TRAINING PACKAGE ON

# CONTENT AND METHODOLOGY OF TEACHING MATHEMATICS IN HIGHER PRIMARY SCHOOLS

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**Development of Training Package on Content and Methodology of Teaching Mathematics and Science for Teachers/Headmasters of Higher Primary Schools in Karnataka**


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# CONTENTS

**Edited by: Dr. N.M. Rao and Dr. N.B. Badrinarayan**

# A NOTE TO THE READER

Mathematics textbook of class VII of Karnataka was developed in the year 1994 and was reprinted after editing and making necessary correction in the year 2000. However the Methodology of Teaching Mathematics could not be incorporated in the textbook due to the restrictions on the size of the textbook. Also, the results in the class VII Public Examinations were quite poor. Hence the authorities of the Education Department of the Government of Karnataka requested the RIE, Mysore to develop a Training Package on the Content and Methodology of Teaching Mathematics and Science at the Higher Primary Levels.

**The Project**

Most of the teachers we met were also seeking help in transacting these topics in the classroom. Hence RIE, Mysore took up this project to develop the Training Packages both in Mathematics and Science for the Higher Primary Schools. Explanations of new concepts, additional illustrations, solutions to difficult problems, graded additional exercise and the units tests on every chapter are some of the special features of the training package in Mathematics.

**Development**

This package was developed during the workshops conducted at RIE, Mysore form 22-31st January 2001 and finalised in another workshop from 26-28 March 2001. Dr. N.B. Badrinarayana, who was the Chairman of the X class textbook committee and Dr. T.S. Kumaraswamy who was the Chairman of the

VII class textbook committee were also involved in writing, scrutinising/editing these materials.

Eventhough the content of this package is based on the topics covered in class VII, the pre-requisites needed for it has been discussed in every chapter either in the form of preview or as revision exercises. Therefore it is suggested that during the training programme, all the participants should be asked to do the revision exercises and the resource persons should see that the concepts of pre-requisites are quite clear.

**The Package**

Units covered: All topics (chapters 2 to 16) of VII class Mathematics Text Book of Karnataka.

Addressed to: Training Faculty/Key Resource Persons/Teachers

| | | |
|---|---|---|
| Format Used: | I | (i) Preview |
| | | (ii) Revision Exercises |
| | II | Introduction (including motivation) |
| | III | Concepts and Notation |
| | IV | Teaching Strategies (including activities) |
| | V | Unit Tests and Graded Exercises |

**How to Use this Package**

This training package (with the training package in Science) could be used for conducting a 10-day Teachers Training Programme of Higher Primary School Teachers of Mathematics and Science.

A programme schedule for such a programme is suggested below:

Each of the ten days may be divided into four sessions of 1½ hours each. The session may be a lecture-cum-discussion session, problems session, experiments/laboratory sessions. Mathematics and Science will get about 19 sessions each. The fifteen (15) topics given in the (Mathematics) package can conveniently be completed in these 19 sessions. The participants should be given these packages in advance and asked to work out the preview exercises (in the beginning of the chapter) and the unit tests (in the end of the chapter at home).

**Unit Test**

The contact periods can be used for initiation of the topics, clearing the concepts, verifying the correctness of the answers and working out the difficult problems.

1. We insist that the unit tests should be worked out by the participants individually and the resource team has to check the worked out problems.
2. The problems which have been worked out wrongly by the majority of the participants will have to be discussed by the resource team during the contact sessions.

## SUGGESTED PROGRAMME SCHEDULE

| Day | I Session | II Session | III Session | IV Session |
|---|---|---|---|---|
| I | General Discussion | Numbers and Numerals | Science | Science |
| II | Integers | Science | Exponents | Science |
| III | Fractional/ Decimal/ Percentage | Science | Problems | Science |
| IV | Science | Simple Interest | Science | Sets |
| V | Science | Algebra | Science | Products of Algebraic Expressions |
| VI | Equations | Science | Problems | Science |
| VII | Areas | Science | Mensuration | Science |
| VIII | Construction | Science | Science | Statistics/ Graphs |
| IX | Coordinate Geometry | Science | Science | Science |
| X | Problems on all Chapters | Science | Science | Any other Discussions |

The programme schedule may be adapted as per the situation and local requirements arising out of the general discussions on the difficult topics if necessary.

**Activities**

Many activities have been suggested at the end of every chapter. The teachers are requested to try these activities during the training period and later do them in their schools to make mathematics more interesting.

We are thankful to all those persons who helped us in developing this training package.

We welcome suggestions for further improvement of this training package.

# CHAPTER II

## NUMBERS AND NUMERALS

### Preview

In Hindu-Arabic system the numbers are expressed in base ten system and this system is called decimal system. Here the numerals used are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. In this system the value of the digit increases ten times as it moves from right to left by one place. Ten is a unit in decimal system.

Eg: 444 The value of 4 in units place is $4 = 4 \times 10^0$

The value of 4 in tenths place is $40 = 4 \times 10^1$

The value of 4 in hundredth place is $400 = 4 \times 10^2$

A number can be expressed in other base systems also.

Eg: base 5, base 6, base 2, ...

We know that to express a number in base five

- The digits used are 0, 1, 2, 3 and 4.

  Eg: $234_{(5)}$ is read as two three four base 5.
- Four is the highest digit used in this system.
- In base 5, Five is the unit.
- The value of digit increases 5 times as it moves from right to left by one place.

  Eg: $120_{(5)}$ - the expanded form of this number is

  $1x5^2 + 2x5^1 + 0x5^0$
- The expanded form of $2043_{(5)}$ is $2x5^3 + 0x5^2 + 4x5^1 + 3x5^0$
- To convert 9 into base 5 we use division method.

  Eg: $9 = 14_{(5)}$ (1 is the quotient and 4 is the remainder)

- To convert $12_{(5)}$ into base 10 we use place value method.

  $12_{(5)} = 1x5^1 + 2x5^0 = 1x5 + 2x1 = 5 + 2 = 7$

**Review Exercise**

1. The highest digit in base five is _______.
2. $142_{(5)}$ is read as _______.
3. 12 can be expressed in base five as _______.
4. $100_{(5)}$ is equal to _________ in base ten.
5. The value of $5^0$ is ________.
6. Write $2431_{(5)}$ in the expanded form.
7. In base five 542 is not representing a number. Why ?
8. The place values of 4 and 3 in $3421_{(5)}$ are ________ and ________.

**Concepts**

- A number can be expressed in base five and base two.
- A decimal number can be converted into base 5 and 2 and vice versa.
- Identification of the place value in base five and base two.
- Comparison between base 5 and base two.
- Fundamental operations like addition and subtraction can be done in base 5 and base 2 numerals.
- Addition and subtraction can be verified by converting it into base 10.

**Introduction**

**History**

Indians have contributed decimal number system. Mathematicians have known for years how to make many different kinds of number systems. Almost three hundred years ago, a German mathematician named Gottfried Leibnitz worked with a number system based on two. Computers use this number system. Binary arithmetic (Base two system) is useful in connection with electronic computers, since the digits '0' and '1' can be described electrically as 'off' and 'on'. Human beings in their early life used to count fingers. Thus evolved base 5 system. Let us learn more about base 5 and base 2 numerals.

**Strategies**

As already discussed earlier, in base five system we need only five numerals, they are {0,1,2,3,4}. In base five system the numerals are grouped in powers of five. Five is a unit. Counting starts from 0, 1, ... upto 4, again starts with 10 for 5.

**Activity**

Objects such as flowers, fruits, beads, cards, seeds, match sticks, etc. can be used to arrange in groups of fives.

xxxxx + = $10_{(5)}$ = 5 (Base 10)

xxxxx + x = $11_{(5)}$ = 6 (Base 10)

xxxxx + xx = $12_{(5)}$ = 7 (Base 10)

xxxxx } +
xxxxx } + xxx = $23_{(5)}$ = 13 (Bse 10)

Likewise a chart can be prepared.

| Base 10 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Base 5 | 0 | 1 | 2 | 3 | 4 | 10 | 11 | 12 | 13 | 14 | 20 | 21 |

To convert a decimal number into base 5 we use division method. In this method we divide a decimal number by 5 and write the reminder in the right side. We go on dividing till the reminder becomes less than 5.

Eg: Convert 32 into base 5.

```
5|32
 5|6  → 2
   1  → 1
```

After division, we write the reminders in the reverse order as follows: $112_{(5)}$ (read as one, one, two, base 5).

Answer these questions.

1. Why should we divide a decimal number by five ?
2. We write the reminders from left, i.e. $112_{(5)}$ and not as $211_{(5)}$ why ?

Ex: Convert the following numbers into base five.

(a) 52 (b) 126 (c) 500 (d) 204

To convert a number from base five into decimal system we use place value chart. The value of a digit increases five times as it moves from right to left by one place.

Eg: Convert $1234_{(5)}$ into base 10

| $5^3$ | $5^2$ | $5^1$ | $5^0$ |
|---|---|---|---|
| 125 | 25 | 5 | 1 |
| 1 | 2 | 3 | 4 |

$5^0 = 1$

$5^1 = 5$

$5^2 = 5x5 = 25$

$5^3 = 5x5x5 = 125$

$1234_{(5)} = 1x5^3 + 2x5^2 + 3x5^1 + 4x5^0$

$= 1x125 + 2x25 + 3x5 + 4x1$

$= 125 + 50 + 15 + 4 = 194$

$1234_{(5)} = 194$

Convert 194 into base 5 and verify the above answer.

Ex: Express the following numbers in base ten.

(a) $120_{(5)}$ (b) $2034_{(5)}$ (c) $2124_{(5)}$ (d) $1010_{(5)}$

**Base two numeration system**

In binary system we need only two digits '0' and '1'. In this system, the numbers are grouped in powers of two. The number 2 is written as $10_{(2)}$ and is read as one, zero, in the binary number system. The number three is written as $11_{(2)}$ and read as "one one base two".

Eg: Convert 7 into base 2 system.

This can be done by grouping in twos

```
x x
        +   xx   +   x    = 1x2^2 + 1x2^1 + 1x2^0
x x
```

$= 111_{(2)}$

In short we use division method.

We go on dividing by two and write the reminders from the left to the right, and the last reminder should be less than '2'.

```
2|7
 2|3-1    - 2^0 place
   1-1    - 2^1 place
   |____>   2^2 place
```

We write the answer starting from $2^2$ place.

$7_{(10)} = 111_{(2)}$

**Activity**: Prepare a table of decimal and binary numbers.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Base 10 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Base 5 | 0 | 1 | 10 | 11 | 100 | 101 | 110 | 111 |

Ex: Convert the following numbers into base two.

(a) 20 (b) 51 (c) 100

**Converting a base two into a base 10**

Here also we use place value columns. The value of digit increases twice as it moves from right to left by one place.

Eg: Convert $1010_{(2)}$ into base 10.

| $2^3$ | $2^2$ | $2^1$ | $2^0$ |
|---|---|---|---|
| 8 | 4 | 2 | 1 |
| 1 | 0 | 1 | 0 |

'1' Eight + '0' Fours + '1' Two + '0' Ones

$1x2^3 + 0x2^2 + 1x2^1 + 0x2^0$
$1x8 + 0x4 + 1x2 + 0x1$
$8 + 0 + 2 + 0 = 10$
$1010_{(2)} = 10_{(10)}$

Convert 10 into base two and verify the above.

Ex: Convert the following into decimal system.

(a) $101_{(2)}$ (b) $1111_{(2)}$

**Addition in Base 5 System**

Let us prepare the table of addition.

| $+_5$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 |
| 1 | 1 | 2 | 3 | 4 | $10_{(5)}$ |
| 2 | 2 | 3 | 4 | $10_{(5)}$ | $11_{(5)}$ |
| 3 | 3 | 4 | $10_{(5)}$ | $11_{(5)}$ | $12_{(5)}$ |
| 4 | 4 | $10_{(5)}$ | $11_{(5)}$ | $12_{(5)}$ | $13_{(5)}$ |

$4+1 = 5 = 10_{(5)}$
$4+2 = 6 = 11_{(5)}$
$4+3 = 7 = 12_{(5)}$
$4+4 = 8 = 13_{(5)}$

Eg. 1: Add

```
  1234(5)
 +3210(5)
 --------
  4444(5)
 --------
```

Eg. 2: Carry and Add

```
  432(5)        4+2 = 6 = 11(5)
 +434(5)        1+3+3 = 7 = 12(5)
 -------
 1421(5)        1+4+4 = 9 = 14(5)
 -------
```

Eg. 3: Add $230_{(5)}$ and $342_{(5)}$ and verify by converting into base ten.

| Place value | $5^3$ | $5^2$ | $5^1$ | $5^0$ | Base 10 | Total |
|---|---|---|---|---|---|---|
| | 125 | 25 | 5 | 1 | | |
| | | 2 | 3 | $0_{(5)}$ | 50+15+0 | 65 |
| | | 3 | 4 | $2_{(5)}$ | 75+20+2 | 97 |
| | 1 | 1 | 2 | 2 | 125+25+10+2 | 162 |

Ex. 1: Add

(a) $3421_{(5)} + 1430_{(5)}$

(b) $2404_{(5)} + 4320_{(5)}$

Ex. 2: Add and verify by converting into base 10.

(a) $210_{(5)} + 423_{(5)}$

(b) $432_{(5)} + 204_{(5)}$

**Addition in base two system**

To add in binary numbers you only need to know two facts.

$1 + 0 = 1_{(2)}$ and $1 + 1 = 10_{(2)}$

| Ex: Binary | Decimal | Binary | Decimal |
|---|---|---|---|
| 10 | 2 | 1111 | 15 |
| +101 | +5 | +101 | +5 |
| 111 | 7 | 10100 | 20 |

Ex: Add and verify by converting into base 2 system.

(a) $111_{(2)} + 101_{(2)}$ (b) $1010 + 0101_{(2)}$

(c) $1011_{(2)} + 1101_{(2)}$

**Subtraction in Base Five**

Ex. 1: Subtract $2210_{(5)}$ from $4321_{(5)}$

$$\begin{array}{r} 4321_{(5)} \\ 2210_{(5)} \\ \hline 2111_{(5)} \\ \hline \end{array}$$

Ex. 2: **Subtraction from borrowing**

Simplify $4031_{(5)} - 1234_{(5)}$

We can use place value chart.

| Place value | $5^3$ | $5^2$ | $5^1$ | $5^0$ |
|---|---|---|---|---|
| Borrow | | 10 | 10 | 10 |
| | 4 | 0 | 3 | $1_{(5)}$ |
| | 1 | 2 | 3 | $4_{(5)}$ |
| | 2 | 2 | 4 | 2 |

<u>Base 5</u> <u>Base 5</u>

Step 1: 1+10=11 | 11-4=2

Step 2: 2+10=12 | 12-3=4

Step 3: 0+4=4 | 4-2=2

Step 4: | 3-1=2

**Decimals**

~~1+5~~ = 6-4 = 2

~~2+5~~ = 7-3 = 4

~~0+4~~ = 4-2 = 2

Ex: Work out the following substractions and verify the answer by converting to base 10.

(a) $3201_{(5)} - 2312_{(5)}$

(b) $2000_{(5)} - 1021_{(5)}$

(c) $3021_{(5)} - 2324_{(5)}$

Ex: Simplify and verify by converting to base 10.
$312_{(5)} - 223_{(5)}$

| Place value | $5^2$ | $5^1$ | $5^0$ | Base 10 | T |
|---|---|---|---|---|---|
| | 25 | 5 | 1 | | |
| | 3 | 1 | $2_{(5)}$ | 75+5+2 | 82 |
| | 2 | 2 | $3_{(5)}$ | 50+10+3 | 63 |
| | 0 | 3 | 4 | 0+15+4 | 19 |

**Subtraction in Base 2**

Subtraction with binary numbers is easy if we know three subtraction facts, viz. 1-1 = 0, 1-0 = 1, $10_{(2)}$-1 = 1.

```
Eg. 1: Binary            Decimal
         11                 3
         -1                -1
         --                --
         10                 2
         --                --

       Binary            Decimal
        1010               10
        -101               -5
        ----               --
         101                5
        ----               --
```

Eg. 2: Subtraction $101_{(2)}$ from $1010_{(2)}$

| $2^3$ | $2^2$ | $2^1$ | $2^0$ |
|---|---|---|---|
| | $10_{(2)}$ | | $10_{(2)}$ |
| 1 | 0 | 1 | 0 |
| | 1 | 0 | 1 |
| | 1 | 0 | 1 |

| Base 2 | Decimal |
|---|---|
| Step 1:10-1=1 | 2-1=1 |
| Step 2:0-0=0 | |
| Step 3:10-1=1 | 2-1=1 |
| | = 10 |
| | = 5 |
| | = 5 |

3. Subtract and verfy by converting into base 10

Simplify $1001_{(2)} - 111_{(2)}$

| $2^3$ | $2^2$ | $2^1$ | $2^0$ | Base 10 | Total |
|---|---|---|---|---|---|
| 8 | 4 | 2 | 1 | | |
| 1 | 0 | 0 | 1 | 8+0+0+1 | 9 |
| | 1 | 1 | 1 | 4+2+1 | 7 |
| 0 | 0 | 1 | 0 | 0+0+2+0 | 2 |

Ex.: Subtract and verify by converting into base 10.

(a) $1101_{(2)} + 110_{(2)}$

(b) $11010_{(2)} + 1011_{(2)}$

**Unit Test**

**Blue Print**

Marks: 25
Time : 1 hr

| | | |
|---|---|---|
| OT | 1x5 = | 5 |
| VSA | 2x5 = | 10 |
| SA | 3x2 = | 6 |
| LA | 4x1 = | 4 |
| Total | | 25 |

1x5 = 5

I. (1) What are the common numerals used in base 2 and base 5 ?

(2) Write the equivalent to $11_{(2)}$ in base 10.

(3) The expanded form of $2431_{(5)}$ is ________.

(4) The sum of $101_{(2)}$ and $11_{(2)}$ = __________.

(5) The difference between $42_{(5)}$ and $24_{(5)}$ is _______.

II. (1) Express $1001_{(2)}$ in the decimal system

(2) Express 7 in base 2

(3) Find the value of $1010_{(2)} - 011_{(2)}$

(4) Simplify $231_{(5)} + 432_{(5)}$

(5) Convert 123 to base 5.

III. Simplify and verify by converting to base 10.

(a) $1101_{(2)} + 1011_{(2)}$

(b) $342_{(5)} - 134_{(5)}$

IV. Convert $1101_{(2)}$ to base 5.

**Suggested Activities**

Materials required: KG card sheets, sketch pen, scale, ...

(1) Prepare table showing base 10, base 5 and base 2.

(2) Prepare an addition table in base 5 and base 2.

(3) Prepare a place value chart in base 5 and base 2.

# CHAPTER III

# INTEGERS

## Preview

The counting might have been started through fingers. There are ten fingers in all (both the hands). This might be the origin of the decimal system. The Counting Numbers 1, 2, 3, ... are called the Natural Numbers. Zero was added to it later and the set W = {0,1,2,3,...} is called the set of Whole Numbers. Zero and the Decimal System are the contributions of Hindus. Zero means many things. It can be a starting point, nothing, nil, empty, place holder, etc. We also know the four fundamental operations with whole numbers.

1. **Addition** is the process of combining.

eg: 8+3 = 11 [8 and 3 are the addends and 11 is the sum, + is the notation for addition]

8+0 = 8 [0 is called the identity element with respect to addition]

8+3 = 3+8 [The commutative property of addition]

(2+3)+5 = 2+(3+5) [The Associative property of addition]

2. **Subtraction** is the inverse of addition or the difference between two numbers.

eg: 15-7 = 8 [15 is called Minuend and 7 is the subtrahend, 8 is the difference, - is the notation for subtraction]

8+7 = 15

[Commutative and Associative property do not hold for subtraction]

3. **Multiplication** is repeated addition.

eg: 5x3 = 15, i.e. 5+5+5 = 15 [5 is added 3 times]

5x0 = 0 [Any number multiplied by '0' is zero]

5x1 = 5 [Any number multiplied by '1' ~~then the product~~ is the number itself and '1' is called the identity element with respect to multiplication]

3x5 = 5x3 [Commutative property of multiplication]

(3x5)x4 = 3x(5x4) [Associative property of multiplication]

4. **Division** is the process of finding one of two factors from the product.

eg: 48 - 6 = 8 [48 is the dividend, 6 the divisor and 8 is the quotient]

Division is the inverse process of multiplication and the result is verified by the multiplication.

eg: 6x8 = 48

**Properties of Division**

a. 4 ÷ 1 = 4

If any number divided by 1, then the quotient is the number itself.

b. 4 ÷ 4 = 1

If any number is divided by itself then the quotient is 1, but the divisor should not be zero]

c. 0 ÷ 8 = 0

When zero is divided by a whole number the quotient is 0.

d. 4 ÷ 2 ≠ 2 ÷ 4

Commutative property does not hold with respect to division.

## Preview Exercises

1. Find the quotient and the reminder in

   (a) 458 ÷ 458 (b) 0 ÷ 2475 (c) 515 ÷ 5

2. Find the least number to be added to 9182 to make it divisible by 9.

3. 28034 x 1 = ____________

4. $15 \times 10^5 \times 22 \times 10^7 = 330 \times (\qquad)$

5. 6000 x 1200 = __________

6. 5x6 = 6x5 (state the property)

## Introduction

### History

"God created the Natural Numbers; everything else is man's Handiwork". This is the statement of the famous mathematician Leopold Kronecker (1823-1891). The natural numbers are not sufficient for the present day work.

For example: to subtract a large number from a smaller number yields a different kind of number. Which number when added to 5 gives three ? 5 + (?) = 3

i.e. 3-5 = -2 which is a negative signed number. In 1629 Girard of France gave a Geometric depiction of negative number. Let us learn more about these numbers. The '-' sign reminds us the origin of negative numbers from the successive subtraction of unity.

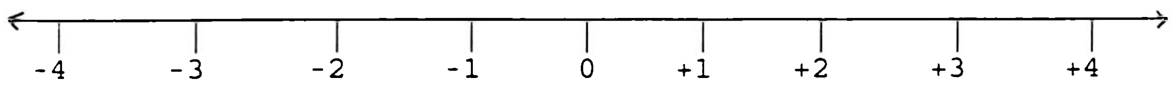
eg: 0 - 5 = -5. Here the minus sign is called a sign of quantity to distinguish it from the subtraction sign which is called the operational sign. In contradiction to negative numbers we can affix a plus sign to any positive number in which case this is a sign of quantity and not the sign of an operation, eg. +2, +3, ..., etc. This difference should be made clear. Prior to introduction of negative numbers there were no positive numbers.

**Concepts**

- The set of signed numbers, i.e. +ve and -ve numbers along with zero is called a <u>set of integers</u> and denoted by the symbol 'Z'.
- Signed numbers can be represented on a number line with a point.
- We can do four fundamental operations on integers.

**Strategies**

Let us draw a number line and represent the set of integers. Z = {..., -4, -3, -2, -1, 0, +1, +2, +3, +4, ...}
A number line is a straight line on which the integers are denoted at equidistant points on either side of a point zero. The line extends indefinitely.




**Properties of Numberline**

(Ask questions to elicit the properties of number line.)

Numbers written to right of zero are positive and left of zero are negative numbers.

As we move from left to right on the number line the numbers are in increasing order and as we move in the opposite direction the numbers are in decreasing order.

eg: $-5<-4<-3<-1<0<+1<+2<+3<+4<+5$ ...

Eg: Draw a number line and denote the following numbers on it.

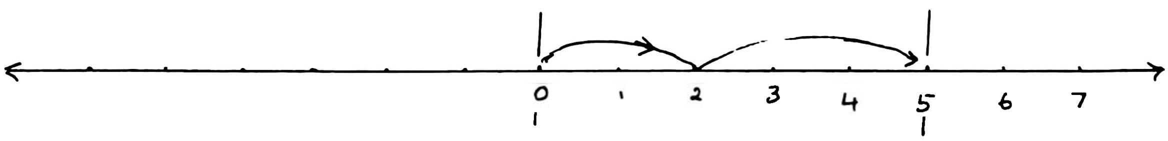
+2, 0, -8, +5, -4, +1, -6, ...

**Addition of Integers**

Ex.1: (+2) + (+3) = ?

Start from zero, walk 2 units to the right. Again walk 3 more units to the right. Then where did you reach ?

(Ans. 5) (+2) + (+3) = +5




Ex.2: (-2) + (-3) = -5

Start from zero, walk 2 units to the left and again walk 3 more units to the left. Where did you reach ?

(Ans. -5) (-2) + (-3) = -5


Let us add integers of opposite sign.

Ex.3: (-3) + (+4) = +1 --> Start from zero, walk 3 units to the left, and again walk 4 more units to the right. Where do you reach ? (Ans. 1)

Ex.4: (-3) + (+2) = -1 --> Start from zero, walk 3 units to the left. Then walk 2 more units to the right. Where do you reach? (Ans. -1)

Ex.5: (+4) + (-3) = ? --> Start from zero, walk 4 units to the right. Then, again walk 3 more units to the left. Where do you reach ? (Ans. 1)

Therefore (+4) + (-3) = +1

**Subtraction of Integers**

The subtraction of one number from another can be done as follows (with the convention that the movement to the right is considered to be positive and the movement to the left is negative):

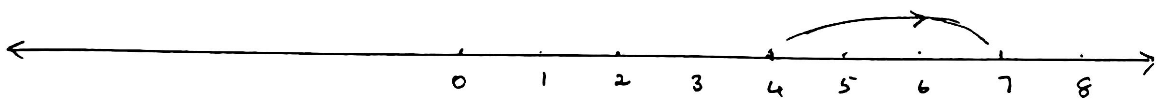
eg: (+7) - (+4) = ?

(a) How many steps I have to move from +4 to reach +7 ?

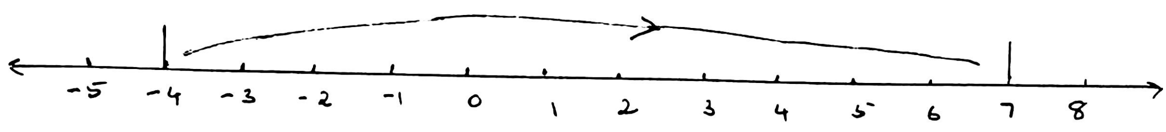
(Ans: 3 units)

(b) Is it to the right or left ? (Ans: Right; positive)

(+7) - (+4) = +3




2. (+7) - (-4) = ?



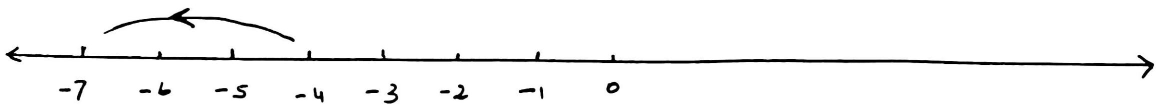

(a) How many units I have to move from -4 to reach +7 ?

(Ans: 11)

(b) Is it to the right or left (Ans: Right)

(+7) - (-4) = +11

3. (-7) - (-4) = ?




Starting from -4 to reach -7, we have to count 3 units towards left, therefore (-7) - (-4) = -3.

4. (-4) - (-4) = (-4) + (+4) = 0

Give reasons for the above subtraction and addition facts.

Ex: Find the results using number line.

1. (+4) + (+6)
2. (-3) + (+8)
3. (+6) - (+8)
4. (-2) - (-6)

**Multiplication of Integers**

Multiplication is the repeated addition.

To fix the sign, the following table is useful.

| x | + | - |
|---|---|---|
| + | + | - |
| - | - | + |

a. When two integers of like signs are multiplied the product is always positive.

eg: 1. (+3) x (+4) = +12

2. (-3) x (-4) = +12

b. When two integers of unlike signs are multiplied the product is always negative.

eg: 1. (+3) x (-5) = -15

2. (-3) x (+5) = -15

**Division of Integers**

As in the case of multiplication, if we divide two like sign integers, the sign of the quotient is positive and negative in the case of unlike signs.

eg: 1. +4 ÷ +2 = +2

-4 ÷ -2 = +2

2. +4 ÷ -2 = -2

-4 ÷ +2 = -2

**Exercise**

**Simplify**

1. (-125) x (-5)
2. (-250) x (+15)
3. (-209) ÷ (-11)
4. (-168) ÷ (+4)

# UNIT TEST

## INTEGERS

Blue print                                                Marks: 25

1x6 = 6                                                   Time : 45 mts
2x4 = 8
3x2 = 6
1x5 = 5

1x6 = 6

I.

