mathematics among many field of endeavour that abound in modern society. Finally we must consider mathematics as a study of possible patterns both in the world around us and in the structure of discipline of mathematics itself. There are regularity and similarities in nature, we can classify the thing of nature into one-dimensional, two-dimensional, --- so on to n-dimensional spaces. The most remarkable fact of about all these is the study of problems in the real world triggers the development of new mathematics. Thus in the teaching of school mathematics we may be able to make an important beginning in helping students to realise the importance of the search for patterns through their own participation in the search.

Mathematics is called science of space and figures; Science of numbers in general, It is a subject which develops thinking among pupils. This thinking leads to reasoning through How and Why – which leads to permanent learning. Its language is called mathematical language and its knowledge is called mathematical knowledge. It uses mathematical knowledge in the daily life situations there by solves the problem and becomes gate and Key for other subjects. It has precision and accuracy, verification of results. All these leads to mental satisfaction.

The subject mathematics though it has been made as the core subject at school education, is the central focal point, many of the students tries to run away from the subject. Citing various reason like- fear, phobia or disliking etc. The major reason is that they have attempted to learn the language of mathematics and then try to understand mathematics. Even the mathematics teacher does not realize this aspect but goes on teaching mathematics mechanically. Mathematics is discipline having its own language and structure. It has more symbolic notations, its own way of reading and writing, its grammar and discourse are different from other disciplines. All this necessitates a different approach to learn mathematics by understanding its own language.

Mathematics though abstract in nature, is widely utilized and applied knowledge. It also facilitates in understanding the other branches of science. Hence Caulson says" Mathematics is the gate and Key to all other Knowledge". Therefore learning of mathematical language becomes more essential. Mathematics is full of notations starting with numeric symbols, fundamental operation symbols- +, - , X,/., , and other symbols like <, >, =, %, @ etc makes it unique. While using these symbols with a set rule, it forms its grammar. With the usage of Variables it has enhanced the arithmetic nature of mathematics and becomes more algebraic with usage of expressions and equations. When these mathematical ideas gets transformed into spatial form it becomes geometrical, which we see abundantly in the physical world in which we are living. With advent of new mathematical ideas and translating into practical reality gives wider scope for evolution of different branches of mathematics. Mathematics has its own vocabulary (notations), grammar, syntax, discourse, community and structure. It is in the form of symbols, variables, axioms, postulates, formulae's, and theorems. Unless one does not understand the context of their usage, then it becomes redundant. But mathematical knowledge has wider utility and applied in other branches of knowledge to make it more meaningful. Mathematics is the language of science and mathematics is the means for communication between scientists. If we emphasize the language role that mathematics is fulfilling we not only add interest and importance to subject but we also point out important consideration for learning of other subjects also. The knowledge of Mathematics is gained by adopting various approaches like- Inductive, Deductive, Analytical, synthetical, Heuristic, Inquiry, Activity Based and Problem-solving. Each of these methods helps in either construction or utilization of knowledge. All these approaches have their own merits as well as demerits. Some of these suitable for beginners while some are suitable for gifted learners also. Hence learning of mathematics has wider approaches and methodologies.

Activity Based teaching- learning is also one of the approaches where the earner becomes very active and gets involved in the learning process and either verifies the existing knowledge or tries to visualise the abstract mathematical ideas in concrete form by creating a model. This approach has the following assumptions:-

- Significant learning takes place as perceived by the learners relevant to their

- own purpose
- Learning by doing
- Learning is facilitated by responsible participation in the process
- Self-initiated learning leads to permanent learning.

When the learner is engaged in this approach, we can enhance their higher order thinking skills- i.e., analysis, synthesis, evaluative abilities and skills; totally engaged in the activities; hence it leads to exploration of their own attitudes and values towards mathematics.