1. How much should be added to (-3) to make it (+5).
2. 5 - (-2) = ?      (-5) - (-6) = ?      5 - (-2) ☐ -5 - (-6)

   Insert the appropriate sign from the list (>, =, <, =) in the box above.
3. ☐ ÷ (8) = -56      Fill in the blanks.
4. From the sum of (-80) and (+50) subtract ~~or~~ (-100).
5. Supply the missing number in (-3) x (...) x (-1) = -15.
6. The product of two negative integers is always a ______.

II.                                                        2x4 = 8

1. Using the numberline subtract (-3) from (-5)
2. Arrange the following numbers in ascending order and denote on number line.

   -4, +4, 0, -3, +5
3. Simplify

$$\frac{(-2) - (-12) + (-5)}{(+5) - (-8) + (-8)} \times \frac{+15}{-5}$$

4. By how much (-10) exceeds (-20).

III. 2x3 = 6

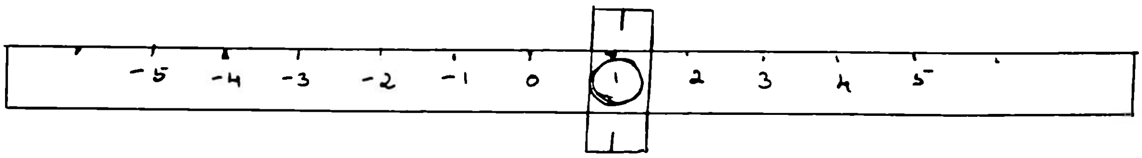
1. The algebraic sum of two integers is 150. If one of them is 250, find the other integer.
2. Add (-3) and (+8) on a number line.

IV. 1x5 = 5

At 1 pm at Ooty on a certain day the temperature was $12^{\circ}C$. By 6 pm on the same day the temperature dropped to $-5^{\circ}C$. Find the drop in temperature on the number line.

**Suggested Activity**

1. Make a sliding chart of a number line and mark the integers to explain addition and subtraction.




2. Construct quiz questions on integers.

# CHAPTER IV

# THE EXPONENTS

## Preview

We already know the meaning of an exponent.

In $10^3$, 10 is the base and 3 is called the index or an exponent.

A number can be expressed in exponential form, eg: $81 = 3x3x3x3 = 3^4$ and read as three to the power of four. The value of a number to the power of zero is 1.

eg: $10^0 = 1$, $8^0 = 1$, $9^0 = 1$

To prove this, consider: $2^3 \div 2^3 = 1$

$$= 2^{3-3} = 2^0 = 1$$

The value of a number to the power of one is the number itself.

eg: $10^1 = 10$, $8^1 = 8$

if the index of a number is 2 then it is called a square.

eg: $3^2$ = (three square)

if the index is 3 then it is called a cube.

eg: $4^3$ = (four cube)

if the index is 4 or more then it is called 'to the power of'

eg: $3^5$ = Three to the power of five.

**Power of negative numbers**

1. The value of the even power of a negative number is ~~even~~ +ve.

   eg: $-5 \times -5 = (-5)^2 = +25$

2. The value of the odd power of a negative number is ~~odd~~ -ve.

   eg: $(-5)^3 = (-5)\times(-5)(-5) = -125$

**Preview exercise**

a. 3x3x3x3; the exponential form of this is __________.

b. -6x-6x-6 = __________

c. Find the value of $(-2)^2 \times (+3)^3$

d. Which is greater among $2^2$ and $3^2$

e. ~~Find~~ Fill in the blanks

   (i) $16 = 4^{\square}$

   (ii) $2^5 = \square$

   (iii) $64 = \square^3$

f. A box contains 12 biscuit packets arranged in 12 rows. Each packet contains 12 biscuits. Find the total number of biscuits in the box.

**Introduction**

The mathematicians thought of a method to express very large numbers in a simple form and this form is called an exponential form. This form is not only easy to read and write but also for calculations. A great mathematician called Napier was the first man who introduced the exponential form of a number.

Now, let us see how it is useful for calculations:

eg:

| Exponent | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Power of 2 | 2 | 4 | 8 | 16 | 32 | 64 | 128 | 256 | 512 | 1024 | 2048 | 4096 |

Multiplication of 32 and 64 takes some time, while it is easy to see that $2^5 \times 2^6 = 2^{11}$ and $2^{11} = 2048$.

Another important use of exponential form is to express very big or very small numbers needed in science. Some of the physical quantities are of very minute like size of the atom, wavelength of a light, etc. Some are of high magnitude like distance between the sun and the earth. Now let us learn how we can express these quantities in exponential form.

**Concepts**

1. Expressing a given number in the exponential form.
2. Simplification of problems having exponentials.
3. Use of exponential form in scientific notations.

**Strategies**

The velocity of light is three hundred million metre per second, i.e. three followed by 8 zeros. This we write as 300,000,000 mt/sec. This can be written as $3 \times 10^8$ mt/sec in exponential form, which is easy to read and write.

eg: Express 625 in exponential form.

In this case 5 is selected as base

The factors ~~5~~ are

5 | 625
5 | 125
5 | 25
5

5x5x5x5

$\therefore 625 = 5^4$

Eg: Express 1724 in exponential form.

Problems: (1) $(-3)^2 = ?$

$(-3)^2 = -3 \times -3 = +9$

(2) $(-3)^3 = ?$

$(-3)^3 = -3 \times -3 \times -3 = -27$

3. Simplify

$$\frac{(-1)^5 + (+2)^3 - (-3)^2}{(+2)^4 + (+3)^3 - (-2)^5}$$

$$\frac{(-1)^5 + (+2)^3 - (-3)^2}{(+2)^4 + (+3)^3 - (-2)^5}$$

$$= \frac{-1 + 8 - (9)}{16 + 27 - (-32)}$$

$$= \frac{-1 + 8 - 9}{16 + 27 + 32}$$

$$= \frac{-2}{75}$$

A number can be written as the product of a number between 1 and 10, and a power of 10.

eg: (1) $35 = 3.5 \times 10^1$ (2) $0.05 = 5 \times 10^{-2}$

This notation is called Scientific Notation.

Area of Karnataka is 1,91,700 sqkm. This can be expressed in scientific notation as

$$1.917 \times 10^5 \text{ sqkm}$$

Ex: Express the following in scientific notation.

1. 656000
2. 0.0005

# UNIT TEST

Blue print

Marks: 25

1x6 = 6
2x5 = 10
3x3 = 9

Time : 45 mts

1x6 = 6

I.

1. Mention the base in $(-2/3)^4$ and the index.
2. Find the value of $3^6$.
3. Express 64 in the exponential form of base 2.
4. Find the value of the cube of (-3/4).
5. Express 1 million in the exponential form.
6. $1^2$ ☐ $10^0$ (use the correct symbol in the box) selecting from {>,<,=,=}

II. 2x5 = 9

1. Simplify $(+5)^2 + (-4)^2 - (-2)^3$
2. Express 56250 in scientific notation.
3. Express the following in exponential form with prime number as base and simplify.

$$\frac{11}{121} \times \frac{729}{81}$$

4. $1.86 \times 10^5$ - Express this in usual form.
5. 0.025 Express this in scientific notation.

III. 3x3 = 9

1. Simplify

$$\frac{2^5}{81} \times \frac{3^2}{32} - \frac{25^2}{125}$$

2. The diameter of the sun = 3140000 km. Express this in scientific notation.

3. The centillion is equal to 1 followed by 303 zeros. Write this number in exponential form.

**Suggested Activity**

1000 millions = 1 billion

1000 billions = 1 trillion

1000 trillions = 1 quadrillion

1000 quadrillions = 1 quintillion

1000 quintillions = 1 sextillion

1000 sextillions = 1 septillions

Prepare the chart expressing them in exponential form of base 10.

# CHAPTER V

## FRACTIONS, DECIMALS AND PERCENTAGES

In the previous classes the concepts of equivalent fractions, comparison of fractions, improper fractions and mixed fractions are discussed. It is requested that the revision of the above concepts may please be taken up first in the orientation trainings before going to the concepts of addition, subtraction, multiplication and division of fractions. Here we discuss these prerequisites in brief.

**Equivalent Fractions**

We know that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \ldots$

They are equivalent fractions and $\frac{1}{2}$ is the simplest of the above equivalent fractions. The simplest form of $\frac{30}{36}$ is $\frac{5}{6}$, because we can cancel 5 from both numerator and denominator.

**Comparison of Fractions**

We know that $\frac{2}{3} > \frac{1}{3}$

i.e. $\frac{2}{3}$ is greater than $\frac{1}{3}$. When the denominators of both the fractions are same, compare the numerators to find out which fraction is greater.

Ex: Compare (i) $\frac{3}{7}$ and $\frac{5}{7}$

(ii) $\frac{5}{11}$ and $\frac{7}{11}$

**When the denominators are different**

Ex: Compare $\frac{4}{7}$ and $\frac{2}{5}$

LCM of 7 and 5 is 35. Multiply $\frac{4}{7}$ by $\frac{5}{5}$ and multiply $\frac{2}{5}$ by $\frac{7}{7}$.

$$\frac{4}{7} = \frac{4}{7} \times \frac{5}{5} = \frac{20}{35}$$

$$\frac{2}{5} = \frac{2}{5} \times \frac{7}{7} = \frac{14}{35}$$

Here,

$\frac{20}{35} > \frac{14}{35}$ Therefore $\frac{4}{7} > \frac{2}{5}$

**Improper Fractions and Mixed Fractions**

$\frac{8}{3}$ is an example of improper fraction. It can be expressed as

$\frac{8}{3} = 2\frac{2}{3}$

```
3) 8 (2
   6
  ---
   2
```

$2\frac{2}{3}$ is the mixed fraction. When the numerator is greater than the denominator we get an improper fraction. The conversion of a mixed fraction into improper fraction and vice versa, should be practised during the training period.

Ex:

1. Write down the following mixed fractions as improper fractions.

   $2\frac{1}{3}$, $3\frac{2}{5}$, $7\frac{1}{7}$, $3\frac{1}{3}$

2. Write down the following improper fractions as mixed fractions.

   $\frac{10}{3}$, $\frac{20}{6}$, $\frac{50}{7}$, $\frac{15}{6}$, $\frac{7}{4}$

**Revision Exercises**

1. Write the three equivalent fractions.

   (i) $\frac{1}{2}$ = ... = ... = ...

   (ii) $\frac{4}{5}$ = ... = ... = ...

   (iii) $\frac{3}{4}$ = ... = ... = ...

2. Write the following in the simplest form.

   (i) $\frac{2}{6}$ = ...  (ii) $\frac{5}{10}$ = ...  (iii) $\frac{3}{15}$ = ...

3. Use the symbol 'greater >', 'lesser <' or 'equal =' between the following.

   (i) $\frac{4}{12}$  $\frac{8}{12}$  (ii) $\frac{1}{2}$  $\frac{3}{4}$

   (iii) $\frac{1}{2}$  $\frac{3}{6}$  (iv) $\frac{5}{7}$  $\frac{25}{35}$

4. Write the following as mixed fractions.

(i) $\frac{4}{3}$ (ii) $\frac{13}{8}$ (iii) $\frac{37}{12}$

5. Write the following as improper fractions.

(i) $4\frac{1}{2}$ (ii) $9\frac{3}{12}$ (iii) $11\frac{1}{8}$

**Introduction**

In this chapter, we are going to study the addition, subtraction, multiplication and division of fractions. In the real life situations there are many examples where, we have to perform these operations on fractions.

For example:

1. What is total length of the fence if the length of the four sides are $10\frac{1}{2}$ m, 11 m, $12\frac{1}{4}$ m and $13\frac{1}{3}$ meters ?

2. From a piece of ribbon $6\frac{3}{4}$ m long, how many pieces of length $1\frac{1}{3}$ m can be cut ?

We discuss more such examples when we come to these concepts.

**Addition of Fractions**

1. When the denominators are same:

Ex: $\frac{1}{3} + \frac{2}{3} = \frac{3}{3} = 1$

$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$

$$\frac{8}{15} + \frac{3}{15} = \frac{11}{15}$$

It is to be brought to the notice of slow learners that when the denominators are same, the numerators can be added directly and the denominator can be written as it is. But this procedure cannot be adopted when the denominators are different.

**2. When the denominators are different**

Ex: $\frac{1}{3} + \frac{1}{4} = ?$ LCM of 3 and 4 is 12.

$$\frac{1}{3} = \frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$$

$$\frac{1}{4} = \frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$$

$$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

(ii) Find the sums: $\frac{1}{4} + \frac{7}{10} = ?$

$$\frac{2}{5} + \frac{3}{10} = ?$$

$$\frac{2}{5} + \frac{2}{7} = ?$$

**3. When the fractions are mixed fractions**

Ex: $2\frac{1}{2} + 3\frac{1}{4} = ?$ LCM of 2 and 4 is 4.

$$2\frac{1}{2} + 3\frac{1}{4} = \frac{5}{2} + \frac{13}{4}$$

$$= \frac{5}{2} \times \frac{2}{2} + \frac{13}{4}$$

$$= \frac{10}{4} + \frac{13}{4}$$

$$= \frac{23}{4}$$

$$= 5\frac{3}{4}$$

**Exercises**: Find the sums:

1. $5\frac{1}{3} + \frac{2}{3} = ?$ (Ans 6)

2. $3\frac{2}{3} + 2\frac{1}{2} = ?$ (Ans $6\frac{1}{6}$)

**Evaluation**: Work out the following problems and bring the answers to the simplest form.

1. $\frac{2}{7} + \frac{3}{7} = ?$ (Ans $\frac{5}{7}$)

2. $\frac{1}{2} + \frac{1}{4} = ?$ (Ans $\frac{3}{4}$)

3. $3\frac{1}{3} + \frac{3}{4} = ?$ (Ans $4\frac{1}{12}$)

4. $4\frac{1}{3} + 2\frac{2}{9} = ?$ (Ans $6\frac{5}{9}$)

**Subtraction of Fractions**

**1. When the denominators of the fractions are the same:**

Ex: (i) $\frac{5}{7} - \frac{3}{7} = \frac{5-3}{7} = \frac{2}{7}$

(ii) $\frac{4}{5} - \frac{3}{5} = \frac{4-3}{5} = \frac{1}{5}$

**2. When the denominators are different:**

Ex: $\frac{3}{4} - \frac{1}{3} = ?$ LCM of 4 and 3 is 12.

$$\frac{3}{4} = \frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$$

$$\frac{1}{3} = \frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$$

$$\frac{3}{4} - \frac{1}{3}$$

$$= \frac{3}{4} \times \frac{3}{3} - \frac{1}{3} \times \frac{4}{4}$$

$$= \frac{9}{12} - \frac{4}{12}$$

$$= \frac{9-4}{12}$$

$$= \frac{5}{12}$$

Ex 2: $\frac{11}{15} - \frac{3}{5} = ?$ LCM of 15 and 5 is 15.

$$\frac{11}{15} - \frac{3}{5} = \frac{11}{15} - \frac{3}{5} \times \frac{3}{3}$$

$$= \frac{11}{15} - \frac{9}{15}$$

$$= \frac{11-9}{15}$$

$$= \frac{2}{15}$$

Work out the following subtractions and reduce the answer to the simplest form.

1. $\frac{9}{10} - \frac{4}{5} = ?$ (Ans. $\frac{1}{10}$ )

2. $\frac{5}{8} - \frac{3}{12} = ?$ (Ans. $\frac{3}{8}$ )

**Subtraction of Mixed Fractions**

Ex.1: $2\frac{1}{2} - \frac{1}{4} = ?$

$2\frac{1}{2} - \frac{1}{4}$ LCM of 2 and 4 is 4

$$= \frac{5}{2} - \frac{1}{4}$$

$$= \frac{5}{2} \times \frac{2}{2} - \frac{1}{4}$$

$$= \frac{10}{4} - \frac{1}{4}$$

$$= \frac{10-1}{4}$$

$$= \frac{9}{4}$$

$$= 2\frac{1}{4}$$

Ex.2: $2\frac{3}{4} - 1\frac{2}{5} = ?$

$$2\frac{3}{4} - 1\frac{2}{5}$$

LCM of 4 and 5 is 20

$$= \frac{11}{4} - \frac{7}{5}$$

$$= \frac{11}{4} \times \frac{5}{5} - \frac{7}{5} \times \frac{4}{4}$$

$$= \frac{55}{20} - \frac{28}{20}$$

$$= \frac{55-28}{20}$$

$$= \frac{27}{20}$$

$$= 1\frac{7}{20}$$

Ex.3: From a can containing $6\frac{1}{2}$ litres of milk, $2\frac{3}{4}$ litres of milk is used. Find the remaining quantity of milk in the can.

Total amount of milk in the can = $6\frac{1}{2}$ lit

Quantity of milk used = $2\frac{3}{4}$ lit

Quantity of milk remaining in the can = $6\frac{1}{2} - 2\frac{3}{4}$ lit

$$= \frac{13}{2} - \frac{11}{4}$$

$$= \frac{26-11}{4}$$

$$= \frac{15}{4}$$

$$= 3\frac{3}{4} \text{ lit}$$

The teachers will have to insist that all the steps will have to be written in working out such problems.

Ex.4: The length and breadth of a room are $6\frac{5}{6}$ meters and $4\frac{7}{8}$ meters. By how many meters, the length exceeds its breadth ?

(Ans. $1\frac{23}{24}$ )

Ex.5: What is the length of the cloth left out if a piece of length $1\frac{2}{5}$ meters is cut from the cloth of length $4\frac{1}{2}$ meters ?

(Ans. $4\frac{1}{2} - 1\frac{2}{5} = 3\frac{1}{10}$)

Note: During the orientation programme, the teachers may insist on the problems in which both addition and subtraction are included as given below:

**Evaluation**

1. $\frac{5}{6} + \frac{1}{12} - \frac{3}{4} = ?$ (Ans. $\frac{1}{6}$ )

2. $15\frac{1}{3} - 10\frac{1}{6} - 3\frac{3}{12} = ?$ (Ans. $1\frac{11}{12}$ )

3. What is the length of the wire left out if a piece of length $14\frac{1}{2}$ is removed from the wire after joining two pieces of length $10\frac{1}{2}$ meters and $8\frac{1}{4}$ meters ?

(Ans. $4\frac{1}{4}$ m)

Multiplication of Fractions

1. Multiplication of a fraction by an integer:

Ex.1: $\frac{1}{4} \times 6 = ?$

$$\frac{1}{4} \times 6 = \frac{1}{4} \times \frac{6}{1}$$

$$= \frac{1}{2} \times \frac{3}{1} = \frac{3}{2} = 1\frac{1}{2}$$

Ex.2: $\frac{3}{7} \times 5 = ?$

$$\frac{3}{7} \times \frac{5}{1} = \frac{15}{7} = 2\frac{1}{7}$$

2. Multiplication of a fraction by another fraction:

Ex.1: $\frac{5}{8} \times \frac{3}{4} = ?$

$$\frac{5}{8} \times \frac{3}{4} = \frac{5 \times 3}{8 \times 4} = \frac{15}{32}$$

Ex.2: $\frac{3}{16} \times \frac{1}{3} = ?$

$$\frac{3}{16} \times \frac{1}{3} = \frac{3}{16} \times \frac{1}{3} = \frac{1}{16}$$

3. Multiplication of mixed fractions:

Ex.1: $2\frac{1}{3} \times 3\frac{1}{5} = ?$

$$2\frac{1}{3} \times 3\frac{1}{5}$$

$$= \frac{7}{3} \times \frac{16}{5}$$

$$= \frac{112}{15}$$

$$= 7\frac{7}{15}$$

Ex.2: What is the cost of $2\frac{1}{2}$ meters of cloth if one meter of cloth costs Rs. $63\frac{1}{2}$ ?

The cost of one meter of cloth = Rs. $63\frac{1}{2}$

The cost of $2\frac{1}{2}$ meters $= 63\frac{1}{2} \times 2\frac{1}{2}$

$$= \text{Rs. } \frac{127}{2} \times \frac{5}{2}$$

$$= \frac{635}{4}$$

$$= \text{Rs. } 158\frac{3}{4}$$

= Rs. 158.75

**Evaluation**

1. $\frac{1}{2} \times 10 = ?$ (Ans. 5)

2. $3\frac{1}{4} \times 5\frac{1}{2} = ?$ (Ans. $17\frac{7}{8}$ )

3. $\frac{7}{8} \times \frac{5}{6} = ?$ (Ans. $\frac{35}{48}$ )

4. If a person walks with the speed of $5\frac{1}{2}$ kms per hour, then what is the distance travelled by him in $3\frac{1}{2}$ hours ?

(Ans. $19\frac{1}{4}$ )

Division of Fractions

1. Division of fraction by a whole number:

Ex.1: $\frac{1}{2} \div 4$

$\frac{1}{2} \div 4$

$= \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

Ex.2: $\frac{9}{16} \div 3$ Verification: $\frac{3}{16} \times 3 = \frac{9}{16}$

$\frac{9}{16} \div 3$

$\frac{9}{16} \times \frac{1}{3}$

$\frac{3}{16}$

Ex.3: $15\frac{1}{8} \div 11$ Verification $\frac{11}{8} \times 11 = \frac{121}{8} = 15\frac{1}{8}$

$15\frac{1}{8} \div 11$

$= \frac{121}{8} \div 11$

$$= \frac{121}{8} \times \frac{1}{11}$$

$$= \frac{11}{8}$$

2. Division of fraction by another fraction:

Ex.1: $\frac{5}{8} \div \frac{5}{16} = ?$ Verification: $\frac{5}{16} \times 2 = \frac{5}{8}$

$$\frac{5}{8} \div \frac{5}{16}$$

$$= \frac{5}{8} \times \frac{16}{5}$$

$$= 2$$

Ex.2: $5\frac{1}{5} \div 2\frac{8}{9} = ?$ Verification: $\frac{9}{5} \times 2\frac{8}{9}$

$$= \frac{9}{6} \times \frac{26}{9}$$

$$= \frac{26}{5} = 5\frac{1}{5}$$

$$5\frac{1}{5} \div 2\frac{8}{9}$$

$$= \frac{26}{5} \div \frac{26}{9}$$

$$= \frac{26}{5} \times \frac{9}{26}$$

$$= \frac{9}{5}$$

$$= 1\frac{4}{5}$$

Ex.3: To stitch one shirt, $1\frac{3}{4}$ meters of cloth is needed. How many shirts can be stitched with 7 meters of cloth ?

$1\frac{3}{4}$ meters of cloth is required for 1 shirt

7 meters of cloth is required for ? shirts

Number shirts $= 7 \div 1\frac{3}{4}$

$= 7 \div \frac{7}{4}$

$= 7 \times \frac{4}{7}$

$= 4$

Verification: $4 \times 1\frac{3}{4}$

$= 4 \times \frac{7}{4}$

$= 7$

**Evaluation**

Work out the following:

1. $\frac{2}{3} \div 8$
2. $\frac{11}{15} \div \frac{33}{50}$
3. $2\frac{1}{8} \div 8$
4. $8\frac{2}{5} \div 3\frac{1}{7}$
5. If each child is to get $\frac{3}{4}$ litres of milk, $11\frac{1}{4}$ litres of milk can be distributed to how many children ?

**Decimals**

The fractions having denominators as powers of ten are called Decimal Fractions. The word decimal might have been derived from the Latin word 'Decem' (Meaning 10) or the Sanskrit word 'Dashama' (Meaning 10)

The decimal system is adopted in making fractions of a litre or a kilometer.

For example:

1 kilometer = 10 deca meter = 1000 meters

Similarly

1 litre = 10 deci litres

1 deci litre = 10 centi litres

1 centi litre = 10 milli litres

1 litre = 10 deci litres

= 100 centi litres

= 1000 milli litres

The decimal system is also adopted in making fraction of a rupee. For ex. 1 Rupee = 100 paise ($10^2$ paise).

The advantages of decimals system is that it makes calculations much easy. The teachers are requested to show these advantages to the students whenever they workout the problems.

In this section, we discuss the conversion of fractions into decimals, comparison of decimals, addition, subtraction, multiplication and division of decimal numbers.

**1. Conversion of fractions into decimals:**

$$\frac{4}{10} = 0.4$$

$$\frac{32}{100} = 0.32$$

$$\frac{88}{10} = 8.8$$

$$\frac{88}{1000} = 0.88$$

$$\frac{7}{10} = \ldots \qquad \frac{12}{100} = \ldots \qquad \frac{375}{1000} = \ldots$$

**2. Conversion of decimals into fractions**

$$0.5 = \frac{5}{10} = \frac{1}{2}$$

$$0.83 = \frac{83}{100}$$

$$0.02 = \frac{2}{100} = \frac{2}{100} = \frac{1}{50}$$

$$1.75 = \frac{175}{100} = \frac{7}{4}$$

Write the following decimals as fractions:

0.9, 0.90, 0.09, 0.009, 9.9

**3. Comparison of decimals**

Ex. 1: 0.3 > 0.2 because $\frac{3}{10} > \frac{2}{10}$

Ex. 2: Compare 0.1 and 0.01

$$0.1 = \frac{1}{10} \text{ and } 0.01 = \frac{1}{10^2} = \frac{1}{100}$$

$$\frac{1}{10} > \frac{1}{100} \qquad 0.1 > 0.001$$

Ex. 3: Compare 0.37 and 0.20.

Ex. 4: Compare 1.24 and 1.42

Ex. 5: Write the following in the increasing order:

2.04, 8.81, 0.003, 0.04, 0.8

(Ans: 0.003 < 0.04 < 0.8 < 2.04 < 8.81)

Ex. 6: Write the following in the decreasing order:

0.8, 8.8, 0.88, 0.008, 0.08

**4. Addition of decimals**

Ex. 1: Add: 0.0901 + 0.1860 + 0.0160

Sum = 0.0901 + 0.1860 + 0.0160

$$\begin{array}{r} 0.0901 \\ 0.1860 \\ 0.0160 \\ \hline 0.2921 \\ \hline \end{array}$$

Ex. 2: $3 + \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} = ?$

$$\text{Sum} = \begin{array}{r} 3.000 \\ 0.300 \\ 0.030 \\ 0.003 \\ \hline 3.333 \\ \hline \end{array}$$

Ex. 3: A person has deposited the following amounts on six days of week: Rs. 8024.56, Rs. 4326.96, Rs. 9762.08, Rs. 10758.75 and Rs. 11030.40. Find the total amount deposited.

$$\begin{array}{rr} & 8024.56 \\ & 4326.96 \\ & 9762.08 \\ & 10758.75 \\ & 11030.40 \\ \cline{2-2} \text{Total Rs.} & 43902.75 \\ \cline{2-2} \end{array}$$

**5. Subtraction of decimals**

Ex. 1: Subtract 23.97 from 25.26

$$\begin{array}{r} 25.26 \\ -23.97 \\ \hline 01.29 \\ \hline \end{array}$$

Ans. 1.29

Ex. 2: The length of a stick is 50.35 cms. A piece of 15.25 cms is cut and removed from the stick. Find the length left out.

| | |
|---|---|
| Ans: Length of the stick ———— | = 50.35 cms |
| Length removed ———— | = 15.25 cms (subtract) |
| The length remaining ———— | = 35.10 cms |

**6. Multiplication of decimals**

Ex. 1: 1.14 x 0.12 = ?

Multiply 114 by 12 and put the decimal point after 4 (2+2) places from the right.

```
1.14 x 0.12
-----------
   0.1368
-----------
```

Ex. 2: 1.04 x 0.906 = ?

```
1.04 x 0.906
------------
      624
     000
    936
------------
  0.94224
------------
```

There are five decimal places (2+3). Hence the answer is 0.94224.

Ex. 3: What is the cost of 4.5 meters of cloth if the cost of 1 meter is Rs. 35.50 ?

(Ans. 159.75)

Ex. 4: If the car can travel a distance of 15.6 kms, then what is distance it can travel in 5.7 litres.

(Ans. 88.92)

## 7. Decimals division

Ex. 1: $6.45 \div 5$

$$= \frac{6.45}{5}$$

$$= 1.29$$

Ex. 2: $2.40 \div 10$

$$= \frac{2.4}{10}$$

$$= 0.24$$

Ex. 3: $2.4 \div 100$

$$= \frac{2.4}{100}$$

$$= 0.024$$

Ex. 4: $1.6 \div 0.8$

$$= \frac{1.6}{0.8}$$

(Remove the decimal of the denominator by multiplying by 10 to both numerator and denominator)

$$= \frac{16}{8}$$

$$= 2$$

Ex. 5: $0.352 \div 0.16$

$$= \frac{0.352}{0.16}$$

$$= \frac{35.2}{16}$$

$$= 2.2$$

Ex. 6: When several metal sheets are placed one upon another, the total thickness of the sheets is 39.9 cms. If the thickness of each sheet is 0.7 cms, then how many sheets are there in total ?

Total thickness = 39.9 cms

Thickness of each sheet = 0.7 cms

$$\text{Number of sheets} = \frac{39.9}{0.7} = \frac{399}{7} = 57$$

**Percentage**

The bank has reduced the interest rate from 13% to 12%; Kamala has scored 98% marks in the VII standard examination; More than 50% people in India are illiterates; These are the statements which the students hear everywhere. They can be given other examples of percentage - profit or loss percent in business, improvement in the results in education, etc.

In this section, we discuss the conversion of fractions into percentage and vice versa.

**1. Conversion of fraction into percentage**

Ex. 1: $\frac{73}{100}$ is $\frac{73}{100} \times 100 = 73\%$

Ex. 2: $\frac{3}{4}$ is $\frac{3}{4} \times 100 = 75\%$

Ex. 3: 63 students out of the total of 75 students in class VI passed this year. What is the pass percentage ?

$$\frac{63}{75} \times 100 = \frac{63}{3} \times 4$$

$$= 21 \times 4$$

$$= 84\%$$

Pass percentage = 84%

Ex. 4: Express the decimal 0.37 as percentage.

$$0.37 \times 100$$

$$= \frac{37}{100} \times 100$$

$$= 37\%$$

**2. Conversion of percentage into fraction**

Ex. 1: Express $37\frac{1}{2}\%$ as a fraction.

$$37\frac{1}{2}\% = \frac{37\frac{1}{2}}{100}$$

$$= \frac{75}{100 \times 2}$$

$$= \frac{3}{8}$$

Ex. 2: 30% of a number is 21. Find that number.

Ans: Let the number be x.

30% of that number is 21

x ---------> ?

$$x \times \frac{30}{100} = 21 \qquad x \times \frac{30}{100} = 21$$

x = ?  $x = 21 \times \frac{100}{30}$

$= 70$

Given number is 70

Verification: $70 \times \frac{30}{100} = 21$

**Evaluation**

1. Convert the following into percentage.

   $\frac{3}{5}, \frac{1}{4}, \frac{6}{25}$  (Ans: 60%, 25%, 24%)

2. Convert the following decimals into percentage.

   0.50, 0.3, 2.35  (Ans: 50%, 30%, 235%)

3. In one class there are 60 students, out of which 30% are girls. Find the number of boys in the class.

   (Ans. 42)

4. Fill in the blanks:

| Fractions | Decimals | Percentage |
|---|---|---|
| $\frac{3}{5}$ | ... | ... |
| ... | ... | $33\frac{1}{3}\%$ |
| ... | 0.57 | ... |

## UNIT TEST

Marks: 1x6

**I. Answer all the questions.**

1. Write two equal fractions of $\frac{3}{4}$

2. Among the numbers 0.375 and $\frac{1}{2}$, which one is larger ?

3. Write the following in descending order.

   1.5, 0.625, 0.5625, 0.75, 0.8

4. Expand 53.768 as the powers of 10.

5. Write $\frac{9}{40}$ in the percentage form.

6. Write 35% as a fraction and also as a decimal number.

II. 2x4

1. Simplify $6\frac{5}{6} - 1\frac{7}{8} - 1\frac{1}{4}$

2. Multiply $2\frac{1}{7} \times 2\frac{4}{9} \times \frac{7}{22}$

3. Divide 13.674 $\div$ 0.53

III. 3x2

1. From a roll of 20 meters, Ramesh purchases $4\frac{1}{2}$ meters of cloth and Mahesh purchases $5\frac{1}{2}$ meters. Find the length of cloth remaining in the roll.

2. I travelled 500 kms, in which 16% was by bus and the remaining was by train. Find the distance I travelled by train.

IV. 5x1

In a bag there are 200 coins, out of which half the coins are 10 paise coins, $\frac{1}{25}$ are 20 paise, $\frac{1}{4}$ are 25 paise and the remaining are 50 paise coins. Find the total value of the money in the bag.

**Suggested Activities**

Fraction discs can be prepared by cutting the disc into required number of sectors and they can be used in teaching the concepts of fractions - their addition,subtraction, etc.

Rectangular plastic sheets in which the appropriate fractions are shown by drawing vertical and horizontal lines and shading the proper squares, can also be used in teaching the concepts of fractions.

# CHAPTER VI

# RATIO AND PROPORTION

**Preview**

In the previous class we have learnt about ratio and proportion. We know that we can compare two quantities of the same kind, eg. distance between two places, weights of persons, values of two articles and areas, etc. can be compared.

The lengths of a straight lines are 2 cms and 3 cms. They are in the ratio 2:3 and read it as "two isto three". Remember that the units of both the quantities have to be the same.

eg: PQ = 4 cms,RS = 6 cms. The ratio between the two is 4:6.

[Note: Two quantities of the same kind is expressed in the form of a quotient, such a relation is called the ratio of one quantity to another. The first quantity (4) is called as anticedent and the second (6) term is called the consequent]

- A ratio is unaltered and does not change its value when both the terms are multiplied or divided by the same number.

eg. 1: $\frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$ here 2:3 = 6:9 [Both the ratios are the same]

eg: 2: 2 kg:250 gms

= 2000gms:250 gms [kg should be converted into gms before comparing]

We can write a fraction in the form of ratio.

eg: $\frac{9}{15}$ = 9:15

The ratio can be reduced to the simplest form.

9:15 [We can divide both sides by 3]

Then 3:5 is the simplest form of 9:15.

**Proportion**

When two ratios are equal then the four qualities are said to be in proportion.

eg: 2:4 = 6:12

Here 2 and 12 are called extremes and

4 and 6 are called the means.

The product of extremes is equal to the product of means.

**Review Exercises**

1. Find the ratios of the following quantities.
   a. Rs. 2 and 50 paise
   b. 173 days and 1 year
   c. 2 Mt and 1 Mt 50 cm
   d. 2 lit 200 gms and 2.250 gms
2. Write in the form of ratio

   a. $\frac{5}{7}$ b. $\frac{18}{7}$
3. Reduce these to the simplest form.

   a. 11:45 b. 60:75

   c. 35:60 d. 26:52
4. Write these ratios in the form of fraction.

   a. 6:8 b. 20:35

   c. 12:18 d. 100:250

## Introduction

The quotient of two numbers is called their ratio. The term ratio is used only when it is required to express one quantity as a fraction of another (homogeneous with the first). Two mutually dependent quantities are proportional if the ratios of their values remain constant.

## Exercises

Express the following ratios in the simplest form.

eg. 1. 25 kg and 45 kg

25:45 (divide both sides by 5)

5:9 is the simplest form

2. Rs. 4 paise 35; Rs. 2 paise 10

Step 1: Convert both of them into paise

435:210 Divide by 5
87:42 (This can found out by finding the HCF)

| | | | |
|---|---|---|---|
| 2 | 435 | 210 | 1 |
| | 220 | 115 | |
| 1 | 115 | 95 | 4 |
| | 95 | 80 | |
| 1 | 20 | 15 | 3 |
| | 15 | | |
| | 5 | 18 | |

3. If eight workers finish a work in 32 days then 16 workers can finish the same work in how many days ?

8:16::x:32

There are four terms here. 8 and 32 are called extremes and 16 and x are called means. Here the product of extremes is equal to the product of means.

$$16 \times x = 8 \times 32$$

$$x = \frac{8 \times 32}{16}$$

$$= 16$$

Ans: 16 workers can complete the same work in 16 days.

4. Find the value of 'x' in the proportion.

18:x::27:3

According to rule the product of means is equal to the product of extremes.

i.e. $27 \times x = 18 \times 3$

$$x = \frac{18 \times 3}{27}$$

$x = 2$

5. Identify whether the following numbers are in proportion.

12, 16, 6, 8

Write the numbers in ascending order in the form of proportion.

6:8::12:16

Find the product of extremes and the product of means.

Then 6x16 = 8x12 = 96

The product are same. Therefore they are in proportion.

**Evaluation Exercises**

1. Express in the form of ratios.
   a. 10 mts and 25 mts
   b. Rs. 20 and Rs. 15 paise 75
2. Identify the 'antecedents' and 'consequents' in the following
   a. 8:6 b. 9:15
3. Say whether the following are in proportion or not.
   a. 2,3,4,6 b. 7,5,2,5
4. Find the value of 'x' in the proportions.
   a. 5:9::x:18 b. x:7::10:14

5. Express the following ratios as fraction.

a. 5:10 b. $0.5:1\frac{1}{2}$

6. The ratio of boys to girls in a class is 3:1. There are 48 girls in the class. How many boys are there ?

**Distance and Time**

The distance travelled in a unit time is called the speed.

1. A train starts at 8 am and reaches as place at a distance of 120 km at 12 noon. Find the speed of the train.

Time from 8 am to 12 noon = 4 hrs

Distance travelled = 120 kms

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}} \text{ or } s = \frac{d}{t}$$

$$S = \frac{120 \text{ km}}{4 \text{ hrs}}$$

$$S = 30 \text{ km/hr}$$

2. Calculate the time taken for a bus to reach a town which is 8 km at a speed of 2 km/hr.

s = 2 km/hr

d = 8 km

t = ?

$$t = \frac{d}{s}$$

$$t = \frac{8 \text{ km}}{2 \text{ km/hr}}$$

$$t = 4 \text{ hrs}$$

3. Find the distance travelled in 5 hrs 30 mts at a speed of 80 km/hr by a van.

$$t = 5 \text{ hrs } 30 \text{ mts} = 5\frac{1}{2} \text{ hrs} = \frac{11}{2} \text{ hrs}$$

s = 80 km/hr

d = ? $\quad\quad$ d = s x t

$$= \frac{80 \times 11}{2}$$

d = 440 kms

**Evaluation Exercises**

1. A bus travels 200 kms in 4 hrs. Calculate distance travelled by the same bus at the same speed in 6 hrs.
2. Calculate the speed of a van if it travels 280 km in 4 hrs.
3. Calculate the time taken to travel by an aeroplane to reach a distance of 2000 km at a speed of 400 km/hr.

**UNIT TEST**

I.

1. Express the following in the form of ratios.

   40 boys and 40 girls

2. Express the following ratio as fraction.

   4:5

3. Insert the suitable sign [=, ≠]

   3:4 ☐ 12:7

4. Fill in the blanks to make a true proportion.

   $2:3 = \frac{1}{2} : \square$

5. Find the value of 'x' in the proportion:

   18:x = 90:105

6. Speed x Time = ____?____

II.

1. Check whether the following are in proportion.

   5:3::15:9

2. Express the following in the simplest form.

   a. 15:45    b. $\frac{64}{144}$

3. Calculate the distance travelled by a scooterist in $3\frac{1}{2}$ hr at a speed of 44 km/hr.

4. A student walks 5 km in first hour and 4 km in second hour, and $3\frac{1}{2}$ km in third hour. What is his average speed ?

III. How long will a journey of 200 km take at an average speed of 40 km/hr. What should be the average speed of the train to reach the same place in 3 hours ?

**Suggested Activity**

1. Compare the lengths of two articles, height of two persons, the weights of two boys, etc. in the classroom.

# CHAPTER VII
# SIMPLE INTEREST

## Preview

In the previous class we have learnt simple interest. Now we will study in detail about simple interest.

In our daily life we require money for starting a business, building a house, marriage and higher studies, etc. In such situations if we require more money, then we have to borrow money from others like money lenders, bankers and banks. We keep money in banks or borrow money from banks. This money is called principal. The charges made for the use of borrowed money is called the interest. They also give interest for our money. The amount of interest depends upon the principal and the time. If the interest is calculated only on the original principal then it is called simple interest.

For eg: Rate of interest 18% per annum - Means the interest
for Rs. 100 for one year is Rs. 18.

Other countries calculate the interest in their own currencies.

For eg: America - Dollar, England - Pounds, Russia - Rubel,
Iran - Rial, Japan - Yen, Srilanka - Rupee,
Bangladesh - Taka.

## Simple interest calculation

Eg: Nalini borrows Rs. 2000 at 18% p.a. Calculate the amount she has to pay after 6 years.

Rs. For 1 year - Interest is Rs. 18

For 1 year = $\frac{18}{100}$

Rs. for 2000 for 1 year = $\frac{18}{100} \times 2000$

6 years = $\frac{18 \times 2000 \times 6}{100}$

The interest is = Rs. 2160

Total amount she has to pay after six years A = P + I

= 2000 + 2160 = Rs. 4160

This can be calculated using the formula:

$$S.I. = \frac{\text{Principal x Rate x Time}}{100}$$

$$S.I. = \frac{P.T.R.}{100}$$

**Review Exercises**

1. What is the formula for the calculation of S.I.
2. Calculate the simple interest on Rs. 4000 for 5 years at 13% p.a.
3. Find the amount I get if I invest Rs. 1650 at $12\frac{1}{2}$ % for $2\frac{1}{2}$ years.
4. Calculate the simple interest on Rs. 10,000 kept in the fixed deposits for 3 months at 10% p.a.
5. What some of money will yield an interest of Rs. 950 in 5 years at $9\frac{1}{2}$ % p.a.

## Introduction

Borrowing and lending money leads to calculation of S.I. and this plays very important role in daily life. Savings also necessary, because it helps as security for future life. There are different kinds of savings like National Saving Certificates, Recurring Deposits, Indira Vikas Patra, etc.

All these savings earn interest and the amount will grow. Therefore it is necessary to know how to calculate simple interest.

Formula for simple interest

$$S.I. = \frac{PTR}{100} \qquad R = \frac{100I}{PT}$$

$$T = \frac{100I}{PR} \qquad P = \frac{100I}{RT} \qquad A = P + I$$

## Solve

1. Rs. 4500 will get interest Rs. 3150 in 5 years. Calculate the rate of interest.

| | | |
|---|---|---|
| Principal | Rs. 4500 | $R = \frac{100I}{PT}$ |
| Interest | Rs. 3150 | |
| Time | 5 years | $\frac{100 \times 3150}{4500 \times 5} = 14$ |
| Rate of interest | 14% | |

2. In what time Indira Vikas Patra of Rs. 5000 amounts Rs. 10,000 in the rate at 20% simple interest.

| | |
|---|---|
| Amount | Rs. 10000 |
| Principal | Rs. 5000 |
| Interest | Rs. 5000 |
| Rate | 20% |

Time $= \frac{100I}{PR} = \frac{100x5000}{5000x20} = 5$ years

3. A certain sum of money amounts to Rs. 7975 in 3 years and Rs. 87000 in 4 years at SI. Find the principal and the rate of interest.

| | |
|---|---|
| Amount in 4 years | = Rs. 8700 |
| Amount in 3 years | = Rs. 7975 |
| Interest for 1 year | = Rs. 725 |
| Interest for 3 years | = 725 x 3 = 2175 |
| Amount in 3 years | = Rs. 7975 |
| Interest for 3 years | = Rs. 2175 |
| Principal | = Rs. 5800 |

P = Rs. 5800 $\quad R = \frac{100I}{PT}$

T = 3 years

$R = 12\frac{1}{2}\%$ $\quad \frac{100 \times 2175}{5800 \times 3}$ or $\frac{100 \times 725}{5800} = 12\frac{1}{2}$

4. A person deposits Rs. 2000 in a bank. If the bank pays at $10\frac{1}{2}\%$ p.a. what is the amount he gets after 146 days.

Principal = Rs. 2000 $\quad S.I. = \frac{PTR}{100}$

Rate $= 10\frac{1}{2}\% = \frac{21}{2}\%$

Time $= \frac{146}{365}$

S.I. $= \frac{2000}{100} \times \frac{21}{2} \times \frac{146}{365} =$ Rs. 84.00

The total amount he gets back = Rs. 2084

## II. Discount

During the festival seasons dealers in some shops announce reduction in price. The reduction in prices is known as 'Discount or Rebate'.

Many times selling a house or a vehicle is done through an agent or broker. He is paid a certain amount of money for the work that he does. The money thus paid to the agent is called the commission. The government also gives commission to the agents for selling N.S.C., I.V.P., etc.

1. The marked price of a radio is Rs. 1700. A dealer allows 15% discount. How much does the customer pay for the radio.

   For Rs. 100 marked price, the discount is Rs. 15

   The discount on Rs. 1700 is $= \dfrac{1700 \times 15}{100} = 255$

   Marked price = Rs. 1700

   Discount = Rs. 255

   Selling price = Rs. 1445

2. Commission

   Ramesh sold a scooter for Rs. 34000. He paid Rs. 680 to an agent. What is the rate of commission ?

   Selling price of a scooter = Rs. 34000

   The commission is = Rs. 680

   Rate of commission is $= \dfrac{680 \times 100}{34000} = 2\%$

## Evaluation Exercises

1. Calculate the simple interest on

   a. Rs. 2400 for 3 years at $16\frac{1}{4}$% per annum. (Rs. 1170)

   b. Rs. 12000 on fixed deposits for 5 years at 10% p.a. (Rs. 6000)

   c. Calculate the rate of SI in which the principal doubles in five years.

2. Solve

   a. A certain sum of money amounts Rs. 3472 in 2 years and Rs. 3808 in 3 years. Find the principal and the rate of interest.

   (Rs. 2800; R = 12%)

   b. Rs. 1500 amount to Rs. 1770 at 6% p.a. SI. Find the time.

   c. Guru gets an amount of Rs. 21300 from his fixed deposit. After 3 years at 14% pa. Find the sum of money deposited in the bank. (Rs. 15000)

   d. A person takes a loan of Rs. 2 lakhs from HDFC to construct a house. Calculate the amount he has to pay after 10 years if the rate of SI 13% p.a. (Rs. 2,60,000)

   e. A TV dealer reduced the price of a TV by 12% and sold it for Rs. 15840. Find the marked price of the TV. (Rs. 18,000)

   f. Savanth received a commission of Rs.2150 for getting a site sold. If the rate of commission is $2\frac{1}{2}$% Find the selling price of the site. (Rs. 86,000)

## UNIT TEST

1x6 = 6

I. Answer the following

1. 18% means _______
2. Rate of interest R = _______
3. Simple interest SI = _______
4. $P = = \frac{100\_\_\_}{R\ \_\_\_}$
5. $T = = \frac{\_\_\_\ I}{P\ \_\_\_}$
6. Marked price = Selling price + _________
7. Amount = __________
8. 2 years 3 months _______ (convert this into year)

II. Solve

1. Find the simple interest at $5\frac{1}{2}$ % for $2\frac{1}{4}$ years on the amount Rs. 8000.
2. A sum of 1800 yields a simple interest of Rs. 243 at 6% p.a. Find the period.
3. A National Saving Certificate of Rs. 100 amounts to Rs. 132 in 3 years. Find the rate of SI.
4. In how many years will a certain sum of money double itself at $12\frac{1}{2}$ p.a. SI.

III.

1. At what rate per cent per annum will Rs. 3400 give interest of Rs. 1700 in 5 years.

2. Marked price of an electric fan is Rs. 1200. A person pays Rs. 960. What is the rate of discount allowed ?
3. Kareem received Rs. 2400 commission for getting a car sold. If the rate of commission is 2%. Find the selling price of the car.

IV.

1. A certain some of money amounts to Rs. 6500 in 3 years. Rs. 7500 in 5 years at SI. Find the principal and the rate of interest.
2. Anitha will be paid commission of 1% for selling National Saving Certificate. If she sells Rs. 2,75,000 worth of certificates. What is the amount of commission she gets ?

**Activity**

1. Give a visit to different banks. Learn different savings bank accounts. Also see that the rate of interest is different for different time intervals.
2. Acquire the knowledge about the exchange rate of currencies of different countries.
3. Visit a readymade garment shop and know about discount sale.

# CHAPTER VIII

## SETS

### Preview

In fifth and sixth classes, some basic ideas of sets have been discussed. These are

(a) Set and elements of a set

(b) Representation of sets - the Roster method and the Rule method (or the set-builder method)

(c) The classification of sets - Finite and Infinite sets, Null set

-- Fifth [(a)–(c)]

(d) Subset of a set - Notations

(e) Universal set - Venn diagram

(f) Equal sets and Equivalent sets

(g) Operations on sets - Union and Intersection of sets

-- Sixth [(d)–(g)]

We briefly summarise these as follows -

(a) A collection of well defined objects is called <u>a set</u>. The objects belonging to the set are called <u>the elements</u> of the set. While sets are represented by capitals A, B, C, etc., the elements are represented by a, b, c, etc. To state that (i) a is an element of the set A (or a belongs to the set A) We write $a \in A$ and (ii) a is not an element of A by $a \notin A$.

Eg: Given $A = \{1, 2, 3\}$, 1, 2, 3 are elements of A.

i.e. $1 \in A$, $2 \in A$, $3 \in A$ but $4 \notin A$ since 4 is not an element of A

(b) A set is represented by listing all the elements of the set or by stating a common property of all the elements of the set. These are respectively the Roster method and the rule method.

For example: The set A = {1, 2, 3} is in the Roster form and the same set in set builder form (Rule method) is A = {x | x is a natural number less than 4}.

(c) The sets are of different types

(i) A set without any element is called a null set or an empty set denoted by $\phi$ or { }. On the other hand if it has atleast one element, then it is not empty, i.e. a non-empty or a non-null set. Thus the set of all natural numbers less than 0 is a null set.

(ii) A set containing a finite number of elements (so that all the elements can be listed) is called a finite set. Thus, for example, the set of all letters of English language (a to z) is a finite set. Let it be A. The number of elements in a finite set A is called its order denoted by O(A). In the previous example O(A) = 26.

(iii) A set which is not finite is called an infinite set. All the elements of an infinite set cannot be listed to the last element. For example, the set of all even integers is an infinite set.

(d) Given two sets A and B, they are said to be equal sets (or identical sets) if A and B have the same elements. Then we write A = B. If A = {the week days},B = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday), then A = B.

(e) Given two sets A and B, if every element of A is also an element of B, then A is called a subset of B and we write $A \subset B$. Suppose A = The set of all the letters of the word 'TERM' and B = The set of all the letters of the word 'TERMINAL'.

Then A = {T, E, R, M}; B = {T, E, R, M, I, N, A, L} and each element of A belongs to B also. Hence $A \subset B$. Observe that (i) if A = B, then $A \subset B$ and $B \subset A$ and (ii) if $A \subset B$ then A may or may not be equal to B.

(f) In any problem on sets, the sets discussed are subsets of some fixed set called the universal set denoted by U. Then all the elements belonging to various sets in the problem, belong to the universal set U. For instance, if the sets discussed are sets with integers as elements, we may take U = the set of all integers.

(g) Finite sets having the same number of elements each are called Finite <u>equivalent</u> sets. In the case of such sets A and B, the elements of A can be paired with the elements of B.

Then if A = {1, 2, 3} and B = {a, b, c}

We can pair the elements these ways.

| | | |
|---|---|---|
| 1 <-----> a | | 1 <-----> b |
| 2 <-----> b | or | 2 <-----> c |
| 3 <-----> c | | 3 <-----> a |

This is possible because A and B have the same number of elements. Hence $A \not\subset B$ and $B \not\subset A$ eventhough $A \neq B$.

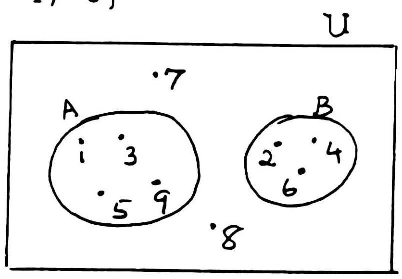
Equal finite sets have the same number of elements. Hence equal sets are equivalent but equivalent sets may or may not be equal.

(h) The ideas connected with sets - sets of different types can all be diagrammatically represented through diagrams called Venn diagrams. (due to John Venn, an English mathematician of 19th century) or more correctly Euler-Venn diagrams (L.Euler - German, 1707-83). Here a rectangular region is used to denote the universal set U and other sets by closed regions (like circles and ovals) drawn inside the rectangular region.

For example, Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}

A = {1, 3, 5, 9}, B = {2, 4, 6}

Then the appropriate Venn diagram is




(i) Just as numbers are added, multiplied, etc. to get the sum or the product of given numbers, we have some operations to form new sets combining the given sets in some way. Union of sets and intersection of sets are two important operations on sets.

(i) **Union of sets**: Given two sets A and B, the set of all elements which belong to A or to B (or to both A and B) is called the union of A and B denoted by A ∪ B.

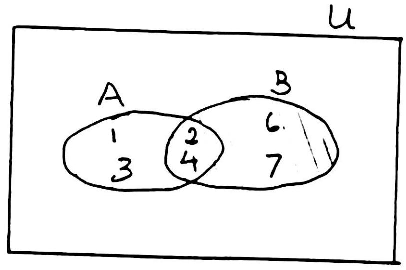
Accordingly, if

A = {1, 2, 3, 4}

B = {2, 4, 6, 7}

Then A ∪ B = {1,2,3,4,6,7}

the operation of union is denoted by ∪




A ∪ B

**A special case**: If A ⊂ B, then A ∪ B = B.

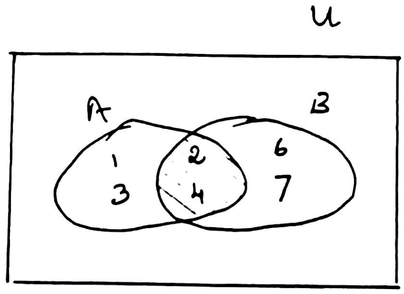
(ii) **Intersection of sets**: Given two sets A and B, the set of all elements belonging to both A and B (i.e. the common elements of A and B) is called the Intersection of A and B denoted by $A \cap B$.

In the same example given above,

$A = \{1,2,3,4\}$ and $B = \{2,4,6,7\}$

so that $A \cap B = \{2, 4\}$

The operation of intersection is denoted by $\cap$.




$A \cap B$

(iii) **Some special cases**

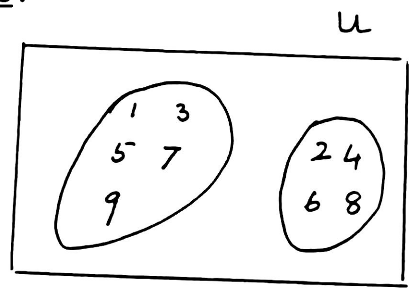
(1) When A and B do not have any common element, then $A \cap B = \emptyset$ (empty set). Then the set A and B are called disjoint sets.

**For example**

$A = \{1, 3, 5, 7, 9\}$

and $B = \{2, 4, 6, 8\}$

are disjoint sets.




$A \cap B = \emptyset$

(A & B are disjoint sets)

(2) When $A \subset B$, then $A \cap B = A$ because, each element of A is a common element of A and B.

**Caution**: While writing a set in the Roster form,

(a) no element of the set should be repeated.

(b) the elements of the set can be written in any order.

**Explanation**: If A = {letters of the word 'SUCCESS'}

then A = {S, U, C, E}

and not A = {S, U, C, C, E, S, S}

Also A can as well be written as A = {C, E, S, U}.

## 1.2 Revision Exercises

(1) Write the set of even natural numbers less than 10 in (a) the Roster form, (b) the set builder form.

(2) Given A = {1, 2, 3, 4, 5} and B = {2, 3, 6, 7, 10}

Fill in (a) $1 \in$ ___ (b) $2 \in$ ___ (c) $4 \in$ ___ (d) $5 \notin$ ___ (e) $7 \notin$ ___ (f) $8 \notin$ ___.

(3) Fill in

(a) When all the elements of a set are listed, the method of writing the set is called _______.

(b) In __________ method, the common property of all the elements of the set is mentioned.

(4) Write the set of letters of each word in the following by Roster method -

(a) TEACH (b) MULTIPLY (c) MATHEMATICS (d) ADDITION (e) NATION (f) EXCELLENT

(5) State whether each of the sets is finite or infinite.

(a) The set of all Rivers flowing in Karnataka State

(b) The set of all Natural numbers

(c) The set of all planets

(d) The set of all points on a line segment

(e) The set of all past prime ministers of our country

(f) The set of all natural numbers less than 100

(g) A null set

(h) The set of all triangles which can be drawn on a sheet of paper

(6) State <u>True</u> (T) or <u>False</u> (F)

(a) Universal set is always infinite

(b) The null set is always a subset of any set

(c) Equivalent sets are also equal sets

(d) For any set A, $A \subset A$ and $A \subset U$

(e) $\phi = \{ 0 \}$

(7) Form the union and intersection of A and B in the following cases.