Strategies used in Activity based approaches are:-

- Discovery approach either using inductive or deductive method
- Appropriate practical work suitable mathematical concepts or generalization
- Use of suitable teaching aids
- Adopting co-operative learning strategy
- Discussion forum to share and exchange of ideas

Activity based teaching of mathematics is essential because learner has to know "Doing Mathematics". Doing mathematics means integrating mathematical thinking, using mathematical knowledge and using mathematical inquiry methods. All these are very essential whenever the learner encounters various problematic situations. Hence doing mathematics become crucial and these can be tested through activities and different strategies can be evolved. Also it provides learner to explore all the mathematical ideas independently and gain confidence of doing mathematics successfully. Whenever the teacher designs any task as an activity, he/she must ensure that it has "richness of task" built in it. The richness of task is to be visualised asits complexity, its novelty or its requirement for analysis, systhesis or evaluation. If these are ensured, then learning becomes more interesting and challenging also.

The NCTM (1998) has identified five imperative needs for all learners.

They are:-

1. Become mathematical problem solvers
2. Communicate the knowledge
3. Reason mathematically
4. Learn to value mathematics
5. Become confident in one's own ability to do mathematics

Even today, these needs are pervasive and one can thing of satisfying these needs through activity based teaching in mathematics to the maximum extent.

References:

1. NCERT (2013) *Pedagogy of Mathematics*(Textbook for Two year B.Ed Course), New Delhi
2. Somashekar T V and et.al (2014), *Methods of Teaching Mathematics,* Neelkamal Publications Pvt Ltd Hyderabad.
3. Susan McDonald & Anne Watson, *Generating mathematically rich activity, University of Oxford, under GCSE mathematics linked pair pilot study, UK*

12. Suggested Activities in Mathematics Laboratory

Activities on Triangles:

1. To show that the area of a triangle is half of the product of its base and height.
2. To verify the triangles on the same base and between the same parallel lines are equal but equal area may not be congruent,
3. To find the in center, circumcenter, orthocenter, medians and centroid of a triangle by paper folding.
4. To verify midpoint theorem by paper folding (i.e the straight line joining midpoints of any two sides of a triangle is parallel to the third side and is half of it).
5. To find the area of a triangle using geoboard activity.
6. To verify that the sum of three interior angles of a triangle is 180_ by using the activity of paper cutting and folding.
7. To verify using coloured strips that (a) the sum of lengths of two sides of a triangle is greater than the length of the third side, (b) the difference of the lengths of two sides of a triangle is less than of the third side.
8. To verify Pythagoras theorem by method of paper folding, cutting and pasting.
9. To find the height of a tree or width of a road or river by using similarity of the triangles.
10. To verify that the ratio of areas of two similar triangles is equal to the ratio of the sequences of their corresponding sides.
11. To find out the sum of perpendicular distances from any arbitrary point inside an equilateral triangle to the sides is always a constant and that it is equal to the altitude of the triangle.
12. To show that the relationship between the radius of the incircle of a right angled triangle and the sides of the triangle is given by r =
13. To show that the midpoint of the hypotenuse of a right angled triangle is equidistant from all the vertices of the triangle.

14. To prove that the area of the equilateral triangle constructed over the hypotenuse of a right angled triangle is equal to the sum of the areas of the equilateral triangles constructed on the other two sides.
15. To calculate the distance between the earth and the moon during the lunar eclipse using similar triangle property.
16. To show that 1 + 3 + 5 + … + (2n-1) = n2 with the help of right isosceles triangles.
17. To prove that in any right angled triangle, the perpendicular drawn from the right angled vertex to the hypotenuse divides the triangle into two triangles and which are similar to each other and to the given triangle.
18. To show that ratio of the in-radii of similar triangles is equal to the ratio of the sides.
19. To see that he converse of Basic Proportionality theorem (i.e. if a line divides any two sides of a triangle in the same ratio, then it is parallel to the third side.
20. To show that medians of a triangle meet at a point which divides the median in the ratio 2:1.
21. To show that a median in a triangle divides the triangle into the triangles of equal area.
22. To show that in a right angled triangle, the height of the perpendicular drawn from the right angle to hypotenuse is equal to the sum of the inradii of the three circles.