(a) $A = \{1, 2\}$, $B = \{0, 2, 4\}$

(b) $A = B = \{a, b, c, d\}$

(c) A = The set of the letters of the word 'GODESS'
B = The set of the letters of the word 'DRESS'

(d) A = The set of natural numbers divisible by 3 and less than 10

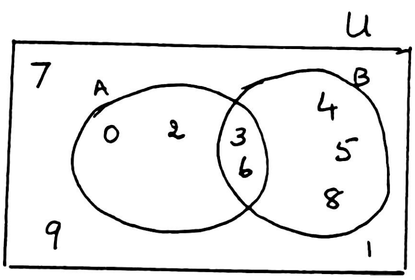
(e) $A = \{1, 2, 4, 6, 8\}$, $B = \{0, 1, 2, 3, 4, 6, 7, 8\}$

(f) A = The set of all natural numbers divisible by 3
B = The set of all natural numbers divisible by 4

(8) How many subsets has $A = \{1, 2, 3\}$ ?

(9) From the Venn diagram given below write down the following sets -

(a) Universal set, (b) A, (c) B, (d) $A \cup B$ and (e) $A \cap B$



(10) In which case are A and B disjoint ?

(a) A = Set of people who are aged less than 25 years
B = Set of people who are aged between 20 years and 50 years

(b) A = Set of all natural numbers
B = Set of all whole numbers

(c) A = Set of isosceles triangles
B = Set of all equivalent triangles

(d) A = Set of all vowels among English letters
B = Set of all consonants among English letters

## 2. Introduction

Sets and related concepts are parts of daily life experiences. We come across collections of different sorts - like set of automobiles, set of educational institutions, set of people living in a place, set of mountains of a country, set of rivers flowing in a country and so on. Besides the language of sets is basic to science and mathematics. The idea of a set is simple yet it has wide applications. Starting from number sets and operations with numbers. Sets of geometric shapes/figures and their properties, we study important branches of mathematics - Algebra and Geometry.

'God gave integers and all else in the handi work of man' said the celebrated mathematician Kronecker (1823-91) and the set of integers is the craddle for other sorts of numbers.

Sets are learnt in standards five and six, and the content of this unit studied in the seventh standard by and large are the ones studied in the previous classes.

### 3. Concepts

(a) Venn diagrams depict sets through diagrams

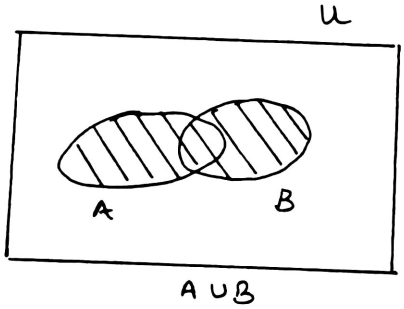
(b) Given two (or more) sets, new sets are formed through operations on sets - Union and Intersection.

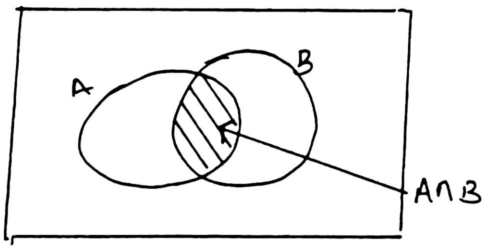
Accordingly, (i) given sets A and B, the set $A \cup B$ whose elements belong to A or B or both A and B is called the union of A and B, while (ii) the set of all common elements is the intersection of A and B denoted by $A \cap B$.

(c) Venn diagram for Union and Intersection of sets - the given sets A and B are shown as circular regions inside a rectangle representing the Universal set U.

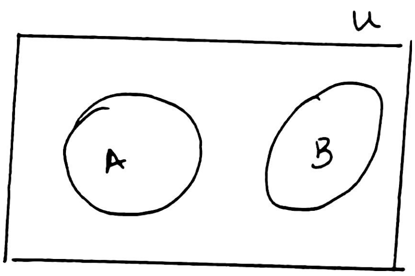
(i) Shading both A and B the same way, we get the union $A \cup B$.

(ii) Shading only the common (overlapping) region we get the intersection A B.




(d) **Disjoint Sets**: Sets having no common elements are called disjoint (or non-intersecting) sets. In the case of such sets, the intersection is an empty set.




Disjoint Sets: $A \cap B = \emptyset$

## 4. Teaching Strategies - Activities

### (a) Through (physical) demonstration of teaching aids

(i) Forming two or more sets using objects available like the set of books of different sizes/on different subjects.

- The sets of students of different ages
- The sets of pencils, etc.

Using these, explaining the formation of union and intersection of sets.

(ii) Preparing charts of Venn diagrams for Union/ Intersection of sets.

(iii) Asking students to give examples of various sets within their experience and eliciting the union and intersection of sets.

(iv) Explaining the operations through examples.

- Simple techniques of formation of unions and intersections of sets like -

**To form the Union of two finite sets A and B**

List all the elements of A (or B) and then those of B without repeating the elements.

**To form the Intersection of two sets A and B**

Marking the common elements and lifting them all.

**(b) Problem Solving**-Illustrated examples (solved problems)

(1) Draw suitable Venn diagrams to represent the following sets -

(a) $A = \{1, 2, 3, 4\}$

(b) $B = \{0, 3, 4, 5\}$

(c) $A \cup B$

(d) $A \cap B$

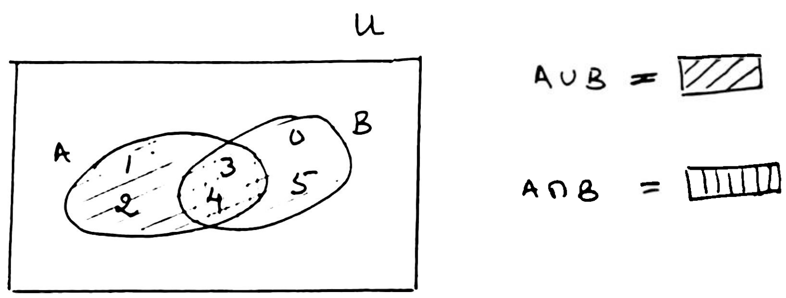
taking the universal set as $U = \{0, 1, 2, 3, 4, 5, 6\}$

**Solution**

Step 1: Draw a rectangle to denote U.

Step 2: Draw two circles to represent A and B inside U such that they overlap (intersect) since the two sets A and B have common elements 3 and 4.

Step 3: Mark the elements of the sets by points appropriately




(2) Form the Union and Intersection of sets in the following cases -

(a) A = {a, b, c, d}, B = {b, d, e}

(b) A = {p, q, r, s, t}, B = {s, t}

(c) A = The set of letters of the word 'DIAMOND'

B = The set of letters of the word 'PEARL'

**Solution**

(i) In forming the union of sets, list all the elements of one set, then those of the other set, without repeating any element.

(ii) In forming the intersection, list only the common elements of the set.

(a) $A \cup B$ = {a, b, c, d, e}; $A \cap B$ = {b, d}

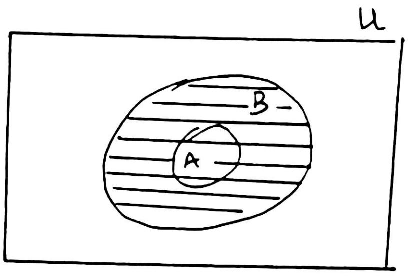
(b) $A \cup B$ = {p, q, r, s, t}; $A \cap B$ = {s, t}

(c) $A \cup B$ = {D, I, A, M, O, N, P, E, R, L}
$A \cap B$ = {A}

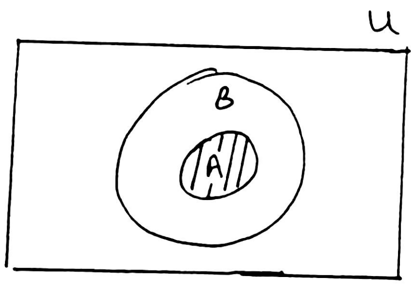
(3) If $A \subset B$, explain how

(a) $A \cup B = B$ and (b) $A \cap B = A$. Write the Venn diagrams also.

**Solution:** $A \subset B$ means every element of A belongs to B. Hence when $A \cup B$ is written, the union contains only the elements of B. Therefore $A \cup B = B$. Since all elements of A are common to both A and B, $A \cap B = A$.




$A \cup B = B$




$A \cap B = A$

(4) Describe the Union and Intersection of the sets A and B.

(a) A = The set of all planets nearer to the Sun than the Earth.
B = The set of all planets farther than the Earth to the Sun.

(b) $A = \{3, 6, 9, \ldots\}$
$B = \{4, 8, 12, \ldots\}$

**Solution**

(a) $A \cup B$ consists of planets which are either nearer to the Sun or far away from the Sun than the Earth.

Hence $A \cup B$ = All the planets except the Earth.

Next, $A \cap B = \emptyset$ because A and B do not have any common element.

(b) $A \cup B$ = {Integers which are multiples of 3 or 4}

$A \cap B$ = {Integers which are multiples of both 3 and 4}

= {Integers which are multiples of 12}

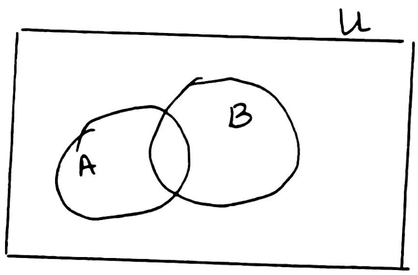
(5) Describe the following sets by Venn diagrams

A = The set of all people who drink Milk or Coffee

B = The set of all people who drink Tea or Milk

Describe $A \cup B$ and $A \cap B$ in words.

**Solution**

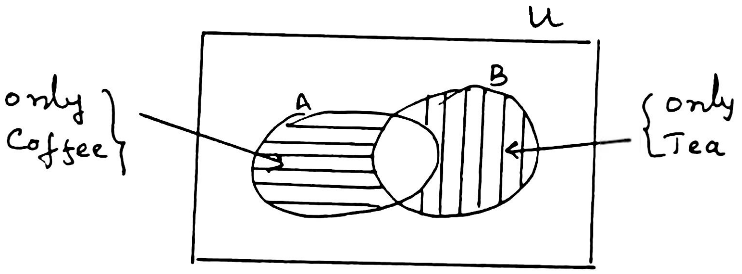



Since people who drink milk belong to A and B both, the sets are as shown in the Venn diagram (not disjoint).

$A \cup B$ = The set of all people who drink coffee or tea or milk

$A \cap B$ = The set of all people who drink only milk

(6) In the previous problem, shade the set of all those who drink (a) only coffee, (b) only tea.



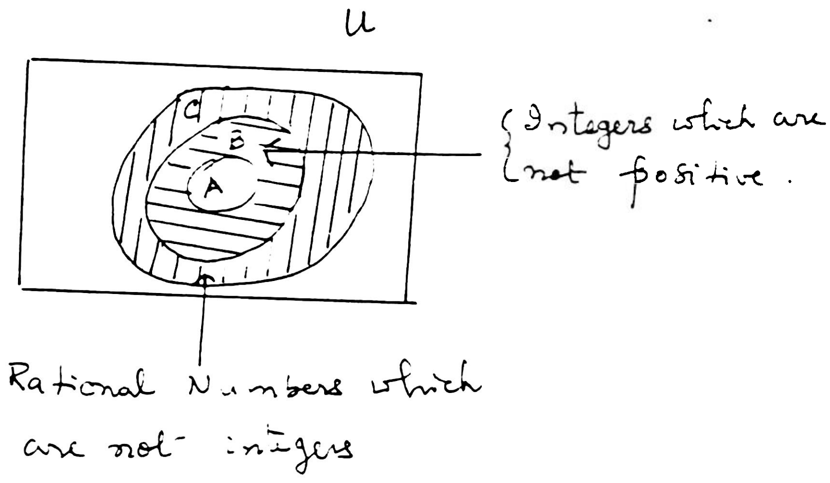

(7) If A = The set of all natural numbers

B = The set of all integers

C = The set of all rational numbers

Draw Venn diagram showing A, B, C. Shade the regions (a) for integers which are not positive, (b) for rational numbers which are not integers. How are the sets A, B, C related ?

Solution




$A \subset B \subset C$

5. **Evaluation and Exercises**

**A unit test on sets (for VII standard)**

**Blue Print:** Topicwise weightages

| Sl. No. | Topic | Question type: 1 mk | 2 mks | 3 mks | 5 mks | Total (mks) |
|---|---|---|---|---|---|---|
| 1 | Sets - Review (pre-knowledge) | 4 | 1 | | | 6 |
| 2 | Venn diagrams | 2 | 1 | 1 | | 7 |
| 3 | Operations on sets | | | | | |
| | (a) Union of sets<br>(b) Intersection of sets | | 2 | 1 | 1 | 12 |
| | Total | 6 x1 | 4 x2 | 2 x3 | 1 x5 | 25 |

# UNIT TEST

Time : 45 min
Marks: 25

Questions: 1 to 6 carry 1 mk each
7 to 10 carry 2 mks
11 to 12 carry 3 mks
and 13 carries 5 mks

**Answer all questions**

1. Write the set of all days in a week in Roster form.

2. State whether $\{a, b, c\}$ is a finite set or infinite set.

3. $A = \{0, 1, 2, 4\}$ Which of the following are true ?

   (i) $1 \in A$, (ii) $4 \notin A$, (iii) $3 \notin A$, (iv) $0 \in A$

4. Write a subset of $\{p, q, r, s\}$ containing two elements.

5. Write Venn diagram for the sets $A = \{-1, 0, 1\}$, $B = \{0,1\}$

6. Name the set of all natural numbers between 0 and 1.

7. Which sets are equivalent ?

   A = {Scale, Protractor, Divider, Compasses}
   B = {Earth, Sun, Moon}
   C = {Dog, Cat, Rat, Lion}
   D = {Rama, Sita, Lakshmana, Bharatha, Hanumantha}

8. Draw the Venn diagram for the sets

   A = {All vegetarians}, B = {All non-vegetarians who are also vegetarians}

9. Draw Venn diagrams and shade $A \cap B$ and $A \cup C$ separately

   given $A = \{0, 1, 3, 4\}$

   $B = \{0, 3, 5\}$

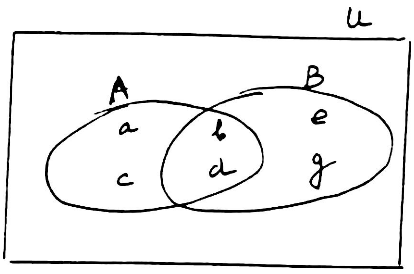
   and $C = \{2, 3, 4, 6\}$

10. $A = \{x : x$ is a positive integer less than $5\}$

    $B = \{x : x$ is a multiple of 3 greater than 4 and less than $10\}$ write

    ~~with~~ $A \cup B$ and $A \cap B$.

11. From the Venn diagram given ~~with~~ write A, B, $A \cup B$ and $A \cap B$.




12. Describe the Union and Intersection of A and B in words.

    Given A = {People whose age is not more than 40}
    B = {People whose age is between 25 and 60}

13. Describe the following sets symbolically using A, B, C, Union (U) and Intersection ( ) symbols.

    Given

    A = {People who speak Kannada}

    B = {People who speak Tamil}

    C = {People who speak Telugu}

    a = {People who speak all the three languages}

    b = {People who can speak at least one of the languages}

    c = {People who can speak Kannada and Tamil or Kannada and Telugu}

    Show these sets by separate Venn diagram shading the sets suitably.

**Exercises**

1. Identify the well-defined collections among the following collections.

   (i) All positive fractions

   (ii) All tall persons

   (iii) All capitals of Indian states

   (iv) All Presidents of India till today

2. Write the sets in Roster form

   (i) Set of all perfect squares between 0 and 10

   (ii) Set of all planets

3. Write the sets in the set builder form

   (i) $\{0, \pm1, \pm2, \ldots\}$

   (ii) $\{a, b, c, \ldots, y, z\}$

4. Identify the finite and infinite sets, and write the order of finite sets.

   (i) A = {All planets}

   (ii) B = {All points on a sheet of paper}

   (iii) C = {All vowels of English language}

5. Which cannot be represented by Venn diagram ?

   (a) Finite set, (b) Infinite set, (c) Universal set,
   (d) Null set

6. Find the (a) union of $\{2, 3, 5\}$ and $(3, 5, 6, 7\}$

   (b) intersection of $\{0,3,6,9\}$ and $\{0,9,12,15\}$

7. If $A \cap B = A$, then B = ?

8. If $A \cup B = A$, then B = ?

9. Represent $A = \{1, 2, 3, 4\}$, $B \{ 2, 4, 5, 7\}$ and

   $C = \{2, 5, 8, 9\}$ in Venn diagram.

10. A = The set of letters of the word 'LETTERS'

    B = The set of letters of the word 'PUPPETTE'

    Represent A, B and their union and intersection by Venn diagram.

11. Suggested Activities

(i) Collect examples of sets from (a) daily life situations, (b) sciences, social sciences and other fields, and discuss how these can be utilised in the classroom to improve your teaching.

(ii) Design charts to explain the concepts related to sets and operations on sets.

(iii) Prepare a unit test on the topic.

(iv) Prepare test items to cover all the aspects of sets typewise (i.e. 1 mk/2 mks/3 mks and 5 mks).

(v) Improvise dummy or working models to explain the basics of sets and operations on sets consulting science teachers and teachers of training institutes.

# CHAPTER IX

# ALGEBRA

## 1. Preview

In Vth and VIth classes basic concepts of Algebra have been discussed. These are

(a) unknown (variable or literal number)

(b) operations - addition/subtraction/multiplication

(c) power of unknown

(d) algebraic expression

(e) simple equation in one unknown and inequality

We briefly recall these basics

(a) A quantity which can take different values over a set of numbers is called an <u>unknown</u> or <u>variable</u>. We denote them by any of the English letters, usually small letters a, b, c, ..., x, y, z.

(i) Suppose the amount of sugar (in kgs) available in a fair price shop changes from time to time as the same is sold, the amount of sugar available may be denoted by x kgs or simply x. Originally if the stock was 100 kgs and at the end of the day all the sugar is soldout, then x takes values over the set of numbers from 0 to 100.

(ii) Suppose there is a cricket match. The number of runs scored from time to time is a variable, likewise the number of over, wickets fallen are variables. Accordingly, the number of runs is x, the number of overs is y and the number of wickets fallen is z.

(b) Just as numbers are added, multiplied, etc. operations on expressions containing one or more variables are defined. While performing these operations on expressions (two or more) it is necessary to group the terms containing the same variable (called like variables) first and then do operations for the terms of each group. While doing so, the coefficient in the terms containing like variables must be combined.

(c) When variable is multiplied by itself once or more than once, powers of the variable are got. Then x x x = $x^2$ ; x x x x x = $x^3$ and so on. x itself is a power and we write x = $x^1$. Here x is called the base and 2, 3 are exponents (or indices) in $x^2$ and $x^3$ respectively.

(d) If x is a variable, $x + 1 = 2$, $2x + 3 = 5$ are examples of simple equations in x. In such an equation x appears in first power (i.e. the exponent is 1) and is called a linear equation in x. An equation of this type is like the statement 'Itis age two years after will be 20 years' (This is an example of an open sentence). Here the unknown is 'Hisage'. If we denote it by x, the statement becomes $x + 2 = 20$. Likewise, for a variable x, the statement x is less than 4 or x greater than 10 take the form $x < 4$ and $x > 10$ respectively. These are examples of linear inequations in x.

**Revision Exercises**

1. Using x for the unknown,rewrite the following statements.

   (a) A basket has some oranges
   (b) My purse contains some rupees
   (c) I have to travel some kilometers today
   (d) He ate some jamoons
   (e) The child has some toys

2. In the above statements find the value of x in each case looking at the following corresponding statements.

   (a) The basket has 10 oranges

   (b) My purse has 50 rupees

   (c) I have to travel 3 kms today

   (d) He ate 8 jamoons

   (e) The child has no toys

3. In the following group the terms with like variables - 5x, +2a, 3x, 4b, 6a, b, 4a, 7x.

4. In the following write the coefficients of the terms - (i) 2x, +3x, 5x, (ii) +3a, +4a, 6a

5. Write the following products as a power.

   (i) x x x x x x x, (ii) x x x, (iii) a x a x a,

   (iv) b x b x ... 5 times, (v) y x y x ... 6 times

6. Write each as a product of the variable by itself

   (i) $x^4$, (ii) $y^3$, (iii) $z^6$, (iv) $a^5$, (v) $b^7$

7. Fill in the blanks

| Product | Power | Unknown (base) | Exponent (index) |
|---|---|---|---|
| 1. x x x x x x x | $x^4$ | - | - |
| 2. - | $a^3$ | - | - |
| 3. - | - | x | 3 |
| 4. y x y x y x y | | | |

8. Which ones are algebraic expression in the following ?

   10+2, 3x+4, $x^2$+1, x-2, $3^2$ - 1

9. Simplify: (a) 4x+3x, (b) 3a-2a, (c) 10y+14y

10. Simplify: (a) (a+2b)+(2a-b), (b) (6x+3y)-(2x+y)

11. Find the value of

    (a) $3m+4n$ when $m = 4$ and $n = 2$

    (b) $4p+3q-1$ when $p = q = 1$

    (c) $a^2+a+1$ when $a = 1$

    (d) $3x+2y+z$ when $x = 1$, $y = 2$, $z = 3$

12. Identify the monomial, binomial and trinomial expressions in the following - (a) $a^2 + b^2$, (b) $3ab$, (c) $a+b+c$, (d) $abc$, (e) $3xy+2yz+x$, (f) $2a^2+3b^3$

13. Fill in

    (a) A binomial expression has _______ terms.

    (b) A monomial expression has _______ terms.

    (c) An expression with three terms is a ____ expression.

14. Find the numerical coefficients in each expression, in the order they appear.

    (a) $3a+2b$, (b) $4a^2+2a+5$, (c) $a^2+2ab+3b^2$, (d) $2m+3mn+5n$

15. Identify the equations and the inequations in the following - (a) $4x > 3$, (b) $3x = 12$, (c) $2x < 9$, (d) $x \geq 1$, (e) $x+4 = 12$.

16. Fill in the blanks

| Sl. No. | Algebraic expression | The variables | Numerical coefficient in order | Name of the expression |
|---|---|---|---|---|
| 1 | $x+2$ | x | 1 | Binomial |
| 2 | $3a+2ab+b$ | a,b | 3,2,1 | Trinomial |
| 3 | $a^2+a+1$ | - | - | - |
| 4 | $m^2+n^2+mn$ | - | - | - |
| 5 | $3a+2b$ | - | - | - |
| 6 | $x+xy-y+y^2$ | - | - | - |
| 7 | $3xyz$ | - | - | - |

## 2. Introduction

In our daily life we come across quantities which keep on changing like, the time shown by the clock, the temperature in a day and during night, the prices of articles, the yields of a crop year by year, the population of a country and so on. Quantities which change are variables. They are also called unknowns. These are denoted by any of the letters as a, b, c, ..., x, y, z (these are also called literal numbers). A variable takes various values over some given set. For example, since the maximum temperature of water is 100$^{o}$C (that of boiling water), if we denote the temperature of water by x, x takes values over the set of positive numbers less than or equal to 100.

Suppose 15 kgs of rice and 2 kgs of dal are required every month by a family; the prices of rice and dal per kg keep on changing so that they are denoted by the variables x and y (in rupees) per kg. Then the amount required to be spent by the family each month is (15x+2y) rupees. The expression 15x+2y is an algebraic expression giving the amount to be spent by the family on the items every month. Since x and y are variables, 15x+2y also varies month by month.

The operations as addition/subtraction and multiplication of algebraic expressions are also meaningful to life. For example, if the monthly requirements of two families are 15 kg rice, 2 kg sugar and 10 kg rice and 3 kg sugar, the amount they spend are respectively (15x+2y) Rs.

and (10x+3y) Rs. Then the total amount spent by the two families is (25x+5y) Rs. (25x+5y) = (15x+27) + (10x+3y). We observe that in finding the sum, the like terms are added separately.

Bhaskara Acharya's book 'Leelavathi' ( ) contains many interesting problems in elementary mathematics involving algebraic equations and their solutions. Different aspects of algebraic expressions, equations and inequations are studied in higher classes and advanced courses in mathematics. Study of algebraic equations from geometrical view point has given rise to an important branch of mathematics called algebraic geometry or analytical geometry. All these have very important applications in different branches of science and technology.

**3. Concepts/Terminology/Notation/Results/Examples and Explanations**

**(a) Basic concepts, terms and notations**

(i) A quantity which keeps changing, taking values over a set of numbers is called an unknown or a variable.

(ii) Variables are denoted by alphabets a, b, c, x, y, z, etc.

(iii) Mathematical statements involving variables are algebraic expressions.

(iv) A variable x multiplied by itself once or more gives powers of x as $x^2$ $(= x \times x)$, $x^3 = (x \times x \times x)$, etc.

(v) An algebraic expression containing powers of a variable of the types $2x+3$, $4x^2+3x+2$, $x^3+2x^2+3x+1$ are called polynomials in x. The first one is a linear polynomial, the second is a quadratic polynomial, the third one is a cubic polynomial and so on.

(vi) When two (or more) algebraic expression containing two or more variables are added, the numerical coefficients of like terms must be added to form the sum. Likewise when an algebraic expression is subtracted from another, the numerical coefficients of the terms of the latter (the subtrahend) must be subtracted from the corresponding coefficients of the first one.

Multiplying algebraic expressions

(vii) When two (or more) algebraic expressions are multiplied, like terms are multiplied. In doing so, the numerical coefficients are multiplied first. This is followed by multiplication of unlike terms as well.

**These concepts are illustrated below**

(1) $x$, $x^2$, $2x$, $3x+2$ are algebraic expressions in x.

(2) $ab$, $a^2+b^2$, $3a+2b-4ab$ are algebraic expressions in a, b.

(3) $abc$, $ab+bc+ca$ are algebraic expressions in a, b and c.

(4) $x^4$, $x^2$, $x^3$ are powers of x with indices 4, 2 and 3 respectively.

(5) $2+3x$, $x+2$, $2x+1$, $3x+2$ are binomials in x.

(6) a, abc, ab are monomials in a; a, b, c and a, b respectively.

(7) $x^2+2x+2$, $4x^2+3x+5$ are quadratic polynomials in x.

(8) $a^2+ab+b^2$ is a quadratic polynomial in a, b.

(9) $3x+2$, $2x-1$ are linear polynomials in x.

(10) $a+b$, $3a+2b$, ... in a and b.

**Addition and Subtraction**

The method of adding two or more algebraic expressions is illustrated by the following examples.

(i) Given 2a, 5a, the sum is $(2+5)a = 7a$

**Caution**

$2a + 3a \neq (2+3)\ (a+a) = 5 \times 2a = 10a$

Likewise $x^2 + 2x^2 + 1 \neq (1+2+1)\ (x^2+x^2)$

**Rule**

Therefore while adding, add only the numerical coefficients of the variable terms in the expression.

(ii) Given a+2 and 3a+1, the sum is

$(a+3a) + (2+1) = 4a + 3$

(iii) Given $a^2+1$ and $3a^2+2$

$(a^2+1) + (3a^2+2) = (a^2+3a^2) + (1+2)$

$= 4a^2 + 3$

i.e. $a^2+1$

$+3a^2+2$

------

$=4a^2+3$

(iv) Given a and b, the sum is a+b.*

***Caution**

It cannot be further simplified, because a and b are two different variables and so two unlike terms.

**An analogy**

3 mangoes and 4 bananas together with 2 mangoes and 3 bananas make 5 mangoes and 7 bananas.

Likewise $(3m+4b) + (2m+3b)$

$= 3m+4b$

$+2m+3b$

------

$= 5m+7b$

However, 3 mangoes together with 4 bananas is just 3 mangoes and 4 bananas and not 7 mangoes or 7 bananas.

**Tip**

In order to add algebraic expressions write the like terms of the addends one below the other and then add.

In order to subtract an algebraic expression from another, write the terms of the subtracted below the corresponding (like) terms of the former.

(v) Subtract 7a in 10a.

$$\begin{array}{l} 10a \\ -7a \\ \hline (10-7)a = 3a \end{array}$$

$10a - 7a = 3a$

(vi) Subtract 2x+4 in 3x+7

$$\begin{array}{l} (3x+7) \\ -(2x+4) \\ \hline (3-2)x+(7-4) = x+3 \end{array}$$

$(3x+7) - (2x+4) = x+3$

(vii) Subtract $m^2 + n^2 + mn$ in $3m^2 - 2n^2 + 3mn$

$$\begin{array}{l} 3m^2 - 2n^2 + 3mn \\ -(m^2 + n^2 + mn) \\ \hline (3-1)m^2 + (-2-1)n^2 + (3-1)mn \\ = 2m^2 - 3m^2 + 2mn \end{array}$$

Observe that the coefficients of like terms are added after changing the signs of the coefficients of the subtractend

**Multiplication of Algebraic Expressions - Illustrations**

(i) Find the product of 3a and 5a.

$$3a \times 5a = 3 \times 5 \times a \times a = 15 \times a^2 = 15a^2$$

In this, while finding the product

**Step 1**: The numerical coefficients are multiplied.

**Step 2**: The variable factors are multiplied. If they are like variables, it is expressed in the exponential form (as a power of the variable).

**Step 3**: The product is written showing the numerical coefficient and variable factor.

If more than two expressions are multiplied, the same method is used.

(ii) Find the product of $5x$, $3x^2$, $-4x^3$

$5x \times 3x^2 = 15x^3$

$5x \times 3x^2 \times -4x^3 = 15x^3 \times -4x^3$

$= -60x^6$

(iii) Multiply $5x$ and $4y$

$5x \times 4y = 5 \times 4 \times x \times y = 20 \times xy = 20xy$

Note: Hence when the variable factors are unlike, they are written as mere product because it cannot be simplified any further.

i.e. $x \times y$ is $xy$

(iv) Simplify: $ab \times bc \times ca$

Here we need to find the product

$ab \times bc = ab^2c = b^2ac$

$ab \times bc \times ca = b^2ac \times ca$

$= b^2aac^2 = b^2a^2c^2$

$ab \times bc \times ca = a^2b^2c^2$

Note: The order of the factors can be changed while writing the product, because for any two variables a,b, ab=ba.

(v) Find $\frac{a}{b} \times \frac{b}{c} \times \frac{c}{a}$

$$\frac{a}{b} \times \frac{b}{c} = \frac{ab}{bc} = \frac{a}{c}$$

$$\frac{a}{b} \times \frac{b}{c} \times \frac{c}{a} = \frac{a}{c} \times \frac{c}{a} = 1$$

Hence $\frac{a}{b} \times \frac{b}{c} \times \frac{c}{a} = 1$

**Caution**

In this a, b, c can take any non-zero value.

**Properties of multiplication**

(a) a x b = b x a (Commutative Property)

(b) (a x b) x c = a x (b x c) (Associative Property)

(c) a x (b + c) = (a x b) + (a x c) (Distributive Property)

for any a, b, c. In (b) a x b x c denotes either of LHS and RHS.

From properties (a) and (b).

a x b x c = b x c x a = c x a x b for any a, b, c

(d) (+a) x (+b) = +(ab); (+a) x (-b) = (-a) x (+b) = -(ab); (-a) x (-b) = +(ab)

| x | +b | -b |
|---|---|---|
| +a | +ab | -ab |
| -a | -ab | +ab |

The multiplication table for all a, b.

eg: If a = 2, b = 3, c = 4,

then (i) a x b = b x a

because 2 x 3 = 3 x 2

(ii) (axb)xc = ax(bxc)

because (2x3)x4 = 2x(3x4)

=> 6x4 = 2x12

=> 24 = 24

and (ii) ax(b+c) = axb + axc

because 2x(3+4) = 2x3 + 2x4

=> 2x7 = 6+8

=> 14 = 14

**4. Teaching Strategies - Activities**

(a) Through teaching aids - Demonstrations

(i) The idea to develope (1) Addition means putting together

(2) Subtraction means taking away

Adding a set of blue marbles (say) to another set of blue marbles.

Here the objects added are alike.

Accordingly, the total number of marbles in the two sets put together is the sum of the number of blue marbles.

(2) If a set has some blue and some red marbles and another has some blue and some red marbles then the two sets put together contains blue and red marbles. How many ?

The total number of blue marbles

and the total number of red marbles

Hence you cannot express the sum of blue and red marbles since these are unlike objects.

Thus 4 blue marbles + 7 red marbles is neither 11 blue marbles nor 11 red marbles.

It is just (4 blue + 7 red) marbles.

More generally, a + b = a + b only.

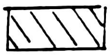
**(3) Multiplication Properties**

(i) a x b = b x a

In the figure,

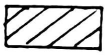
3x4 = Number of

of square shaded

as  = 12

4x3 = Number of

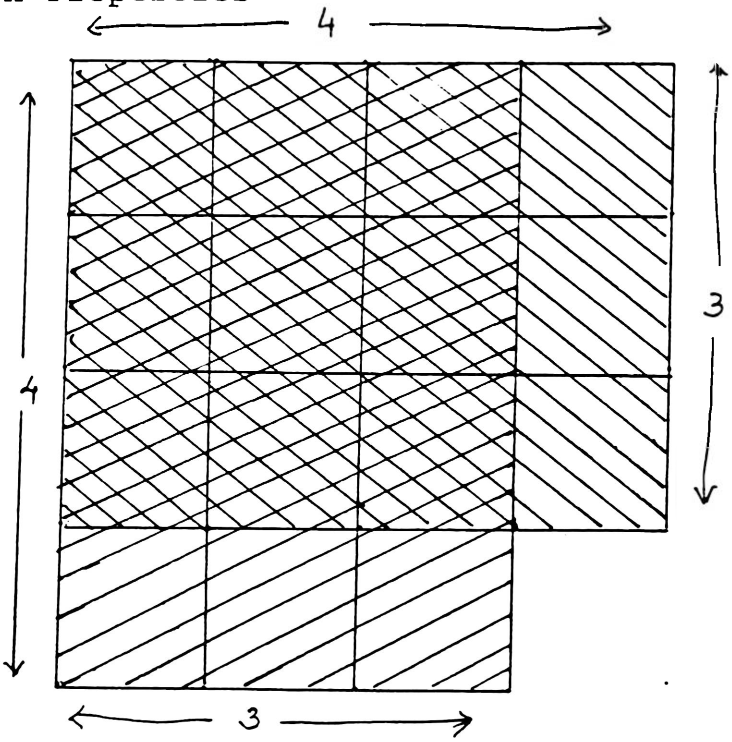
squares shaded

as  = 12

The two are the

same (i.e.

3x4 = 4x3)




Instead take a and b units and you can see a x b = b x a.

Note: (i) a x b = The area of a rectangle of length a and breadth b.

(ii) a x b x c = The volume of a rectangular box of length a, breadth b and height c.

(2) (axb)xc = ax(bxc)

(axb)xc = (Area of the face with sides a and b) x c

= (Area of the base with sides a and b) x height c

= Volume of the box (i) .......(i)

ax(bxc) = a x Area of the face with sides b and c

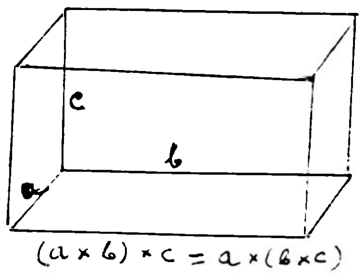
= height a x (area of the base with sides b and c)

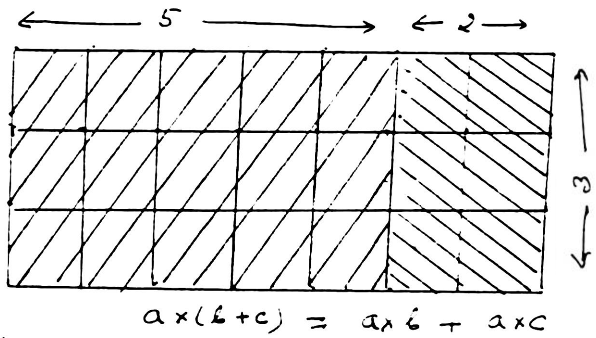
= Volume of the box .......(ii)

Since (i) and (ii) are both volumes of the box,

$$(a \times b) \times c = a \times (b \times c)$$

(3) $a \times (b+c) = a \times b + a \times c$




Total area = 3 x (5+2) = 21

Also total area = 3x5 + 3x2 = 15 + 6 = 21

Then 3 x (5+2) = 3x5 + 3x2

Instead the dimensions may be taken as a, b, c, when we see ax(b+c) = axb + axc.

**Problem Solving**

(1) Add: (a) 10a, 5a, (b) 6a, 3a, 2a

**Solution:** (a)

```
 10a
+5a
--------------
(10+5)a = 15a
```

(b)

```
  6a
 +3a
 +2a
---------------
(6+3+2)a = 11a
```

(2) Add: (a) (2a-3b), (4a-6b) (b) x+y+z, 2x-3y+4z

**Solution:**

(a)

```
  2a - 3b
 +4a - 6b
-----------------
(2+4)a + (-3-6)b
= 6a - 9b
```

(b)

```
  x + y + z
 2x - 3y + 4z
-------------
 3x - 2y + 5z
```

(Adding like terms separately)

(3) Add: (a) a + 2b, 2a - b, 3a + 4b

(b) x + y, y + z, x + z

**Solution**

(a)

```
   a + 2b
 +2a - b
 +3a + 4b
--------
  6a + 5b
```

(b)

```
   x + y
 +     y + z
 + x     + z
--------------
 = 2x + 2y + 2z
```

(4) Add: $2ax + 3by - 4cz$, $5ax + 8by - 10cz$

**Solution:**

$$\begin{array}{l} \quad 2ax + 3by - 4cz \\ +(5ax + 8by - 10cz) \\ \hline = 7ax + 11by - 14cz \end{array}$$

(5) A person bought 5 kg, 3 kg, 1 kg of rice, dal and sugar in one shop and 3 kg, 2 kg, 2 kg of the same articles in another shop. How much rice, dal, sugar has he bought in all ?

**Solution:** Denoting R, D, S by Rice, Dal, Sugar respectively

$$\begin{array}{ll} \text{In 1st shop} & 5R + 3D + 1S \\ \text{In 2nd shop} & 3R + 2D + 2S \\ \hline \text{Totally} & 8R + 5D + 3S \end{array}$$

In total he has purchased 8 kg of rice, 6 kg of dal and 3 kg of sugar.

(6) Subtract

(a) $13x$ from $20x$ (b) $5xy$ from $8xy$

**Solution:** (a)

$$\begin{array}{l} \quad 20x \\ -13x \\ \hline (20-13)x \\ = 7x \end{array}$$

(b)

$$\begin{array}{l} \quad 8xy \\ -5xy \\ \hline \quad 3xy \end{array}$$

(7) Subtract: (a) $2a - 4b$ from $3b - 6a$

(b) $a^2 + 3b$ from $4b - 2a^2$

**Solution:** (a)

$$\begin{array}{l} \qquad 3b - 6a \\ -(-4b + 2a) \\ \hline (3-(-4))b + (-6-2)a = 7b - 8a \end{array}$$

(b)

$$\begin{array}{l} \quad 4b-2a^2 \\ -(3b+a^2) \\ \hline (4-3)b + (-2-1)a^2 = b - 3a^2 \end{array}$$

(8) Subtract: (a) $3x^2 + 5x - 10$ from $5x^2 + 4x + 2$

(b) $x^2 - xy + y^2$ from $3x^2 + 2xy - 2y^2$

Solution: (a) $5x^2 + 4x + 2$

$-(3x^2 + 5x - 10)$

--------------------------------

$(5x^2 - 3x^2) + (4x - 5x) + (2 + 10)$

$= 2x^2 + (-x) + 12 = 2x^2 - x + 12$

(b) $3x^2 + 2xy - 2y^2$

$-(x^2 - xy + y^2)$

---------------------------

$= (3x^2-x^2) + (2xy+xy) + (-2y^2-y^2)$

$= 2x^2 + 3xy - 3y^2$

Note: The coefficients of the subtractend change the sign.

(9) State the properties in the following statements.

(a) $3x + 4y = 4y + 3x$

(b) $(2x+3y)+z = 2x+(3y+z)$

(c) $3x(4y+z) = (3x)(4y) + 3xz$

Solution

(a) It resembles $a+b = b+a$

Hence it is Commutative property with respect to addition.

(b) This resembles $(a+b)+c = a+(b+c)$

Hence it is Associative property with respect to addition.

(c) This property is Distributive property since it resembles $a(b+c) = ab + ac$.

(10) Multiply

(a) $3x$, $6x$ (b) $x$, $2x^2$, $3x^3$

(c) $3a$, $4b$, $6c$ (d) $4pq$, $6qr$, $pr$

**Solution**

(a) $3x \times 6x = (3 \times 6) \times (x \times x) = 18x^2$

(b) $x \times 2x^2 \times 3x^3 = (1x2x3)(x \times x^2 \times x^3) = 6x^6$

(c) $3a \times 4b \times 6c = (3x4x6) \times (abc) = 72abc$

(d) $(4pq) \times (6qr) \times (pr) = (4x6)(pq)(qr)(rp)$

$= 24\ (pxp)(qxq)(rxr)$

$= 24p^2q^2r^2$

(11) Find the product of

(a) $2a$ and $(a+3b)$

(b) $(a+2b)$ and $(2a-b)$

**Solution**

(a) $2a \times (a + 3b)$

$= 2axa + (2a) \times (3b)$

$= 2a^2 + 6ab$

(b) $(a+2b) \times (2a-b)$

$= (a+2b)x2a + (a+2b)(-b)$

$= ax(2a) + 2bx(2a) + ax(-b) + (2b) \times (-b)$

$= 2a^2 + 4ab - ab - 2b^2$

$= 2a^2 + 3ab - 2b^2$

(12) Using the terms

(a) $2a$ and $3b$, state the Commutative property with respect to multiplication

(b) $2a$, $3b$ and $4c$, state the Associative property with respect to multiplication

(c) $2a$, $3b$ and $4c$, state the Distributive property

**Solution**

(a) $(2a) \times (3b) = (3b) \times (2a)$

(b) $2a\times(3b\times4c) = (2a\times3b)\times4c$

(c) $2a\times(3b+4c) = (2a) \times (3b) + (2a) \times (4c)$

(13)

(a) Find the area of a rectangle whose length and breadth are $(2a+3b)$ and $(a+2b)$.

(b) Find the volume of rectangle box whose length, breadth and height are $x+y$, $y+z$ and $x+z$ respectively.

**Solution**

(a) The area of the rectangle = Length x Breadth

$= (2a+3b) \times (a+2b)$

$= (2a)a + (2a)(2b) + (3b)(a) + (3b)(2b)$

$= 2a^2 + 4ab + 3ab + 6b^2$

$= 2a^2 + 7ab + 6b^2$

(b) The volume of the box = Length x Breadth x Height

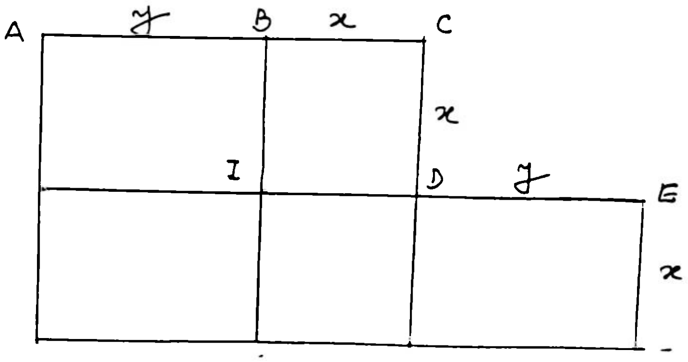
$= (x+y)\ (y+z)\ (x+z)$

$= (xy + xz + y^2 + yz)\ (x + z)$

$= xy^2 + xyz + x^2z + xz^2 + xy^2 + y^2z + xyz + yz^2$

$= xy^2 + x^2y + yz^2 + y^2z + xz^2 + x^2z + 2xyz$

(14) Find the area of the figure shown below :



Solution: The total area of the figure

= Area ABGH

+ Area BCDI

+ Area IEFG

= ABxBG + BCxCD + IExEF

= y x 2x + x x x + (x+y) x x

= $2xy + x^2 + x^2 + xy$

= $2x^2 + 3xy$

(15) Find the volume of the rectangular box of dimensions x + y, y + 2z, 2x + z.

Volume = Length x Breadth x Height

= (x+y) (y+2z) (2x+z)

= $(xy+2xz+y^2+2yz)(2x+z)$

= $2xyz + 4x^2z + 2xy^2 + 4xyz + xyz + 2xz^2 + y^2z + 2yz^2$

= $7xyz + 2xy^2 + 2yz^2 + y^2z + 2xz^2 + 4x^2z$

(5) **Evaluation and Exercises**

UNIT TEST

| | | |
|---|---|---|
| Questions: 1 to 6 --> 1 mk each | | Time : 45 m |
| 7 to 10 --> 2 mks each | | Marks: 25 |
| 11 & 12 --> 3 mks each | | |
| 13 --> 5 mks | | |

**Answer all questions**

1. Write down the numerical coefficients of x, y, z in 4x - 7y + 3z.

2. Which one is a binomial ?

   (a) ab (b) a+b (c) $a^2b$ (d) 3ab

3. $14xy + 3xy = ?$

4. Subtract: $6a^2 + 5$ from $10a^2 + 6$

5. Verify Associative property with respect to + using any three numbers of your choice.

6. Multiply: $2x$, $-3y$ and $-4z$

7. Add: $5a^2 + 3a + 4$ and $2a^2 - a - 3$

8. What should be added to $2a+3b$ to get $3a+2b$ ?

9. Subtract $4xy - 3x^2$ from $x^2 - 2xy$

10. Multiply: $x$, $(x+1)$ and $(x+2)$ and express it in terms of

    $x^3$, $x^2$, $x$ and a constant.

11. Expand $(a+b)^3$ step by step, stating the property used in each step.

12. Apple, orange and grapes cost x, y, z (in Rs) per kg. Find the expression for the amount spent to buy 4 kg, 3 kg and 2 kg of these fruits respectively. If $x = 30$, $y = 15$, $z = 40$, find the amount spent in rupees.

13. Fill in the blank in the following multiplication table.

| x | 2a | 3b | 4c | Row total |
|---|---|---|---|---|
| a | | | | |
| 2b | | | | |
| 3c | | | | |
| Column Total -----> | | | | Grand total = |

What does the grand total denote ?

**Exercises**

1. Add

   (i) $6a$, $5a$, $10a$

   (ii) $a^2+1$, $2a^2-1$, $2-a^2$

   (iii) $2a+3b$, $3a-b$, $a+b$

   (iv) $a^2+ab-b^2$, $2ab+2b^2-3a^2$

   (v) $7a-3b+c$, $2a+b-4c$

2. Subtract

(i) 15a from 17a

(ii) 2a-3b from a+4b

(iii) $m^2+mn-n^2$ from $2m^2+3mn+n^2$

(iv) xyz+x+y+z from 2x+3y-z-xyz

(v) ab+2bc-3ca from bc+2ca-4ab

3. Fill in

| | ① | + ② | = ③ Sum |
|---|---|---|---|
| 1 | 2a+b | 3b-2a | ______ |
| 2 | $x^2+y^2$ | ______ | $2x^2+z^2$ |
| 3 | $x^2y+xy^2$ | $4xy^2-X^2y$ | ______ |
| 4 | ______ | $m^2+2m-1$ | $3m^2-4m+5$ |

4. Find the product of

(i) 2x, $3x^2$, $4x^3$ (ii) a, 2a+1

(iii) a+2b, 2a-b (iv) $a^2$, a+1

(v) (x+2y), (2x+y) (vi) (a+b), (a+b+c)

5. Multiply

(i) 3x and 4y (ii) $3a^2$ and $2a^2$

(iii) ax and bx (iv) m, mn, n

(v) a, a+b, b (vi) a+b, a-b

6. Verify

(a) (a+b)+c = a+(b+c)

(b) (axb)xc = ax(bxc)

(c) ax(b+c) = axb + axc

by taking numerical values of your choice for a, b, c.

7. Fill in

(a)

| x | +b | -b |
|---|---|---|
| +a | | |
| -a | | |

(b)

| x | +1 | -1 |
|---|---|---|
| +1 | | |
| -1 | | |

8. Find the coefficients of $x^2$, x and the constant term in the product of (2x+1) and (x-3).

9. Find the coefficients of $x^3$, $x^2$, x and the constant term of (i) x, (x+1), (x+2)

   (ii) $(x^2+1)$, (x+1)

10. Show that (i) $(a+b)(a-b) = a^2-b^2$

    (ii) $(a+b)^2 = a^2+2ab+b^2$

    (iii) $(a-b)^2 = a^2-2ab+b^2$

    and (iv) $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$

(6) Suggested Activities for (a) Self learning and (b) Enrichment

(i) Prepare charts to explain (a) addition, (b) subtraction and (c) multiplication of algebraic expressions

(ii) Prepare a unit test on the unit for 45 mins with a maximum 25 marks.

# CHAPTER X

# PRODUCT OF ALGEBRAIC EXPRESSIONS

## Preview

In chapter IX, algebraic expressions and operations as addition, subtraction and multiplication have been discussed. The chapter IX itself presupposes basic ideas as variables and the associated ideas.

A variable is a changing quantity. An expression involving one or more variables besides the signs +, - and x is an algebraic expression. While adding/subtracting algebraic expressions, like terms are grouped and operations are performed. Multiplication of algebraic expressions have been discussed in detail in the previous chapter.

## Introduction

To solve many problems arising in daily life, we need to formulate algebraic expressions and deal with them keeping the operations suggested by the problems. Some standard products keep coming up in problems. In the chapter these are studied in the form of certain identities. While multiplying two or more expressions results in a single expression called the product, the original expressions are called factors of the single expression, namely the product. The process of expressing an algebraic expression as a product of two or more expressions is factorisation. In the chapter, we learn some techniques of factorisation.

Concepts/Terminology/Techniques

(a) Product of two binomials of the type (x+a) and (x+b).

= (x+a)(a+b)

(b) The identities (i) $(a+b)^2 = a^2 + 2ab + b^2$

(ii) $(a-b)^2 = a^2 - 2ab + b^2$

(iii) $a^2 - b^2 = (a+b)(a-b)$

(c) Techniques of factorisation

(i) splitting a term and grouping

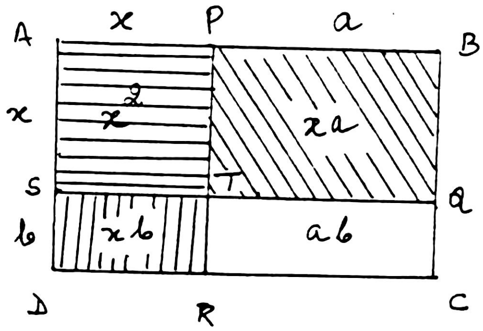
(ii) use of identities

(iii) factorisation of trinomials of the type $ax^2 + bx + c$

**Teaching Strategies**

**(a) Demonstration by charts (geometrical proofs of identities)**

(i)




Area of rect. ABCD = (x+a)(x+b) .......(1)

Area of ABCD = Area APTS + Area PBQT + Area STRD + Area TQCR

Area ABCD $= x^2 + xa + xb + ab$ .......(2)

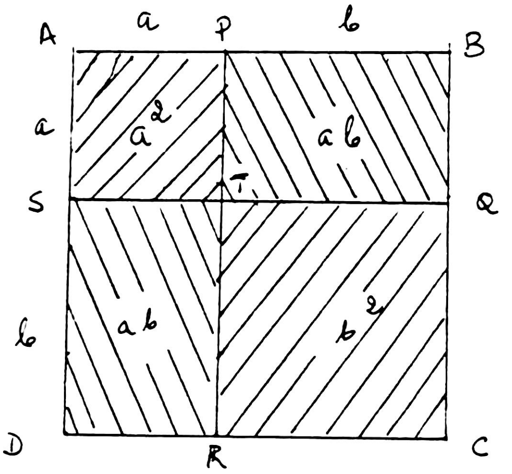
(1) = (2)

=> $(x+a)(x+b) = x^2 + xa + xb + ab$

or $\boxed{(x+a)(x+b) = x^2 + (a+b)x + ab}$

This formula is useful in some factorisation problems.

(ii)




Area ABCD = (a+b)(a+b) = $(a+b)^2$ ........(1)

Area ABCD = Area APTS + Area PBQT +

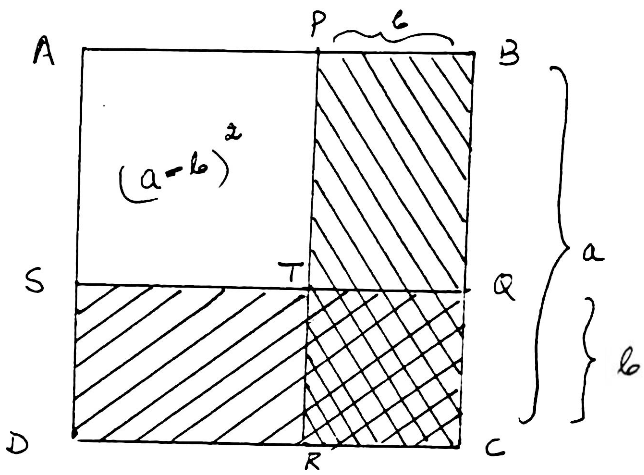
Area STRD + Area TQCR

Area ABCD = $a^2 + ab + ab + b^2$

= $a^2 + 2ab + b^2$ ........(2)

(1) = (2) => $\boxed{(a+b)^2 = a^2 + 2ab + b^2}$

(iii)




Area ABCD = Area ABQS + Area PBCR + Area STRD - Area PBRT

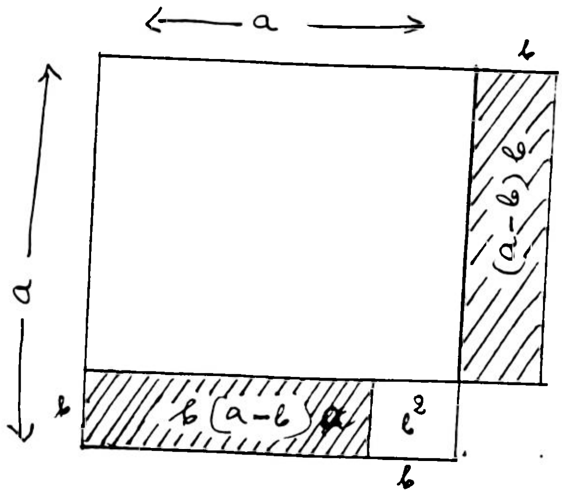
=> $a^2 = ab + ab + (a-b)^2 - b^2$

= $2ab + (a-b)^2 - b^2$

=> $a^2 - 2ab + b^2 = (a-b)^2$

or $\boxed{(a-b)^2 = a^2 - 2ab + b^2}$

(iv)



In the figure ABCD is a square of side AB = a

CSQR is a square of side QR = b

Hence Area ADSQRB = Area ABCD - Area CSQR

$= a^2 - b^2$ ......(i)

Area PQSD = PQ x PD = (a-b)b

Area BMNR = BR x BM = (a-b)b

Area PQSD = Area BMNR ......(ii)

Hence Area ADSQRB = Area ABRP + Area PQSD

= Area ABRP + Area BMNR

= Area AMNP

= AM x AP = (a+b)(a-b)

Area ADSQRB = (a+b)(a-b) .....(iii)

From (i) and (iii), $a^2 - b^2 = (a+b)(a-b)$

This proof is the basis of the following demonstration to show the identity $a^2 - b^2 = (a+b)(a-b)$.

(1) Take a square shaped card board of side a.

(2) Cut out a smaller square of side b from a corner what remains now is of area $(a^2 - b^2)$ .....(i)

(3) Remove a rectangle from this piece so that the remaining part is a rectangle (of sides a and (a-b)).

(4) Attach the removed piece to the remaining rectangular part so that it becomes a bigger rectangle. This rectangle has the side (a+b) and (a-b) so that in area is (a+b)(a-b). .....(ii)

(i) = (ii). Hence the identity is demonstrated.

**(b) Problem Solving**

(1) Show that $\frac{1}{2}[(a+b)^2 + (a-b)^2] = (a^2 + b^2)$

**Solution:** $(a+b)^2 = a^2 + b^2 + 2ab$

and $(a-b)^2 = a^2 + b^2 - 2ab$

---

Adding $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$

$$\frac{1}{2}[(a+b)^2 + (a-b)^2] = a^2 + b^2$$

(2) Sum of two numbers is 7 and difference is 1. Find the sum of the squares of the numbers.

**Solution:** Let the numbers be a and b.

Then $a + b = 7$

and $a - b = 1$

$$a^2 + b^2 = \frac{1}{2}[(a+b)^2 + (a-b)^2]$$

$$= \frac{1}{2}[7^2 + 1^2] = \frac{1}{2}(49+1) = \frac{50}{2} = 25$$

$a^2 + b^2 = 25$

(3) Expand (a) $(x+4)^2$ (b) $(a+2b)^2$ (c) $(2a-3b)^2$ (d) $(3a+2b)^2$

**Solution:**

(a) $(x+4)^2 = x^2 + 2 \times x \times 4 + 4^2 = x^2 + 8x + 16$

(b) $(a+2b)^2 = a^2 + 2a \times 2b + (2b)^2 = a^2 + 4ab + 4b^2$

(c) $(2a-3b)^2 = (2a)^2 - 2(2a)(3b) + (3b)^2 = 4a^2 - 12ab + 9b^2$

(d) $(3a+2b)^2 = (3a)^2 + 2(3a)(2b) + (2b)^2 = 9a^2 + 12ab + 4b^2$

(4) Find the value of

(a) $101^2$ (b) $98^2$ (c) $(10.1)^2$ (d) $9.8^2$

using the appropriate identities.