Activities on Quadrilaterals:

23. To show that a diagonal of a parallelogram divides it into two congruent triangles.
24. To verify that the diagonals of a parallelogram bisects each other.
25. To explore similarities and differences in the properties with respect to diagonals of quadrilaterals like parallelogram, square, rectangle and rhombus.
26. To verify that the quadrilateral formed by joining the midpoints of the sides of a quadrilateral is a parallelogram.

27. To verify that the area of a rhombus is obtained by taking half of the product of the lengths of its diagonals.
28. To show that the area of a parallelogram is the product of its base and height.
29. To derive the formula for area of a trapezium in different ways.
30. To find the area of different quadrilaterals like square, rectangles, parallelogram, rhombus …etc by using Geoboard activity.
31. To verify using activity of paper cutting and folding that the sum of four angles of a quadrilateral is 360o .
32. Using paper folding activity to verify that the sum of either pair of opposite angles of a cyclic quadrilateral is supplementary (i. e. 180o).
33. Using paper folding activity to verify that in a cyclic quadrilateral the exterior angle is equal to the interior opposite angle.
34. To show that for any quadrilateral whose four sides are tangential to any given circle, the sum of the opposite sides are equal.
35. Tangrams – To form the geometrical shapes like square, rectangle, hexagon, trapezium, ..etc, from the given pieces and to improve the mental ability of students.
36. To convert any polygon into a square of a same area as that of a polygon.
37. To show that if a circle touches all the four sides of a quadrilateral, then sum of the angles subtended by the opposite sides of a quadrilateral at the centre of circle are supplementary.
38. To find the golden ratio through golden rectangle (the golden ratio is $(1 + \sqrt{5})/2$ and a rectangle with these dimension is called as golden rectangle).
39. To verify the properties of parallelogram using wooden scale model of a parallelogram.
40. To find the sum of the areas of the squares obtained by joining the midpoints of previous squares.

Activities on Circle:

41. To verify that the line drawn through the centre of circle to bisect a chord is perpendicular to chord.
42. To verify that the perpendicular drawn from the centre of the circle to a chord which is not a diameter bisects the chord.
43. To verify that the perpendicular bisector of a chord of a circle passes through the centre of the circle.
44. To verify that that chords equidistant from the centre of a circle are equal.
45. To verify that equal chords of a circle are equidistant from the centres.
46. To verify that chords of a circle subtend equal angles at the centres of the circle.
47. To give suggestive demonstration of the formula that the area of a circle is half the product of its circumference and radius.
48. To locate the position of the centre of a circle whose circumference is given, using paper folding and cutting method.
49. By paper cutting, pasting and folding, verify that the angle subtended by an arc at the centre of the circle is twice the angle subtended by the same arc at any other point on the remaining part of circle.
50. To verify that the angles in the same segment of a circle are equal, using the paper folding, cutting and pasting.
51. To verify that the angles in a semicircle is 900.
52. Using the method of paper cutting and folding to show that the angle in a major segment is acute and angle on a minor segment is obtuse.
53. To verify that the length of tangents drawn to a circle from an external point are equal by using the method of paper folding.
54. To find the area of circle using the area of circles.
55. To show that in case of concentric circles any chord of the larger circle, which is tangential to the smaller circle is bisected at the point of contact.
56. To show that if three circles of equal radius touch each other externally then the triangle formed by joining the centres of these circles is

equilateral triangle.

57. To illustrate that the path of the moving chord of constant length inside a circle is a circle and to find out the radius of this inner circle.
58. To show that the locus of the centres of the circle passing through two given points is the perpendicular bisector of line segment joining the points.
59. To illustrate that when two circles are tangents to each other, then their centres and the point of contact of circles are collinear.
60. Developing the teaching aid to show the relation between ellipse and circle.
61. To show that the circle drawn touching out three semicircle is one sixth of the diameter of the bigger circle.
62. To verify the property of intersection of chords of circle.
63. To find the area inscribed by four equal circles.
64. To show that when a chord is parallel to a tangent of circle, the triangle formed by joining the two ends of the chord and the point of tangency is an isosceles triangle.