**Solution**

(a) $101 = 100 + 1$

$101^2 = (100+1)^2 = 100^2 + 2x100x1 + 1 = 10000 + 200 + 1$

$= 10201$

(b) $98 = 100 - 2$

$98^2 = (100-2)^2 = 100^2 - 2x100x2 + 2^2$

$= 10000 - 400 + 2$

$= 9602$

(c) $10.1^2 = (10+.1)^2 = 10^2 + 2x10x(0.1) + (0.1)^2$

$= 100 + 0.2 + 0.01 = 100.21$

(d) $9.8^2 = (10-0.2)^2 = 10^2 - 2x10x(0.2) + (0.2)^2$

$= 100 - 0.4 + 0.04 = 99.64$

(5) Expand (a) $(a+b+c)^2$ (b) $(a+b-c)^2$

**Solution**

(a) $(a+b+c)^2 = a^2 + 2a(b+c) + (b+c)^2$ Considering

$= a^2 + 2ab + 2ac + b^2 + 2bc + c^2$ b+c as a

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$ single term

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc - 2ca$

(B) $(a+b-c)^2 = a^2 + 2a(b-c) + (b-c)^2$ Considering

$= a^2 + 2ab - 2ac + b^2 - 2bc + c^0$ b-c as a

$= a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$ single term

$(a+b-c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$

(6) Show that $(a+b+c)(a+b-c) = a^2 + b^2 - c^2 + 2ab$

Solution

$(a+b+c)(a+b-c)$

$= (a+b)^2 - c^2$

$= a^2 + 2ab + b^2 - c^2$

$= a^2 + b^2 - c^2 + 2ab$

Considering a+b as a single term and using $(a+b)(a-b) = a^2 - b^2$

(7) Prove that $(x-y)(x+y)(x^2+y^2) = x^4 - y^4$

Solution: $(x-y)(x+y) = x^2 - y^2$

Multiplying both sides by $(x^2 + y^2)$

$(x-y)(x+y)(x^2+y^2) = (x^2-y^2)(x^2+y^2)$

$= (x^2)^2 - (y^2)^2$

$= x^4 - y^4$

$(a^2)^2 = a^{2x2} = a^4$

(8) Using the formula $(a+b)(a-b) = a^2 - b^2$, find the value of 1.01 x 0.99.

Solution: $(1.01 \times 0.99) = (1 + 0.01)(1 - 0.01)$

$= 1^2 - (0.01)^2 = 1 - 0.0001 = 0.9999$

(9) Using $(x+a)(x+b) = x^2 + (a+b)x + ab$, factorise

(a) $x^2 + 5x + 4$ (b) $x^2 + 10x + 21$ (c) $4x^2 + 8x + 3$

(d) $x^2 - 7x + 10$

Solution

(a) $x^2 + 5x + 4$

$= (x^2 + x) + (4x + 4)$

$= x(x+1) + 4(x+1)$

$= (x+1)(x+4)$

(b) $x^2 + 10x + 21$

$= x^2 + 3x + 7x + 21$

$= x(x+3) + 7(x+3)$

$= (x+3)(x+7)$

Note: In all such problems, the coefficient of x and the constant term, must be put in the form (a+b) and (axb) respectively. Then the formula $(x+a)(x+b) = x^2+(a+b)x + ab$ is used.

**Tip**: When the coefficient of $x^2$ is 1 and the constant term is a positive number express the coefficient of x as the sum of two numbers whose product is the constant term. If this is not possible, factorisation cannot be done.

(c) $4x^2 + 8x + 3$

$= 4x^2 + 2x + 6x + 3$

$= 2x(2x+1) + 3(2x+1)$

$= (2x+1)(2x+3)$

Write the coefficient of x $= 8 = 2 + 6$ since $2\times 6 = 4\times 3 =$ coefficient of $x^2$ x constant term

**Tip**: When the coefficient of $x^2$ is not equal to 1, express the coefficient of x as the sum of two numbers such that their product is equal to the product of the coefficient of $x^2$ and the constant term.

(d) $x^2 - 7x + 10 = x^2 - 2x - 5x + 10$

$= x(x-2) - 5(x-2)$

$= (x-2)(x-5)$

(10) Factorise (a) $x^2 + 5x - 24$ (b) $x^2 - 4x - 21$

(c) $2x^2 + 3x - 2$ (d) $12x^2 - 7x - 12$

(e) $3a^2 - 7ab + 4b^2$ (f) $8x^2 - 2xy - 15y^2$

**Solution**

(a) $x^2 + 5x - 24$

$= x^2 + 8x - 3x - 24$

$= x(x+8) - 3(x+8)$

$= (x+8)(x-3)$

Hence the constant term = -24 (negative number). Express the coefficient of x as the difference of two numbers whose product is 24. i.e. $5x = 8x - 3x$

(b) $x^2 - 4x - 21$
$= x^2 - 7x + 3x - 21$
$= x(x-7) + 3(x-7)$
$= (x-7)(x+3)$

Tip: When the constant term is -ve Express the coefficient of x as the difference of two numbers whose product is the numerical value of the constant form.

(c) $2x^2 + 3x - 2$
$= 2x^2 + 4x - x - 2$
$= 2x(x+2) - 1(x+2)$
$= (x+2)(2x-1)$

Here express the coefficient of x as the difference of two numbers. Whose product is equal to the product of the coefficient of $x^2$ and the numerical value of the coefficient.

(d) $12x^2 - 7x - 12$
$= 12x^2-16x+9x-12$
$= 4x(3x-4) + 3(3x-4)$
$= (3x-4)(4x+3)$

Here $-7 = -16+9$
and $16x9 = 155 = 12x12$

(e) $3a^2 - 7ab + 4b^2$
$= 3a^2 - 3ab - 4ab + 4b^2$
$= 3a(a-b) - 4b(a-b)$
$= (a-b)(3a-4b)$

(f) $8x^2 - 2xy - 15y^2$
$= 8x^2 - 12xy + 10xy - 15y^2$
$= 4x(2x-3y) + 5y(2x-3y)$
$= (2x-3y)(4x+5y)$

## (5) Evaluation and Exercises

A unit test on algebra - Product of algebraic expressions and factorisation

Questions: 1 to 6 --> 1 mk each
7 to 10 --> 2 mks each
11 & 12 --> 3 mks each
13 --> 5 mks

**Answer all questions**

1. Simplify $x(x+2)$
2. Simplify $(x+1)(x-2)$
3. Simplify $(a+2b)^2$
4. Expand $(3a-2b)^2$
5. Factorise $4a^2 + 6a$
6. Express $9a^2 - 4a^2$ as the product of sum and difference of two monomial terms
7. Find the numerical coefficient of $x^2$ in $x(x+1)(x+2)$
8. Without direct multiplication find the value of $(1000)^2 - (999)^2$
9. Simplify: $\left(x + \frac{1}{x}\right)^2 - \left(x - \frac{1}{x}\right)^2$
10. Simplify: $(a-b)(a+b)(a^2+b^2)(a^4+b^4)$
11. Find all the linear factors of
    (a) $(x^2 + 8x + 7)$
    (b) $(2x^2 + x - 6)$
12. Find the volume of a rectangular box whose dimensions (i.e. length/breadth/height) are $(2a+b)$, $(2a-b)$ and $(4a^2+b^2)$.
13. The area of a rectangular field is $6x^2+x-6$. Find its perimeter, given that in length and breadth are each of the form $ax+b$.

## Exercises

1. Multiply

   (a) $(a+1)(2a-3)$ (b) $(2a+3)(3a-2)$ (c) $(x^2-1)(3x^2-2)$

2. Expand: (a) $(2a+3b)^2$, (b) $(x^2+\frac{1}{x})^2$, (c) $(2x-\frac{3}{x}\;{}^2$

   (d) $(a^2-\frac{1}{a^2})^2$

3. Expand: (a) $(x^2+x+1)^2$ (b) $(x^2-x+1)^2$

4. Show that: (a) $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

   (b) $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$.

5. Find the value of (a) $(9.9)^2$ (b) $(10.01)^2$

   (c) 10,000 (d) $100^2 - 98^2$, using appropriate identities

6. The sides of a cuboid are of lengths $x+y$, $x+y$ and $x-y$. Find the areas of the faces and the volume.

7. Expand: (a) $(a^2+b^2)^2$ (b) $(x^2+x+2)(x+1)$

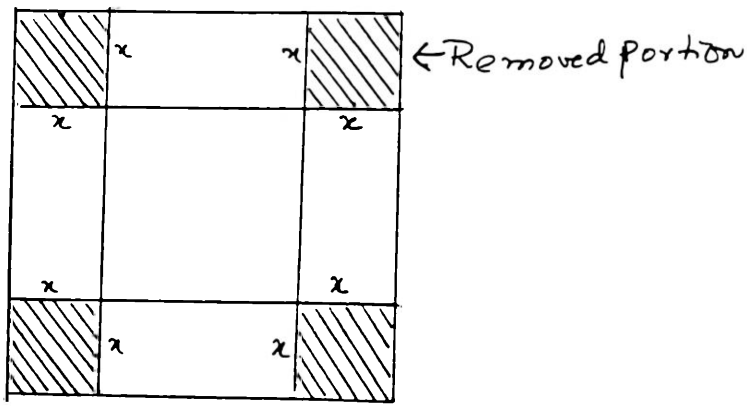
8. If $x + \frac{1}{x} = \frac{5}{2}$, find $x^2 + \frac{1}{x^2}$ and $x^2 - \frac{1}{x^2}$

9. Factorise: (a) $a^2 + 7a + 6$ (b) $m^2 + 8mn + 15n^2$

   (c) $4x^2 - 8xy + 3y^2$ (d) $105x^2 - 103x + 2$

   (e) $5x^2y^2 - 8xy - 4$ (f) $9x^3 - 6x^2 - 3x$

10. A square sheet of metal with side length 30 cm is converted into a rectangle box, by removing equal squares of side x cm from each corner and then folding up the flaps. Find the volume of the box as a polynomial in x.



Suggested Activities for (a) self learning, (b) enrichment

(1) Construct teaching aids to demonstrate the validity of the formulas (a) for $(a+b)^2$, $(a-b)^2$ and $a^2-b^2$.

(b) $(a+b+c)^2$, (c) $(a+b)^3$

[Hints: Use charts or card boards or wooden rectangular blocks]

(2) Expand $(a+b)^4$ [Use the formula for $(a+b)^2$]

(3) Given $a+b+c = 6$, $ab+bc+ca = 11$ and $abc = 6$ express $(x+a)(x+b)(x+c)$ as a polynomial in x with numerical coefficients.

# CHAPTER XI

# EQUATIONS

**Preview**

In the V standard you have studied about

(a) Number facts like 4+1 = 5 or 2+6 = 8

(b) Equations in one variable like x+3 = 8, 5 = 8-x, 3y = 6, $\frac{y}{3} = 6$ and $\frac{3}{y} = 6$.

(c) Left hand side (L.H.S.) and Right hand side (R.H.S.) of an equation.

(d) Explanation of an equation and its solutions with the help of a balance with two pans.

**Review Exercises**

(a) Identify L.H.S. and R.H.S. of the following equation (i) 9 = 8+x, (ii) 3y = 10, (iii) 7 = z-10

(b) Solve the following equations: (i) y+2=10, (ii) 10=8+x, (iii) 3 = z - 10, (iv) -5 + x = -10

(c) Solve the following equations: (i) 4x = 10, (ii) 7 = 3x, (iii) -5x = 10, (iv) -4 - 12 = -4x, (v) -4x - 12 = 10

(d) find the value of the variable 'y': (a) $\frac{3}{y} = 7$, (b) $-5 = -\frac{7}{y}$

(c) $-5 = \frac{-10}{-y}$ (d) $\frac{-7}{y} = 10$

**Answer to review questions**

(a) (i) 1 (ii) $\frac{10}{3}$ (iii) 17

(b) (i) 17 (ii) 2 (iii) 13 (iv) -5

(c) (i) $\frac{5}{2}$ (ii) $\frac{7}{3}$ (iii) -2 (iv) 3 (v) -3

(d) (i) $\frac{3}{7}$ (ii) $\frac{7}{5}$ (iii) -2 (iv) $-\frac{7}{10}$

## Introduction

What we will study in this chapter is called linear equations in one or two variables. They are useful to us because they help us in

(i) solving every day life problems, and

(ii) studying Chapter XV (Coordinate Geometry) on linear equations, their graphs and solving simultaneous equations using graphs. In coordinate geometry the graph of a linear equation is a straight line.

## Concepts and Notations

(a) An equation is a mathematical statement showing the equality of two quantities or two expressions. For example, $x-10 = 2$ is an equation showing the equality of the expression $x-10$ and the quantity 2. Similarly the equation $x+3 = 2x$ is an equation showing the equality of two expressions $x+3$ an $2x$.

(b) The symbol "=" means "is equal to" i.e. is a short from of the phrase "is equal to". For example, $5 = 2+3$ means that 5 is equal to 2+3. '$a = b$' means that '$a$ is equal to $b$'.

(c) L.H.S. and R.H.S.: These are short forms for the phrases 'left hand side' and 'right hand side' respectively of an equation. For example, in the equation $5x+2 = 4$, $5x+2$ is the left hand side (i.e. L.H.S.) and 4 is the right hand side (i.e. R.H.S.)

(d) The phrase 'solve' means 'find the value of the variable if'. For example, 'solve $5x+2 = 6$' means 'find the value of the variable $x$ if $5x+2 = 6$'. This also means that the value of $x$ when substituted for $x$ ~~means~~ makes L.H.S. = R.H.S.

(e) Simultaneous Equations means a set of two or more equations containing more than one variable for which we have to find common values for the variables (unknowns). For example, to solve the simultaneous equations $x+y = 10$, $x-y = 6$ we have to find common values of $x$ and $y$ so that these two equations may be true.

(f) Transposing means bringing a term of an equation (i) from L.H.S. to R.H.S. and (ii) from R.H.S. to L.H.S. according to certain rules. For example, $2x+3 = 4x$ means the same $2x = 4x - 3$ after transposing 3 from L.H.S. to R.H.S. (by afixing - sign before 3).

(g) Linear equation: An equation of the form $2x+3 = 0$ or $2x+3y = 10$ is called a linear equation in one or two variables respectively. An equation in two variables contains two different variables (i.e. more than one variable). For example, in the equation $2x+3y = 10$, $x$ and $y$ are two different variables.

Linear equations are also called simple equations.

(h) Unknowns: Variables in an equation are also called unknowns. For example, in the equation $7x+3y = 18$, $x$ and $y$ are also called unknowns.

(i) Problems describing everyday life situations or mathematical situations in spoken language wherein we need some equation or equations to solve, to find answers to the problems are called verbal problems or word problems. The following examples are examples of word problems -

(i) The sum of two numbers is 12. Their difference is 8. Find the numbers.

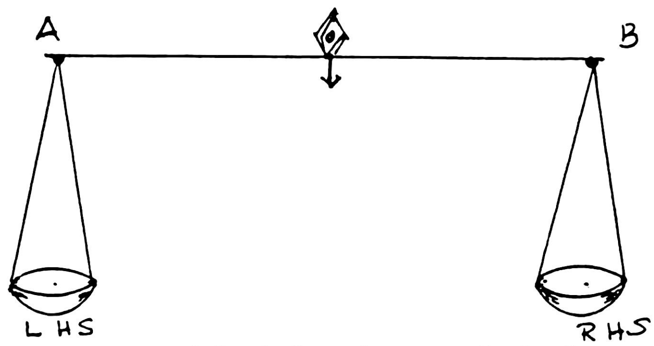
(ii) I am twice as old as I was 20 years before. How old am I at present ?

Here (i) is a verbal problem with a mathematical situation, (ii) is a verbal problem describing everyday life situation.

**(IV) Teaching Strategies (including activities, etc.)**

The teachers should start teaching this chapter on equations by asking the class several review questions as given in (I) to make sure whether students know all the previous knowledge needed for learning this chapter.

Then the teacher can illustrate the four basic rules for solving equations (or sometimes called self-evident truths) with the help of a two-sided horizontal balance with two pans. The teacher can say that the two pans of the balance are like the L.H.S. and R.H.S. of an equation each pan representing one of the two sides (L.H.S. and R.H.S.) of an equation.




A two-sided horizontal balance

Four basic rules for solving an equation are given below with illustrations.

(i) If the same term is added to both sides (L.H.S. and R.H.S.) of an equation, the resulting equation will be equivalent to the starting equation. "Two equations being equivalent" means that both will have the same values for the variable.

For example the equation $2x-3 = 5$ is the equivalent to $2x-3+3 = 5+3$ which is again equivalent to $2x-0 = 5+3$, i.e. $2x = 8$ or $x= 4$ is the value of x for both equations.

(ii) If the same term is subtracted from both sides of an equation, the resulting equation is equivalent to the starting one. For example, consider the equation $2x+4 = 5$ which is equivalent to the equation $2x+4-4 = 5-4$ which is equivalent to the equation $2x+0 = 5-4$ which is equivalent to the equation $2x = 1$.

(iii) If sides of an equation are multiplied by a non-zero number then the resulting equation is equivalent to the original one. For example, the equation $3 + \frac{2x}{5} = 7$ is equivalent to $(3 + \frac{2x}{5})x5 = 7x5$ which is equivalent to $15+2x = 35$. Here both the sides of the original equation is multiplied by the non-zero number (value) 5.

(iv) If both two sides of an equation are divided by a non-zero number then the resulting equation is equivalent to original one. For example the equation $10x = 5$ is equivalent to the equation $10x \div 10 = 5 \div 10$ which is equivalent to the equation $x = \frac{5}{10}$ which is equivalent to the equation $x = \frac{1}{2}$

The first two basic rules described above are collectively called transposition or transposing which means bring a term (being shifted) from one side of the equation to the other side by any of the above mentioned basic rules. While multiplying or dividing by a term all the terms of both the sides should be multiplied or divided.

Now we will study how to solve linear equations by solving some equations as given below.

(i) Solve $\frac{x-2}{3} - \frac{x-3}{4} = \frac{x-5}{6}$

Multiply each term by the L.C.M. 12 (of denominators 3, 4 and 6)

**Solution**: Then the resulting equation is

$$\left(\frac{x-2}{3}\right) \times 12 - \left(\frac{x-3}{4}\right) \times 12 = \left(\frac{x-5}{6}\right) \times 12$$

$4(x-2) - 3(x-3) = 2(x-5)$

By removing the brackets

$4x-8 - 3x+9 = 2x-10$

$x+1 = 2x - 10$

$x+1-2x = 2x-10-2x$ (subtracting 2x from each side)

$-x+1 = -10$

$-x = -10-1$ (transposing 1 to R.H.S.)

$-x = -11$

$x = 11$

(ii) Solve the simultaneous equations

$6(2x+1) = 5(y+3)$

$7(3x-2) = 4(2y-1)$

**Solution**: The given equations are equivalent to

$12x+6 = 5y+15$ ....(1)

$21x-14 = 8y+4$ ....(2)

$12x-5y = 15-6$

$21x-8y = 4+14$

$12x-5y = 9$ ....(3)

$21x-8y = 18$ ....(4)

To solve the simultaneous equations (3) and (4) the techniques to eliminate one of the variables x and y from (3) and (4) so that the resulting equation is a linear equation in one variable x or y. From this linear equation we may find the value of one of the two variables -x or y.

To eliminate y from (3) and (4) multiply (3) by 8 and (4) by -5. Then

$12x-5y = 9$ ....(3) x 8

$21x-8y = 18$ ....(4) x (-5)

$96x-40y = 72$ ....(5)

$-105x+40y = -90$ ....(6)

Adding (5) and (6),

$-9x = -18$

$$x = \frac{-18}{-9} = 2$$

Substituting the value of x in the equation (1) we get

$$12 \times 2+6 = 5y+15$$

$$24+6 = 5y+15$$

$$30 = 5y+15$$

$$30-15 = 5y$$

$$15 = 5y$$

$$y = \frac{15}{5} = 3$$

$$x = 2, \ y = 3$$

**Verbal Problems and Equations**

Solving a verbal problem with the help of equations means translating the condition(s) of the problem into an equation or two simultaneous equations and then solving the equation(s) to find the answer to the problem. The teacher may follow the steps given below in the order indicated.

**Step 1**: Let the student read the problem once, twice or more times till he finds the meaning of each word of the problem. Then he may understand the problem completely. In case of difficulty, the teacher may help the student.

**Step 2**: Let the student find out the unknown quantities and known quantities in the problem.

**Step 3**: Let him assume one of the unknown quantities as 'x' and express the remaining unknown quantities in terms of x.

In case of some verbal problems two unknowns may be termed as x and y. Then the remaining unknowns are expressed in terms of x and y.

**Step 4**: Let him form an equation or equations according to the given condition(s) to find relationship(s) between known and unknown quantities.

**Step 5**: Let him solve the equation(s) and (so) arrive at the answer to the verbal problems.

These steps are illustrated in the following example.

**Problem**: The sum of two integers is 24. One integer is thrice the other integer. Find the integers.

**Solution**

**Step 1**: The student must know the meanings of the terms - integers and thrice.

**Step 2**: Integers are unknown, but their sum is known.

**Step 3**: Let the smaller of the two integers by x. Then the larger integer = 3x according to the condition of the problem.

**Step 4**: The given condition can be written as an equation in x -

$$x + 3x = 24$$

**Step 5**: $x+3x = 24$

$$4x = 24$$

$$x = \frac{24}{4} = 6 \text{ and } 3x = 3x6 = 18$$

So the smaller integer is 6 and the larger integer is 18.

In this problem we can also suppose the two integers to be x and y. Then the conditions of the problem lead to the simultaneous equations.

$$x + y = 24$$

$$x = 3y$$

which have common values of x and y --- x = 18, y = 6. But this method is lengthy and involve more calculations than the given one here.

**Some more illustrative word problems**

(a) **Problem**: The sum of thrice a number and 4 times the same number is 49. Find the number.

**Solution**: Here the number is unknown and so let the number be x. Then thrice a number = 3x and four times the same number = 4x. So the sum of these two will be equal to 3x+4x. So the condition of the problem is represented by the equation

$$3x + 4x = 49$$

$$7x = 49$$

$$x = \frac{49}{7} = 7$$

So the required number is 7.

(b) **Problem**: The sum of three consecutive numbers is 33. Find them.

**Solution**: Here three numbers are unknowns and their sum is known. So let x be the smallest of the three numbers. Then other two numbers are x+1 and x+2.

So the condition of the problem leads to the equation

$$x + (x+1) + (x+2) = 33$$

Removing the brackets we have

$$x + x + 1 + x + 2 = 33$$

$$3x + 3 = 33$$

$3x = 33 - 3$ by transposing 3 to R.H.S.

$$3x = 30$$

$$x = \frac{30}{3} = 10$$

$x+1 = 10+1 = 11$ and $x+2 = 10+2 = 12$

So the required numbers are 10, 11 and 12.

(c) **Problem**: The cost of an eraser and 2 pencils is Rs. 5.
The cost of 2 erasers and 3 pencils is Rs. 8.
Find the cost of each eraser and each pencil.

**Solution**: Let the cost of an eraser = x rupees
and the cost of a pencil = y rupees

Then the cost of 2 pencils = 2y
the cost of 2 erasers = 2x
and the cost of 3 pencils = 3y
-- when x and y denote the amount in rupees

Then the conditions of the problem lead to

$x + 2y = 5$ ....(1)

$2x + 3y = 8$ ....(2)

Now to eliminate x multiply the equation (1) by 2. This leads to the equation

$2x + 4y = 10$ ....(3)

$2x + 3y = 8$ ....(4)

Subtracting (4) from (3) we have

$y = 2$

Substituting 2 for y in (1) we have

$x + 2x2 = 5$

$x + 4 = 5$

$x = 5-4 = 1$

So $x = 1$ and $y = 2$

So the cost of an eraser is 1 rupee and the cost of a pencil is 2 rupees.

(d) **Problem**: Mrs. Shabana is 20 years older than her son. Ten years hence, she would be twice as old as her son. Find their present ages.

**Solution**: Let Mrs. Shabana's present age be x and her son's present age be y. Then the conditions of the problem lead to the equations

$$x = y + 20 \qquad \ldots.(1)$$

$$x + 10 = 2(y+10) \qquad \ldots.(2)$$

Here we have the equation (2) because ten years hence (i.e. after) Mrs. Shabana's age will be x+10 and her son's age will be y+10.

Subtracting (1) from (2) we have

$$(x+10)-x = 2(y+10) - (y+20)$$

$$x + 10 - x = 2y + 20 - y - 20$$

$$10 = y$$

Substituting 10 for y in (1) we have

$$x = 10 + 20 = 30$$

$$x = 30,\ y = 10$$

So Shabana's age is 30 years and her son's age is 10 years.

**(V) Unit Test and Graded Exercises**

UNIT TEST

Time : 45 mts
Marks: 25

1. What is x if

(a) $x - 3 = 15$ (b) $\frac{3}{2} = 2x$ (c) $3x - 17 = 4$

2. Find the value of the variable x if

(a) $3(3x+1) - 5(2x+5) = 8(x-4) - 8$

(b) $\frac{x-2}{3} - \frac{x-3}{4} = \frac{x-4}{6}$

3. Solve the simultaneous linear equations

(a) $3a + 5b = 25$
$3a - 2b = 11$

(b) $5x + y = 7(x-3)$
$x - 2 = 3y - 1$

4. Two supplementary angles differ by $20^{\circ}$. find the measure of each angle.

5. The sum of two numbers is 38. Their difference is 14. Find the numbers.

6. The total of the father's age and his son's age is 52 years. The difference between their ages is 28 years. Find their ages.

**Marks allotted questionwise**

| Question No. | Marks allotted |
|---|---|
| 1 | 1x3 = 3 |
| 2 | 2x2 = 4 |
| 3 | 3x2 = 6 |
| 4 | 4x1 = 4 |
| 5 | 4x1 = 4 |
| 6 | 4x1 = 4 |
| Total | 25 |

## GRADED EXERCISES

1. Solve the following

   (a) $x-3 = 12$   (b) $25x = -75$

2. Solve: (a) $3x+4 = 19$   (b) $18x-20 = 10x+4$

   (c) $2(x-4) = 16$   (d) $3(7-x) = 4(x+7)$

3. Find the value of x if

   (a) $3(2x+1) - (x-1) = 4(x+8)$

   (b) $\frac{x}{4} + \frac{x}{6} = \frac{x}{8} + 3\frac{1}{2}$

4. Solve the simultaneous equations

   (a) $5a - 2b = 18$
   $3a - 2b = 10$

   (b) $5x-7y = 22$
   $11x-7y = 82$

5. Find the values of x and y if

   (a) $3x+5y = 19$
   $5x = 4y+7$

   (b) $4x+7y = 4$
   $5x-3y = 52$

6. If 40 is added to thrice a number, the result is 52. What is that number ?

7. The sum of two numbers is 42. One of them is 17. Find the other.

8. The sum of two numbers is 42 and one number is twice the other number. Find the numbers.

9. The total of the father's age and his son's age is 52 years. The difference between their ages is 28 years. Find their age.

**Answer to Graded Exercises**

(1) (a) 9   (b) -3
(2) (a) 5   (b) 3   (c) 12   (d) -1
(3) (a) 28   (b) 12
(4) (a) $a = 4$, $b = 1$   (b) $x = 10$, $y = 4$
(5) (a) $x = 3$, $y = 2$   (b) $x = 8$, $y = -4$
(6) 4
(7) 25
(8) 28 and 14
(9) 40 years and 12 years

## (VI) Suggested Enrichment Activities for Teachers

(a) Given an equation, many a time we can find life situations which translate into that equation. For example, given the equation $2x+12 = 30$, helps in solving the following verbal problem:

Twice the present age of a boy becomes 30 years after a period of 12 years. What is his present age ?

So now frame every day life problems wherein the situations (conditions) of the problems will translate into the following equations.

(i) $4x + 10 = 58$

(ii) $3x - 10 = 14$

(iii) $\frac{x}{3} + 10 = 20$

(b) How will you illustrate the steps involved in solving the equation $2x+4 = 10$ through the two pans of a two-sided horizontal weighing scale ?

(c) Form verbal problems stating the number facts so the verbal problems lead to the equations (i), (ii) and (iii) of enrichment activity (a).

(d) Prepare three charts corresponding to suggested enrichment activity (c) wherein the steps are related to the two-sided weighing balance illustrated.

(e) Construct two problems, one containing number facts and the other describing everyday life situations for each of the following simultaneous equations:

(i) $x+y = 90$, $x-y = 30$

(ii) $2x+3y = 15$, $3x+5y = 21$

# CHAPTER XII

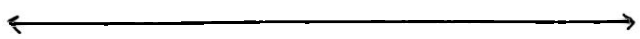
# AREAS

## Preview

We see many objects all around us. They will be having different shapes, sizes and positions. If we can understand their characteristics and properties, we will be able to appreciate the beauty in them and use them in our daily life. This will help us in enhancing our faculty of mind in designing and planning. Let us try to remember the geometrical concepts and principles studied in the previous classes.

## Line

A point is a dot marked on a sheet of paper. A straight line is a set of points resembling long meter scale. A straight line has a direction and can be extended indefinitely. A line segment is a part of a line between any two points on the straight line. A ray is a part of a straight line on one side of a point on the line. The point is called the initial point of the ray. A ray starts from a point and extends indefinitely. Lines which are not straight are called as curved lines. Planes are surfaces with indefinite number of points. Two straight lines may intersect or may not intersect. Parallel lines do not intersect. Points which lie on a straight line are called collinear points.

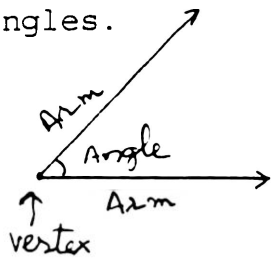


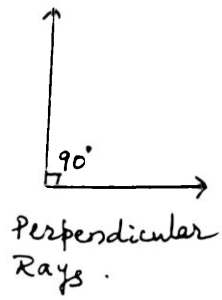
(A line)

## Angle

An angle is formed by two rays having the same initial point called the vertex of the angle. The rays are the arms of the angle. Protractor is used to measure an angle. When the angle ~~mean~~ measure is 90°, the arms of the angle are said to be perpendicular. The angle is called a right angle.

There are different types of angles.




Acute angle - less than 90°

Abtuse angle - less than 90°

Right angle - is equal to 90°

Straight angle - is equal to 180° and may be formed at a point on either side of the straight line.

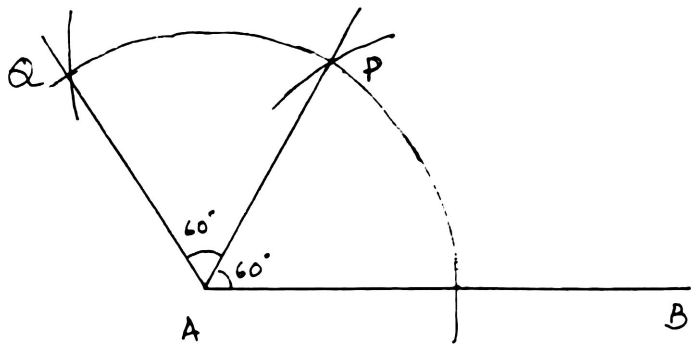
**Complementary angles** - Two angles at a point whose sum is 90°.

**Supplementary angles** - Two angles whose sum is 180°.

**Construction of angles of 60° and 120° at a point A, with a given arm AB (using ruler and compass)**

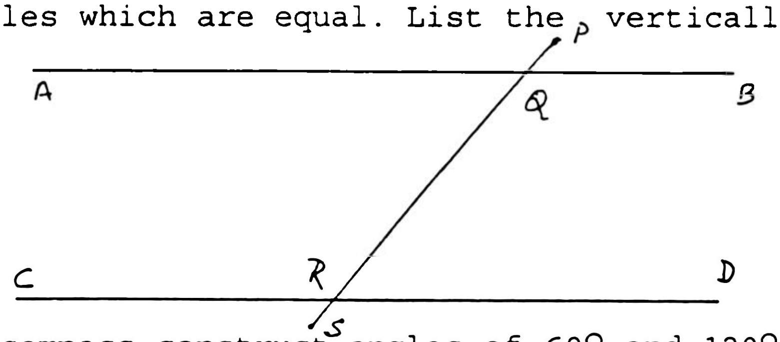
AB is a straight line. To construct an angle of 60° and 120° at A, with A as centre draw an arc with a convenient radius to cut AB at C. With C as centre and with the same radius cut the arc at P, and with P as centre, cut the same arc at Q. Join AP and AQ. Now $|\underline{PAB}$ = 60°

$|\underline{QAB}$ = 120°



## Exercises

1. Draw a line segment. Name its end points x and y. Measure the distance between x and y.
2. How many lines can pass through one point ?
3. Using a protractor and a scale, construct angles measuring $30^{o}$, $45^{o}$, $60^{o}$, $90^{o}$, $120^{o}$, $180^{o}$ and name them. Identify acute angles, obtuse angles and straight angles.
4. Write the names of the angles formed in the figure. Identify the angles which are equal. List the vertically opposite angles.




5. Using ruler and compass construct angles of $60^{o}$ and $120^{o}$. Name them.

## Circle

**Introduction:** It is well known that geometry took its birth in Egypt and developed by eminent mathematicians of Greece. Thales introduced the study of geometry in Greece. Many principles concerning triangles, circles were found out by him. He insisted proofs rather than intuition in geometry. Pythogoras was a student of Thales. It is said that when Pythogoras discovered the proof of his theorem, that bears his name, he was supremely happy. More than a hundred proofs have been given for this theorem. He was fond of geometrical relations. He called the sphere as the most beautiful of all solids and the circle as the most beautiful of all plane figures. He was the first to discover that the earth is a sphere.

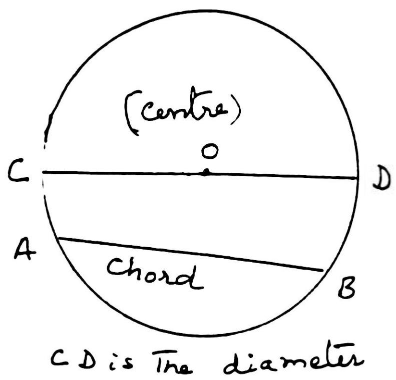
Later on Plato (429 BC to 348 BC) established his school in 389 BC. "Let no man ignorant of geometry enter my doors" was the inscription over the entrance of his academy. Many concepts were developed pertaining to prism, pyramid, cylinder, cone, etc. in his academy.

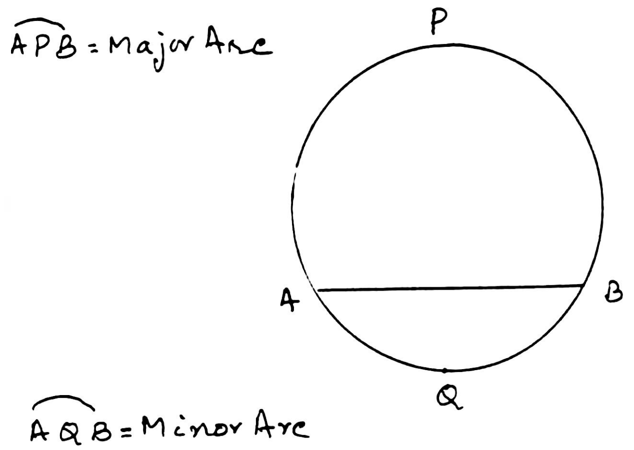
Ptolemy founded the University of Alexandria in 338 BC. Alexander brought back from India and Babylon many manuscripts and tablets of mathematical content after the warlike excursions. It was here that Euclid and his pupil Appolonius (300 BC) lived, who established many geometrical theorems. Archemedes lived and contributed a lot during this time.

The Indian mathematicians dealt with numbers, the Greeks with form.

A circle is a set of points in a plane at a fixed distance from a fixed point. The fixed point is called as the centre and the perimeter of the circle is called the circumference. Compasses are used to draw circles. A chord is a line segment drawn by joining two points on the circumference of a circle. The circumference of a circle is circular in nature and is formed with reference to a fixed point. The distance between the fixed point and any point on the circumference is called as the radius. A chord of a circle passing through the centre of the circle is called a diameter. The length of a diameter of a circle is twice the radius. A diameter divides the circle into two equal parts and each part is called as a semi-circle.

A part of the circumference of a circle is called an arc. Any chord divides a circle into segments.




**Exercise**

Draw a circle with a convenient radius. Draw a chord, diameter, radius. Name them and measure them.

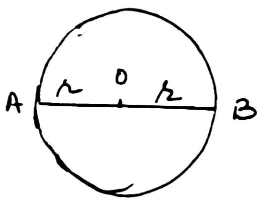
**Properties of Circles**

1. In a circle the diameter passes through its centre and is twice the radius of the circle.

   Diameter AB = AO + OB

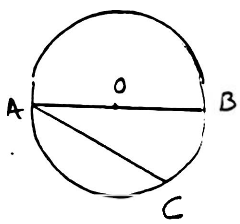
   d = r + r

   d = 2r




2. In a circle, the diameter is the longest chord.

   The diameter AB is the longest chord of the circle, i.e. AB > AC



## Measuring the Circumference of a Circle

Draw a circle on a Thick sheet of paper and cut it. Roll it completely (one round) ~~that is~~ from a point marked on its circumference, over a thread stretched straight. The length of the thread is measured with a scale. This length becomes the circumference of the circle.

## Relationship between the diameter and the circumference of a circle

Draw circles with different radii and measure their circumference. It is found that the circumference of a circle increases as the diameter increases.

It can be calculated that the ratio of

$$\left[\frac{\text{Circumference of a circle}}{\text{Diameter of the same circle}}\right] \text{ is a constant}$$

This ratio is equal to 3.14 or 22/7 approximately. This constant is denoted by a symbol $\pi$ called 'pie'.

Aryabhata, an Indian mathematician gave a more accurate value of as 3.1416. An approximate value of $\pi$ may be taken as 3.14 or 22/7.

$$\frac{\text{Circumference of a circle}}{\text{Diameter}} = \pi$$

i.e. $\frac{c}{d} = \pi$ [C = Circumference, d = Diameter

or $c = \pi d$

Exercises

1. Calculate the circumference of each of the following circles whose radii are 21 cm and 56 cm taking $\pi = 22/7$.

$C = \pi d$ Taking $= 22/7$

$= \frac{22}{7} \times 42$ ( $d = 22$)

$C = 132$ cm

$C = \pi d$

$= \frac{22}{7} \times 112$

$C = 352$ cm

2. Calculate the diameter of a circle whose circumference is (i) 154 cm (ii) 682 cm

$c = \pi d$

$d = c/\pi$

$d = \frac{154}{22/7}$

$= \frac{154}{22} \times 7$

$d = 49$ cm

$c = \pi d$

$d = c/\pi$

$d = \frac{682}{22/7}$

$= \frac{682}{22} \times 7$

$d = 217$ cm

3. A cycle wheel has a circumference of 132 cm. How many revolutions does it make when it moves a distance of 264 metres.

$$\text{Number of revolution} = \frac{\text{Distance travelled}}{\text{Circumference}}$$

$$= \frac{264 \times 100}{132}$$

$$= 200$$

4. A wheel makes twenty revolutions in moving 110 metres. Find the diameter of the wheel.

$$\text{Number of revolution} = \frac{\text{Distance travelled}}{\text{Circumference}}$$

$$\text{Circumference} = \frac{\text{Distance travelled}}{\text{Number of revolutions}}$$

$$= \frac{110 \times 100}{20}$$

$$= 550 \text{ cm}$$

$$C = \pi d$$

$$\text{Diameter } d = \frac{C}{\pi}$$

$$= \frac{550}{22/7}$$

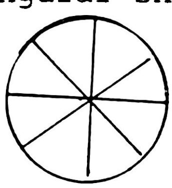
$$= \frac{550}{22} \times 7$$

$$= 175 \text{ cm}$$

$$= 1.75 \text{ mt}$$

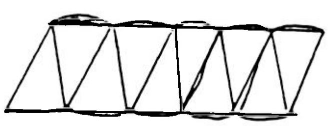
**Area of a Circle**

Any circle can be divided into equal sectors. If these sectors are placed side by side, we can see that a rectangular shape is formed.



Area of a rectangle = Length x Breadth

Here the length is half the circumference of the circle and breadth is the radius of the circle.



Area of the rectangle so formed = Area of the circle

Area of the circle = $\frac{1}{2} \times \pi d \times$ radius

Area of the circle = $\pi r^2$ Square measures

(The value of $\pi$ may be taken as 22/7.)

**Another method**

A circle of radius $r$ is divided into a large number (n) of equal sectors so that each sector works like an isosceles triangle with base as the arc of the sector.

Let c = The circumference of the circle

then each arc = $\frac{c}{n}$

Area of each sector = $\frac{1}{2}$ base x height

$$= \frac{1}{2} \left(\frac{c}{n}\right) \times r$$

Area of the circle = n x Area of a sector

$$= n \times \frac{1}{2} \left(\frac{c}{n}\right) r = \frac{1}{2} c r$$

But $c = 2 \pi r$

Area of the circle = $\frac{1}{2} (2\pi r) r = \pi r^2$

Area of a circle of radius = $\pi r^2$

**Exercises**

1. Calculate the area of a circle whose radii are 7 m, 21 m.

Area of a circle = $\pi r^2$ [ $\pi = \frac{22}{7}$ ; r = radius]

$$= \frac{22}{7} \times 7 \times 7 = 154 \text{ sqcm}$$

Area of a circle = $\pi r^2$

$$= \frac{22}{7} \times 21 \times 21 = 1386 \text{ sqm}$$

2. Calculate the area of a circle whose circumference is 88 cm.

Area of a circle = $\pi r^2$

Circumference of a circle = $\pi d$

$c = \pi d$

$d = c/\pi$

$$d = \frac{c}{\pi} = \frac{88}{22/7} = \frac{88x7}{22} = 28 \text{ cm}$$

$2r = 28$

$$r = \frac{28}{2} = 14 \text{ cm}$$

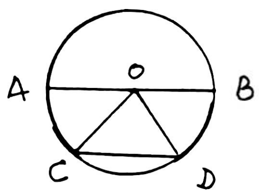
Area of the circle = $\pi r^2$

$$= \frac{22}{7} \times 14 \times 14$$

$$= 616 \text{ sqcm}$$

**Exercises**

1. Identify the centre, chord, sector, segment, diameter of the circle given here. [Note: A sector is the region between two radii and an arc of a circle.]




2. Calculate the radii of the circles whose diameters are 4 cm, 6 cm and 8 cm.

3. Draw a circle and divide it into four equal parts, by drawing diameters suitably.
4. Calculate the area of the circle whose diameter is 49 mts and 70 min.
5. In 15 revolutions a wheel covers a distance of 82.5 mts. Find the diameter of the wheel.
6. Calculate the area of a circular ring whose internal and external radii are 21 cm and 56 cm.

**Self-Learning Activities**

1. Measure the circumferences of circular objects such as bangles, plates, wheels of different vehicles.
2. Many problems can be designed pertaining to circles with the help of students.

## UNIT TEST

Time : 45 min
Marks: 25

**I. Fill in the blanks** 6x1 = 6

1. A chord of a circle that contains the centre of that circle is _______.
2. All the radii of a circle are _______.
3. Circumference of a circle is ________.
4. A line segment of a circle which does not pass through the centre is called _______ of the circle.
5. A diameter of a circle is twice ______.
6. A part of the circumference of a circle is called as ______.

**II. Solve the following problems** 4x2 = 8

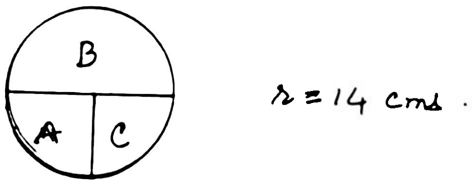
7. Calculate the circumference of the circles whose diameter is 7 cm and 14 cm.
8. Find the diameter of (a) a circle if the circumference is 154 cm, (b) a wheel of 132 cm as its circumference.
9. Calculate the circumference of each of the circles whose radii are 21 cm and 56 cm.
10. Calculate the area of the circle whose diameters are 28 cm and 49 mts.

**III. Solve** 2x3 = 6

11. A cart wheel has a diameter of 154 cm. Find the distance covered when it makes 150 revolutions.
12. How many times will the wheel of a car rotate in a journey of 3850 m, if its radius is 90 cm.

IV. 1x5 = 5

Calculate the area of each sector in the following diagram.

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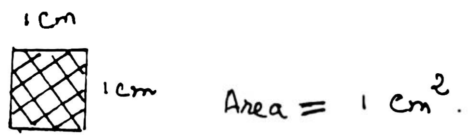
# CHAPTER XIII
## MENSURATION

### Preview

We are familiar with certain plane figures such as a quadrilateral, rectangle, parallelogram, square, triangle, rhombus and circle.

We know, how to calculate the areas of various figures.

The area of a square whose side is 1 cm is a unit of area and is called one square cm and is written as 1 $cm^2$.




Area of any rectangle is the sum of the total unit squares present in it.

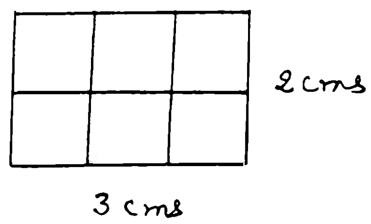
Area of the rectangle ABCD = Length x Breadth

= 3 x 2 = 6 sqcm

Area of a parallelogram = Base x Height

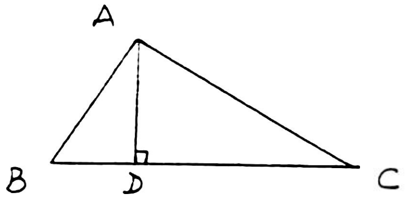
Parallelogram ABCD = 3 x 2

= 6 sqcm



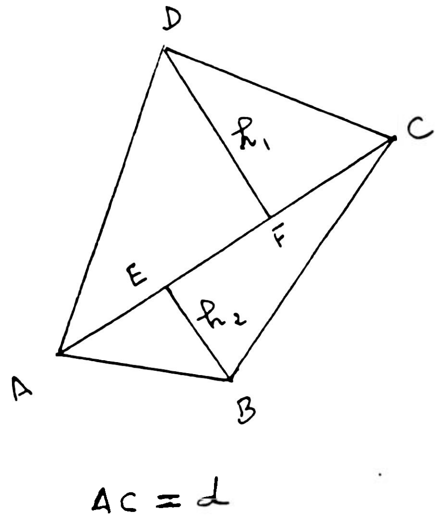

Area of a square of side $\ell$ is square, $\ell^2$ units.

Area of a triangle is half its base multiplied by its height.




Area of $\triangle ABC = [\frac{1}{2} \times BC \times AD]$

Area of a quadrilateral can be calculated by dividing the figure into triangles.




Area of ABCD

= Area ABC + Area ADC

$= \frac{1}{2} AC \times BE + \frac{1}{2} AC \times DF$

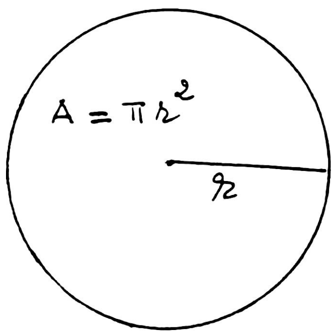
$= \frac{1}{2} (BE + DF) \times AC$

Area of ABCD

$= \frac{1}{2}$ [Diagonal] x [Sum of distances of the diagonal from the other two vertices]

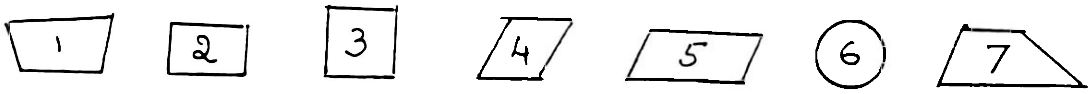
or $$A = \frac{1}{2} d(h_1+h_2)$$

Area of a circle is $\pi r^2$ where $r$ = the radius of the circle.



**Review Exercises**

1. Find the area of a square table whose side is 30 cm.
2. Calculate the area of a rectangle whose sides are 4 cm and 2.5 cm.
3. Calculate the area of a parallelogram whose base is 6 cm and its altitude is 5 cm.
4. Find the area of circles whose radii are 14 cm and 28 cm.
5. Name the following diagrams and identify them.




6. Draw different types of quadrilaterals. Measure the sides and angles of each figure. Mention the figures whose sides are equal. Identify the equal angles.
7. Find the area of the quadrilateral ABCD given AC = 6 cm, BE = 4 cm, DF = 5 cm when BE, DF are perpendicular to AC.

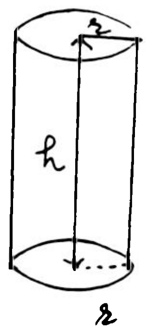
**Concepts**

1. Cylinder - Curved surface area and its total surface area
2. Cone - Curved surface area and its total surface area

## CYLINDER

**Introduction**

We see many cylindrical objects around us. For example, pipes, drums, rollers, etc. Any cylindrical object has two circular congruent faces and a curved surface. The line joining the centres of the circular faces is called the axis of the cylinder and its length is denoted by 'h'. The radius of the circular base of the cylinder is denoted by 'r'.



## Curved Surface Area of a Cylinder (C.S.A.)

### Activity

A rectangular sheet of paper is rolled over the curved surface of a cylinder so as to cover it completely. The area of the curved surface of the cylinder will be the area of the rectangle used to cover it.

(i) The area of the curved surface area of the cylinder = The area of the rectangular sheet = The length of the sheet x The breadth

= (the circumference of the base circle) x (the height of the cylinder)

[ Area of the rectangle = length x breadth]

C.S.A. of the cylinder = $2\pi r \times h$

**Hence C.S.A. = $2\pi rh$** ( Circumference = $2\pi r$)

(ii) Total surface area of a cylinder (T.S.A.)

= Curved surface area of the cylinder + Area of the two circular faces

T.S.A. of the cylinder = $2\pi rh + 2\pi r^2$

T.S.A. of the cylinder = $2\pi r(h+r)$

h = Height of the cylinder

r = Radius of the base

Observe (from (i) and (ii)) that

$$\text{T.S.A.} = \text{C.S.A.} + 2\pi r^2$$

### Exercise

Find the CSA and TSA of the following cylinder whose radius and height are given.

(i) r = 14 cm, h = 30 cm

(ii) r = 21 cm, h = 40 cm

Example

Area of the curved surface of the cylinder $= 2\pi rh$

Taking $\pi = \frac{22}{7}$

$$= 2 \times \frac{22}{7} \times 14 \times 30$$

$= 2640$ sqcm

Area of the total surface of a cylinder $= 2\pi r(h+r)$

$$= 2 \times \frac{22}{7} \times 14\,(14+30)$$

$= 44 \times 2 \times 44$

$= 3872$ sqcm

## CONE

### Concepts

<u>Cone</u> - Area of the curved surface

- Area of the total surface

### Introduction

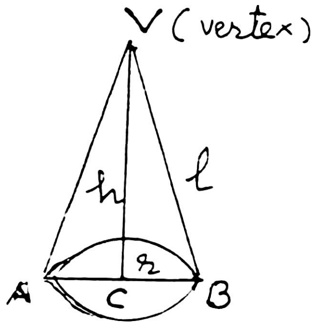
We see conical shapes in many objects. Crowns of idols of Gods/Goddesses we see in temples, a funnel, a heap of grain on a level ground - are some examples of cones. Icecream cones are familiar to the children. A cone has a plane circular base and a curved lateral surface. The sharp point of the cone is called its vertex. The line joining the vertex and the centre of the base is called the height of the cone. The distance between the vertex and any point on the circumference of the base is the slant height of the cone.

In the figure,

$V_C$ is the height (h)

$V_B$ is the slant height ($l$)

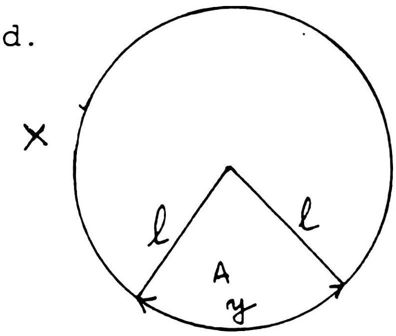
$C_B$ is the radius of the base ($r$)

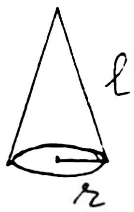



**Area of the curved surface of a cone**

**Activity 1**

A sector is cut from a circular sheet of paper having a radius '$l$'. By bringing the radii of the sector to coincidence, a cone is made from the sector. Let the circular base of this cone be 'r'. The slant height of the cone so formed is 'l'. Here the area of the sector (A), the area of the circle (x), the length of the arc (y) and the circumference of the circle form a proportion. This fact is assumed.




The ratio of the area of the sector (A) to that of the circle with radius $l$ is equal to the ratio of the arc length ($y$) to the circumference of the circle. The arc length of the sector, when converted into a cone will be having a circular base with radius r.

$$\frac{\text{Area of the curved surface of the cone}}{\text{Area of the circle}} = \frac{\text{Arc length of the sector}}{\text{Circumference of the circle}}$$

$$\frac{\text{Area of the sector A}}{\pi \ell^2} = \frac{2\pi r}{2\pi \ell}$$

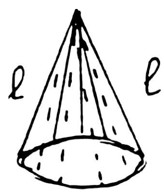
∴ Area of the sector A or the

$$\text{Area of the curved surface of the cone} = \pi \ell^2 \times \frac{r}{\ell}$$

$$\text{Area of the curved surface of the cone} = \pi r\ell$$

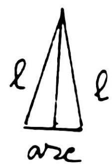
**Activity 2**

The curved surface of a cone is divided into a number of small sectors. Each sector becomes a triangle approximately. The base of this triangle is the arc and the slant height is the height.




$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{Area of each triangle} = \frac{1}{2}\ (\text{arc length}) \times (\text{height})$$




Area of all the triangles formed will be the Area of the curved surface of the cone.

$$\text{Area of the curved surface of the cone} = \frac{1}{2}\ (\text{sum of the arc length}) \times (\text{slant height})$$

$$\text{Area of the curved surface of a cone} = \frac{1}{2} \times (\text{circumference of the base}) \times (\ell)$$

( Sum of the arc lengths becomes the circumference of the base of the cone)

$$\text{Area of the curved surface of the cone} = \frac{1}{2} \times (2\pi r) \times \ell$$

$$= \pi r\ell$$

**Area of the total surface of the cone**

Area of the total surface of the cone = (Area of the curved surface) + (Area of the base)

$= \pi rl + \pi r^2$

( $\therefore$ Area of a circle $= \pi r^2$)

Area of the total surface of the cone $= \pi r(l+r)$

$= \pi r(r+l)$

**Example**

The radius of the base of a cone is 14 cm and its slant height is 50 cm. Calculate the area of the curved surface and the area of the total surface.

Here $r = 14$ cm and $l = 50$ cm

Area of the curved surface of the cone $= \pi rl$

$= \frac{22}{7} \times 14 \times 20$

= ~~2200~~ 880 sqcm

Area of the total surface of the cone $= \pi r(r+l)$

$= \frac{22}{7} \times 14\ (14+50)$

$= 22 \times 2 \times 64$

= 2816 sqcm

**Exercise**

A conical tent of slant height of 8 mts and the radius of the base is 14 mts is to be made. Calculate the canvas required for the tent.

## UNIT TEST

Time : 45 min
Marks: 25

**I. Fill in the blanks** 1x6 = 6

1. Area of the curved surface of a cylinder = __________
2. Area of the total surface of a cylinder = __________
3. Area of the curved surface of a cone = __________
4. Area of the total surface of a cone = __________
5. A cylinder has ________ curved and _________ circular surfaces.
6. A cone has _________ surface and _________ circular surface.

**II. Solve** 3x4 = 12

7. Find the area of the curved surface and the area of the total surface of a cylinder having a radius of 7 cm and a height of 21 cm.
8. The radius of the base of a cylindrical tin is 12 cm and its height is 30 cm. Find the area of the paper required to cover the tin.
9. Find the curved surface area and the total surface area of a cone whose slant height is 35 cm and the radius of the base is 14 cm.
10. A conical tent has a base of diameter 4.2 mts. If its slant height is 3.5 mts, calculate the canvas required for the tent.

III. Solve 1x7 = 7

11. The diameter of the base of a conical tent is 8.4 mts. If the vertical height of the tent is 5.6 mts, find the area of the canvas used for the tent.