Activities on Algebra

65. To find the square root of natural number using its geometrical representation.
66. To obtain length segments corresponding to square roots of natural numbers using a model of graduated wooden sticks.
67. To find the factors of quadratic polynomial $ax^2 + bx + c$ where $c \neq 0$.
68. To verify the algebraic identities by using working models.
69. Introduction to one of the many useful calculation methods from vediv mathematics.
70. To obtain the condition for consistency of system of linear equation in two variable by graphical method.
71. To verify that the given sequence is a Athematic progression by paper

cutting and pasting activity.

72. To verify that the sum of first n natural numbers is n(n + 1)/2
73. To verify that the sum of first n odd natural numbers is n^2 by using cutting and pasting method.
74. To find the formula for 8:;term and to obtain the sum of an arithmetic progression geometrically.
75. Conversion of numbers from Denary to Binary system through a mod4el.
76. To enable the students to solve the quadratic equation using quadratic equation solver.
77. Triangular numbers, Pythagorean numbers, square numbers, pentagonal numbers and tetrahedral numbers.
78. To find the square root of a number by Guess average method.
79. To enable the students to understand the conjunction and disjunction of two statements and draw their truth table.
80. To show the physical meaning of Fibonacci sequence.(i.e the sequence 1,1,2,3,5,8,13,21,34,55,89,… is Fibonacci sequence).
81. To enable the students to understand the construction of irrational numbers on number line in the spiral form.
82. To find the product of two numbers of two digits and three digits in a different way.
83. Nomo gram – To perform addition operation of numerals with respect to base 8.
84. To present the geometrical models for the finite geometric series.
85. To show that the total number of different square boards of all sides in a board of size n x n is equal to Σn^2.
86. Easy multiplication by Napier's strips.
87. Addition and subtraction of integers using number line.
88. To multiply any two numbers by diagonal relationship method.
89. Miracle addition puzzle.
90. To teach the mathematical operation by using Spike Abacus.

Activities in three-dimensional geometry:

91. To locate a point in three dimensional space.
92. To obtain the formula for total surface area of Right triangular prism and Pyramid.
93. To make a right circular cylinder of given height and circumference of the base.
94. To obtain the formula for the curved surface area of a right circular cylinder of given radius of its base and height.
95. To obtain the formula for the total surface area of a closed right circular cylinder
96. To give a suggestive demonstration of the formula for the volume of a right circular cylinder in terms of its height and radius of its base.
97. To make a right circular cone of a given slant height and the base circumference and to get the formula for the area of the lateral surface of a cone by using paper cutting and folding activity.
98. To relate the volumes of a right circular cone and a right circular cylinder and obtain a suggestive formula for the volume of a right circular cone.
99. To give a suggestive demonstration of the formula for surface area and volume of a prism.
100. To study the shape of the solids obtained by revolving different geometric shapes around an axis and around any given line.
101. To see the relation between the volume of the original sphere and the volume of the interior of the simple cube constructed from the sphere.
102. To find the number of cubes of all sizes in a given cube
103. To find the surface area and the volume of a torus

Activities in probability

104. To familiarize with the idea of probability of an event through an activity of throwing a pair of dice.
105. To show that the marbles flowing through a series of nails in the form of

Pascal's triangle, will settle down in the shape of a normal probability curve using a wooden model.

1. To show that if we toss N coins simultaneously, the probability of getting head and tail are equal to N/2 by using coin tossing machine.

106. To introduce the concept of probability through probability disc.

Activities in Trigonometry

107. To make a clinometer and use to measure the height of an object

108. Activity on building the trigonometry tables

109. To measure vertical heights and horizontal distances using a Stadia tube.

13. Activities on Paper folding

1. Create a Line: Take a sheet of paper. Fold the paper once. The fold got is a straight line.

2. Create a Point using two intersecting lines: Fold the paper twice so that one fold cuts the other fold. The intersection of two folds (two lines) is a point (The point of intersection of two lines).

3. Show that one and only one line passes through two given distinct points: Mark two points on a sheet of paper. Fold it so that, it contains both the marked points. A single fold (only one line) is got.

4. Create a perpendicular to a line: Fold the sheet. One line l is got. Refold the sheet so that the fold passes through a point on l and the path of the line are brought to conclude. The second fold got is the line l_1 perpendicular to the given line l.

5. Create a perpendicular to a given line through a point (a) on the given line (b) outside the given line:

(a) Fold the sheets to get the given line l. Mark a point on l. Fold the paper through P perpendicular to l. The $l1$ so got passes through P and is perpendicular to l. **(b)Mark P outside the line** l (obtained by folding the sheet). Fold the sheet through P perpendicular to line l. The line l_1 so got passes through P and perpendicular to l.

6. Fold a pair of parallel lines and create a parallelogram: Fold the sheet of line$l_1$on a rectangular sheet of paper. Fold the sheet again so that the fold got l_1 is parallel to the earlier fold l_1. Now l_1 is parallel to l_1 Similarly create two parallel linesl_2 and l_2 where l_1 and l_2 intersect on the sheet. The two pairs of lines l_1, l_1,l_2, l_2 form a parallelogram.

7. Create (a) The perpendicular bisector of a line segment, (b) the angle bisector of a given angle:

(a)Mark a line $l$and mark points A and B on l. Fold the sheets so that A and B are brought to coincidence. The fold so got is perpendicular bisector of AB.

(b) Create an angle with vertex at O. Fold so that the crease passes through O1and the arms of the angle are brought to coincidence. The fold got is the angle bisector.

8. Getting the (a) Centroid,(b) Orthocenter (c) Incentre and (d) the circumcentre of triangle. Mark a triangle ABC on a sheet of paper.

(a) Mark the mid points of the sides BC,CA and AB as D, E, F respectively**.** Fold the sheet trice passing through A, D, D, E and E, F. The folds AD, BE and CF which are the medians pass through the centroid G of the triangle.

(b) Fold the sheet trice so that the folds pass through the vertices and perpendicular to the opposite sides. These folds are the altitudes of the triangle passing through the orthocenter of the triangle.

(c) Fold thrice to get the angle bisector of the triangle which passes through the incentre of the triangle.

(d) Fold thrice to get perpendicular bisectors of the sides. These pass through the circumcentre of the triangle.

REGIONAL INSTITUTE OF EDUCATION, MYSURU-570006

CONTENT-CUM- PEDAGOGY ENRICHMENT PROGRAMME FOR KRP's ON MATHEMATICS AT SECONDARY LEVEL (Telangana, Andhra and Karnataka states)

(From 24th September to 28th September, 2018)

Coordinator : Prof V S Prasad

	9:30 - 11: 00	11:15 - 1 :00	1:00-2:00	2:00 - 3 :00	3:45 - 5: 15
24/09/18 **(Monday)**	Registration and Inauguration	**V.S. Prasad -** Number System, operations and Polynomials	**Lunch Break**	**Ajaykumar** - Statistics and Probability	**B Madhu-** Circle and its properties
25/09/18 **(Tuesday)**	**V S Prasad-** Puzzles and games in mathematics	**B S P Raju** 2D and 3D geometry-I		**B Madhu-** Geogebra-1	**T K Vijayan-** Geogebra-2
26/09/18 **(Wednesday)**	**T K Vijayan** Geogebra applications in geometry-I	**B S P Raju** 2D and 3D geometry-II		**Sharath Sure-** Teaching of Mathematics-I	**T P Prakashan** Problem solving-I
27/09/18 **(Thursday)**	**Sharath Sure** - Teaching of Mathematics-II	**N M Rao** - Activity based teaching-I		**B Madhu** Geogebra applications in geometry-II	**T P Prakashan** Problem solving-II
28/09/18 **(Friday)**	**T V Somashekar-** Teaching of mathematics	**N M Rao-** Activity based teaching-II		**V S Prasad-** Progressions	Valedictory function