(Hint: $l^2 = r^2 - h^2$)

Self Learning Activity

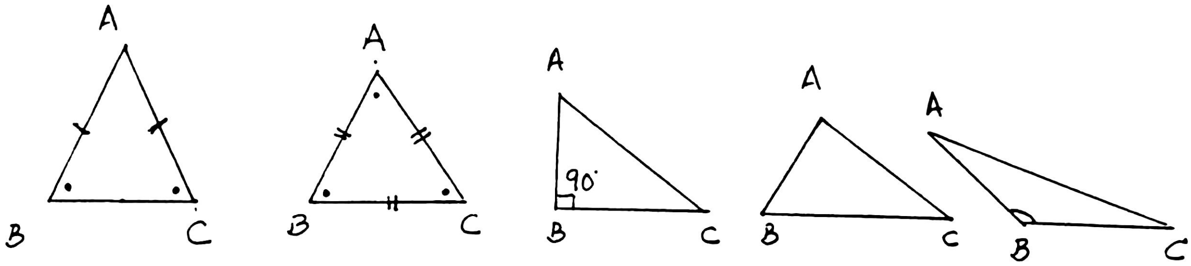
1. Make a list of cylindrical objects at home and calculate the area of the curved surface in each case. If it is in the form of a cylinder, find its total surface ~~of~~ area.
2. Make different conical shapes with paper and find the area of the curved surface in each case.

# CHAPTER XIV

# CONSTRUCTIONS

## Preview

We know that a triangle is a figure formed by three line segments called the sides of the triangle. A triangle has three angles inside with their sum 180°. The vertices of a triangle are represented by capital letters and the sides by small letters. We also know that there are five (5) different types of triangles. They are as shown below:




Isosceles triangle | Equilateral triangle | Right angled triangle | Acute angled triangle | Obtuse angled triangle

(i) In an isosceles triangle two sides are equal and the two angles which are opposite to those sides are equal.

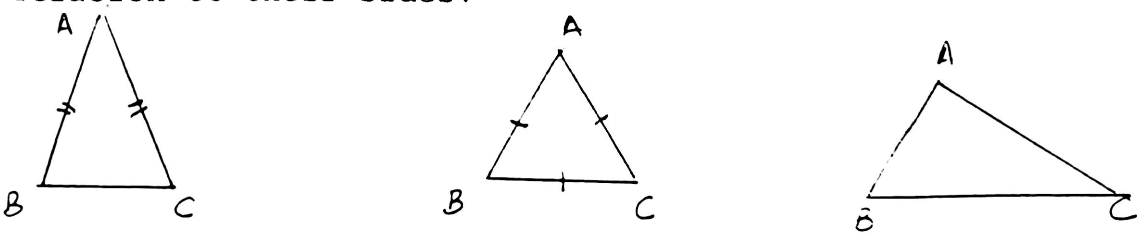
(ii) In an equilateral triangle all the sides are equal and angles are equal.

(iii) In a right angled triangle one angle is 90°. The side opposite to the right angle is called the hypotenuse.

(iv) In an acute angled triangle each angle is acute (less than 90°).

(v) In an obtuse angled triangle, one angle is obtuse (more than 90°).

Triangles are also classified into three (3) types in relation to their sides.



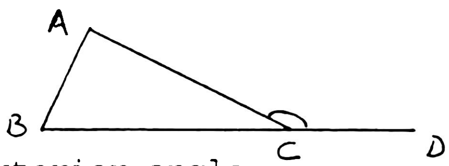

Isosceles triangle

Equilateral triangle

Scalene triangle

[Note: In any triangle, the sum of its two sides must be greater than the third side.]

We know that an exterior angle is formed when a side of a triangle is extended. In the triangle ABC




$\angle ACD$ is the exterior angle

$\angle ACB$ is the interior angle. BAC and ABC are the interior opposite angles.

$\angle ACD = \angle BAC + \angle ABC$

In a triangle the exterior angle is equal to the sum of the two interior opposite angles.

**Exercises**

1. Draw a triangle. Name the triangle. Measure the sides and the angles. Find the sum of all the angles.
2. Fill in the blanks:
   - (i) An _________ triangle is also equi angular.
   - (ii) In an isosceles triangle ________ sides are equal.
   - (iii) In an obtuse angled triangle _______ angles are acute.
   - (iv) If the sides of a triangle are unequal then it is called a _______.

(v) If an angle of a triangle is $90^{\circ}$, then it is called ________.

3. A ladder makes an angle of $60^{\circ}$ with a vertical wall. What angle does it make with the ground.
4. In a triangle ABC, $\underline{|A} = 65^{\circ}$, $\underline{|B} = 45^{\circ}$. Find the angle $\underline{|C}$.
5. In a triangle ABC, $\underline{|A} = 40^{\circ}$ and the exterior angle $\underline{|ACD} = 120^{\circ}$. Find $\underline{|ABC}$ and $\underline{|ACB}$.

**Construction of a Triangle**

A triangle can be constructed if the following conditions are given.

1. Three sides are given.
2. Two sides and the included angle are given.
3. One side and two angles are given.
4. Hypotenuse and a side of a right angled triangle are given.
5. Two sides which form a right angle of a right angled triangle are given.

[Note: (i) A rough diagram should be drawn before constructing the required triangle.

(ii) Ruler, compass and protractor are used in the construction.]

**Pythogoras Theorem**

**Activity**

Draw a number of right angled triangles. Measure the hypotenuse and the sides of each triangle.

Calculate the squares of the sides and hypotenuse. It is found that the sum of the squares on the sides is equal to the square on the hypotenuse.

In the triangle ABC, $|\underline{B} = 90^{\circ}$

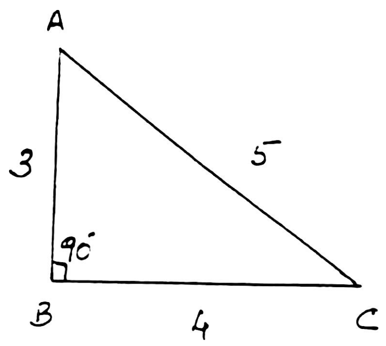
$AB^2 + BC^2 = AC^2$

$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

Similarly $5^2 + 12^2 = 13^2$

$8^2 + 15^2 = 17^2$




In a right angled triangle if two sides are given, the third side may be calculated.

**Example**

In the triangle ABC

$|\underline{ABC} = 90^{\circ}$

AB = 5 cm and AC = 13 cm, BC = ?

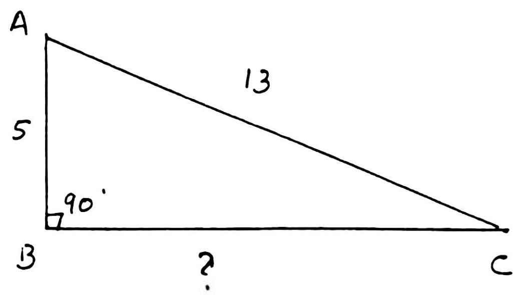
We know $AB^2 + BC^2 = AC^2$

$5^2 + BC^2 = 13^2$

$BC^2 = 13^2 - 5^2$

$BC^2 = 144$

$BC = \sqrt{144} = 12$ cm

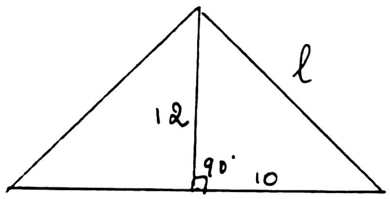


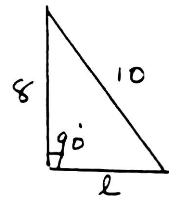

"In a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides of the triangle" was established by Pythogoras and is called as Pythogoras theorem.

**Exercises**

1. Construct a triangle ABC given AB = 6 cm, BC = 10 cm and AC = 8 cm. Measure the angles.
2. Construct a triangle DEF given $|\underline{F} = 110^{\circ}$, EF = 4.5 cm and DF = 6.8 cm. Measure the remaining two angles.

3. Construct a triangle ABC given AB = 5.5 cm, $\angle A = 45^\circ$ and $\angle B = 85^\circ$.
4. Construct a right angled triangle XYZ. Given hypotenuse XZ = 9 cm and YZ = 5 cm. Measure the remaining side and the angles.
5. Find the value of 'l' in the following figures!




6. A rectangular plot has a length of 16 mts and breadth of 12 mts. Calculate the length of the diagonal.

**Activity**

Construct several right angled triangles and make the students realise the relationship between the sides and hypotenuse.

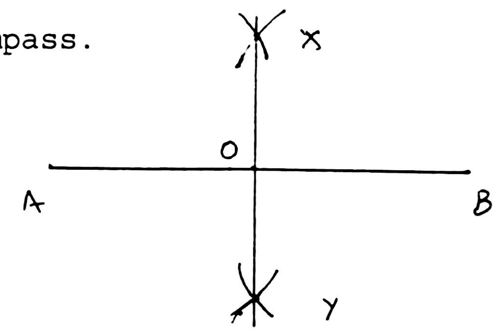
**CIRCUM CIRCLE**

**Perpendicular bisector of a line segment**

**Example**

Draw a line AB = 5.5 cm. Draw a perpendicular bisector of AB using compass.

**Construction**

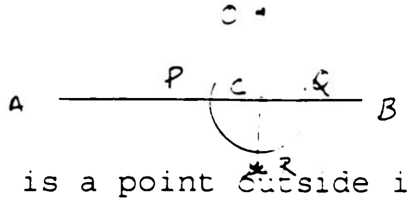



Draw AB = 5.5 cm with A and B as centres, more than half of the length AB as radius, draw arcs on either side of AB; which intersect at X and Y. Join XY. XY is the perpendicular bisector of AB ~~at O~~.

$\angle AOX = \angle BOX = 90^\circ$ AO = BO

Perpendicular to a line from a point located outside the line

**Exercise**



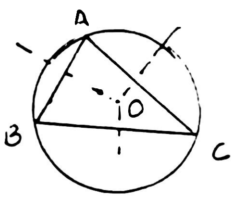

AB is the line segment. O is a point outside it. With O as centre draw an arc to intersect AB at P and Q. With P and Q as centres draw arcs to intersect at R, with a convenient radius. Join OR. OR is perpendicular to AB at C.

1. A circle which is circumscribed is called a circum circle. The centre of a circum circle is called the circum centre.

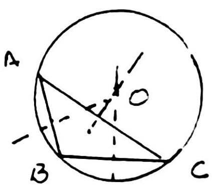
**Activity**

Draw a triangle ABC. Draw the perpendicular bisectors of all the sides. It is found that the perpendicular bisectors meet at a point (O) called the circum centre. Circum centre is equidistant from the vertices of the triangle. With O as centre, OA or OB or OC as radius draw a circle. This circle passes through A, B and C. It is the circum circle of the triangle ABC.

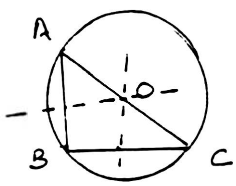
2. The circum centre of a circum circle is located either inside or outside any triangle except in the case of right angled triangle, where it lies on the midpoint of the hypotenuse.




Acute angled triangle (O is inside)




Obtuse angled triangle (O is outside)




O is on the mid point of the hypotenuse in the right angled triangle

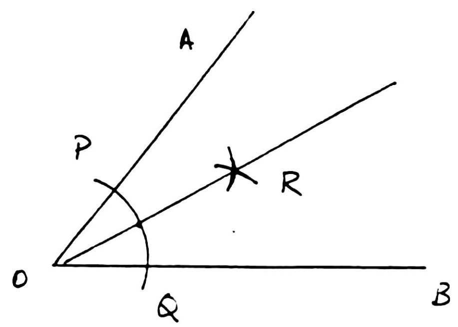
[Note: The circum centre of a circum circle can be located by constructing the perpendicular bisectors of any of the two sides of a triangle.]

**Exercises**

1. Draw a triangle ABC. Construct its circum circle. Write the construction and measure the circum radius.
2. Draw a triangle where the circum centre is on the hypotenuse.
3. Name the point which is equidistant from the vertices of a triangle.
4. Draw a triangle where the circum centre lies outside the sides.

**Incircle**

**Bisector of an Angle**




AOB is an angle. With O as centre intersect OA and OB at P and Q, with P and Q as centres, and with a convenient radius, arcs are drawn to intersect at R. OR is joined. OR is the bisector of the |<u>AOB</u>. Here |<u>AOR</u> = |<u>BOR</u>.

**Exercise**: Draw an angle and bisect it. Measure the angles.

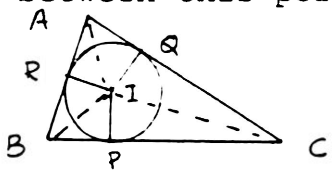
**Incircle**

A circle touching all the sides of a triangle is called as the incircle of the triangle as it is inscribed. The centre of the incircle is called as the incentre.

**Activity**

Draw a triangle ABC. Bisect the angles. The bisectors meet at I. Draw IP perpendicular to BC. IQ, perpendicular to AC and IR perpendicular to AB. It is found that IP = IQ = IR with I as centre and IP as radius of circle is drawn. It touches all the three sides of the triangle. It is the incircle of the triangle ABC and I is the incentre.

[Note: An incircle of a triangle is drawn by bisecting any two angles of the triangle and a perpendicular drawn to a side of the triangle from the meeting point of the bisectors. The distance between this point and a side is the inradius.]




**Exercises**

1. Draw a triangle and construct an incircle. Measure the inradius. Write the construction.
2. Name the point which is equidistant from the sides of a triangle.

**Activity 1**

Draw an equilateral triangle ABC having AB = 8 cm. Draw circum circle. The point of intersection of the perpendicular bisectors of the sides is the centre of circum circle. Also draw the incircle - The point of intersection of the angle bisectors is the centre of incircle. In this case, you will find that both the centres coincide.

**Activity 2**

The circles having common centre are called concentric circles. Draw three concentric circles of radii 2, 3 and 5 cms.

# UNIT TEST

Time : 45'

Marks: 25

**I. Fill in the blanks** 1x6 = 6

1. The circum circle passes through the ______ of the triangle.
2. The circum centre is _________ from the vertices of the triangle.
3. The circum centre of the right angled triangle is at the _________ of the hypotenuse.
4. The incentre always falls ________ the triangle.
5. The incentre is equidistant from the ________ of the triangle.
6. In a right angled triangle the square on the hypotenuse is equal to the ________ of the squares of the other two sides.

II. 2x4 = 8

7. In a triangle ABC, BC = 5 cm and AB = AC = 4.5 cm. Construct the triangle and give the name of it.
8. Construct a triangle XYZ with XY = 6 cm, $\angle X = 60^\circ$, $\angle Y = 30^\circ$. Measure the third angle.
9. Draw a right angle $\angle PQR$ and bisect it. Write the construction.
10. Draw a line AB = 7 cm. Draw a perpendicular from a point 'O' located outside the line AB, using a ruler and compass.

III. 3x2 = 6

11. A vertical poll 12 mts high is at a distance of 5 mts from a peg on the ground. Find the length of the wire required to connect the top of the pole and the peg.

12. Draw a circum circle of the triangle ABC, given AB=5 cm, BC = 6 cm and AC = 7 cm. Measure its circum radius.

IV. 5x1 = 5

13. Construct a triangle PQR where PQ = 45 cm, QR = 8 cm, PR = 6.5 cm. Draw an incircle. Measure its inradius. Write the construction.

**Self Learning Activity**

Construct different triangles with different dimensions and draw an incircle or circum circle to each of them. Measure the radius in each case.

# CHAPTER XV

# STATISTICS AND GRAPHS

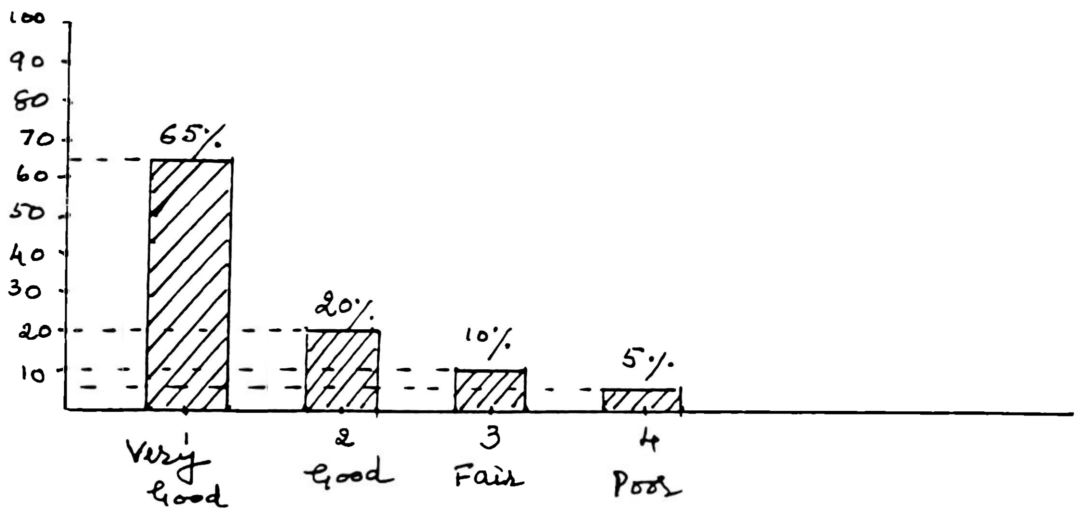
Preview

In VI standard you have already studied about:

(a) **Bar graph** is a way of representing the statistical data (information) wherein vertical rectangles are drawn on equal basis such that the measures of their areas are proportionate to these quantities. An example is the following.

**Problem**: The following table represents the rating of a certain TV programme. Draw a bar graph to represent the same. (Scale: 10% = 1 cm)

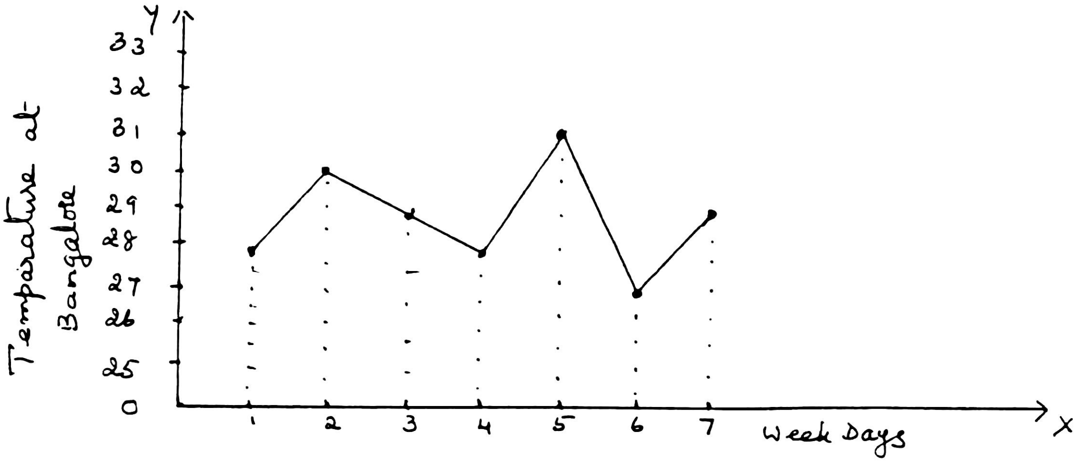
| Rating | Percentage |
|---|---|
| 1. Very good | 65% |
| 2. Good | 20% |
| 3. Fair | 10% |
| 4. Poor | 5% |



(b) **Line graph** is a way of representing the continuous data by joining the line segments. An example of line graph is given below.

**Problem**: Draw the line graph of the temperature (in centigrade) recorded at Bangalore on different days of a week.

| Days of the week | 1 Sun | 2 Mon | 3 Tue | 4 Wed | 5 Thu | 6 Fri | 7 Sat |
|---|---|---|---|---|---|---|---|
| Temperature | $28^{o}$ | $30^{o}$ | $29^{o}$ | $28^{o}$ | $31^{o}$ | $27^{o}$ | $29^{o}$ |




(c) **Sector graph** is a way of representing the statistical data by the sectors of a circle, the area of sectors being proportional to the quantities the total number represented by the area of the circle. An example is given below.

**Problem**: Vijay spends the time everyday (24 hrs) as follows.

1. Sleeping - 8 hrs
2. School and home work - 6 hrs
3. Play - 4 hrs
4. Other activities - 6 hrs

------

24 hrs

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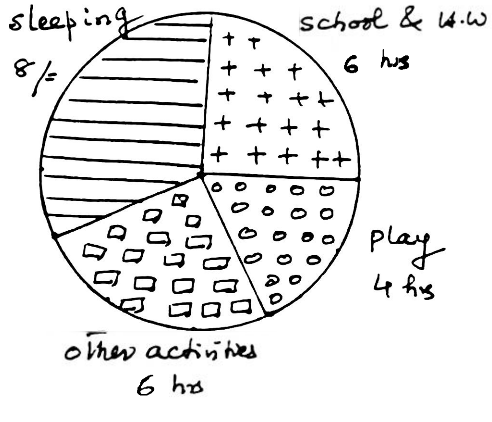


Sector graph is drawn.

Sector graphs are also called **Pie-charts**, as the quantity Pie ( $\pi$ ) is intimately connected with the properties of a circle.

**Review Exercises**

1. Draw bar graph to represent the following data (~~sub~~ scale 10 years = 1 cm)

Life expectancy of animals

| Name of animal | Length of life (years) |
|---|---|
| Elephant | 68 |
| Camel | 28 |
| Horse | 20 |
| Goat | 15 |

2. The marks scored by ten students in mathematics are given below.

| | | | |
|---|---|---|---|
| a. Latha | - 65 | f. Singh | - 35 |
| b. Ramesh | - 40 | g. Meera | - 60 |
| c. Vijay | - 50 | h. John | - 50 |
| d. Subhash | - 60 | i. Hamid | - 35 |
| e. Chandra | - 70 | j. Parimala | - 40 |

Arrange the marks in ascending order: Find out who has scored on the highest marks and draw a line graph.

3. In a class, number of children playing different games is given below. Draw a sector graph to represent the data.

| | | |
|---|---|---|
| a. Cricket | - | 60 |
| b. Football | - | 30 |
| c. Hocky | - | 20 |
| d. Other games | - | 10 |
| | | 120 |

## II. Introduction

Statistics is a study of collection, organisation, examination, analysis and interpretation of data. These data (or information) are collected from a sample (a small part) of a bigger set. Statistics helps us to predict about the bigger group by studying the data related to the smaller part (sample). It saves us from collecting the data for the whole of bigger group. Statistics is used in many fields like census, population, production and consumption and education.

In olden days kings collected data regarding the social and economic conditions of their subjects to fix up taxes and for other administrative purposes. This is described in Doomsday Book of England and in Arthashastra by Kautilya of India. In present days, the works of Prof. P.C. Mahalanabis and the Indian Statistical Institute in statistical research and its applications have earned international recognition.

## III. Concepts and Terms

Data means information.

Raw score is the data directly collected from observation of the sample.

Tabulation of data means putting the raw data collected in the form of a table as shown below.

Following are the marks obtained by 30 boys of a class in a test (put in a tabular form):1

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 14 | 7 | 5 | 10 | 7 | 12 | 9 | 1 | 5 | 13 |
| 12 | 11 | 3 | 19 | 18 | 14 | 13 | 17 | 15 | 10 |
| 17 | 4 | 13 | 19 | 12 | 11 | 16 | 8 | 10 | 13 |

The numbers of the data (here marks) collected directly from the sample are called raw scores. The above data in tabular form consists of raw scores.

In the data there are some scores (here marks) which occur more than once in the table. The total number of its occurrence in the table is called the frequency (denoted by f) of the score.

The frequency table is essential for the analysis of the data. It is a table which is prepared as follows.

Find out the highest and lowest scores in the raw data. The difference between the highest score (symbolically represented as H) and the lowest score (symbolically represented as L) in the group is called the range. For the above example of data (in tabular form).

Range = Highest score - Lowest score

$= H - L = 19 - 1 = 18$

Depending upon the size of the range, the scores are divided into groups. Such groups are called Class Intervals (i.e. C.I.).

For example, in the present case, we may have four groups starting from the lowest score 1.

| Class Intervals | Size of the C.I. |
|---|---|
| 1-5 | 5 |
| 6-10 | 5 |
| 11-15 | 5 |
| 16-20 | 5 |

Note that the class intervals are of equal size.

Frequency Distribution Table is a table (a rectangular array) which tells us how many scores in raw score table are from a particular class interval. For example, the frequency distribution table for the given raw score table is given below.

**Frequency Distribution Table**

| Class Interval (Scores) | Tallies | Frequency (f) |
|---|---|---|
| 1-5 | \|\|\|\| | 4 |
| 6-10 | ~~\|\|\|\|~~ \|\|\| | 8 |
| 11-15 | ~~\|\|\|\|~~ ~~\|\|\|\|~~ \|\|\| | 13 |
| 16-20 | ~~\|\|\|\|~~ | 5 |

Frequency distribution table consists of three vertical columns -- (1) Class interval, (2) Tallies and (3) Frequency (f). Class interval and frequency have already been explained. Now we will discuss, what are 'tallies or what is tallying.

<u>Tallying</u> is a counting process. In olden days a shephard used to keep a count of the sheep by tallying the number of sheep with the number of stones. To start with, he used to pick one stone from a stone heap from every sheep and used to keep them separately from the heap. Thus at the end of this process he used to have a collection of stones in which these were exactly the same number of stones as the number of sheep. While returning after grazing the sheep, the shephard used to throw one stone from his collection for every sheep. If at the end two stones remain in the collection, he will know that two sheeps are missing. If at the end the last stone thrown corresponds with the last sheep of the flock, it means that the number of sheep counted equals the number of stones. This process is called tallying sheep with stones. Similarly here we will tally every score counted with a vertical bar like | upto four scores and fifth score with a horizontal bar — across the four horizontal bars so that at the end of every fifth score counted we will have a tallying symbol like 卌. These tallying symbols are called tally marks or simply tallies. So now a count of 4 corresponds to |||| and that of 5 corresponds to 卌. Similarly counts of 8, 10 and 12 correspond with tallies as shown below.

<table>
<tr><th>Counts</th><th>Tallies</th></tr>
<tr><td>8</td><td>卌 |||</td></tr>
<tr><td>10</td><td>卌 卌</td></tr>
<tr><td>12</td><td>卌 卌 ||</td></tr>
</table>

Note: The number of tally marks against a class is always equal to the number of scores tally in that class interval.

For example, in the frequency table, we find against the class 1-5, the number of tally marks 4 = the number of scores. Looking at the data the scores tally in the class 1-5 are 5, 1, 3 and 4.

The mid-point of a class interval is supposed to represent the score of that class. For example, in the class interval 1-5, the scores are 1, 2, 3, 4, 5. The mid-point of this C.I. is $\frac{1+5}{2} = \frac{6}{2} = 3$. Similarly for the class interval 6-10, the mid-point is $\frac{6+10}{2} = \frac{16}{2} = 8$. We can find the mid-points of other C.Is. exactly in the same manner.

In the frequency distribution table, the highest frequency (13) is for the class interval 11-15. The mid-point of this C.I. is said to indicate the approximate average score of the entire group. So $\frac{11+15}{2} = \frac{26}{2} = 13$ is the approximate average score. This means that a large number of students have obtained around 13 marks in the group.

From the given frequency distribution table, the following conclusion can be drawn.

1. The standard of the group is above the average as 18 (13+5) students, i.e. more than half($= \frac{30}{2} = 15$) the students have scored more than the average score of 13 marks.
2. 12 (=30-18) students have scored less than the average score (of 13).

Pictograph

Pictograph is a way of representing the data by means of pictures, the number of pictures in each division being proportional to the relevant quantity in the data. For example, the sale of eggs by a merchant in 4 months (January to April) can be shown as follows:

| | | |
|---|---|---|
| January | 0 0 0 0 0 | One circle (egg) |
| February | 0 0 | represents |
| March | 0 0 0 | 500 eggs |
| April | 0 0 0 0 | sold. |

Pictograph of sale of eggs from January to April in 2000 if the sales are as follows: Jan - 2500, Feb - 1000, Mar - 1500, Apr - 2000.

## IV. Teaching Strategies

Most of the teaching strategies are incorporated in the explanation of concepts and terms in III.

Besides, the teacher can ask the students to look for graphs of sales of commodities in newspapers, temperature graphs of patients in hospitals and also for pie-charts in different life situations.

Wherever possible, the teacher should train the student in drawing the different types of graphs.

The following problem on frequency distribution table has been fully discussed in the section on explanation of concepts and terms.

**Problem**: Following are the marks scored by 30 boys of a class in a science test. Prepare a frequency distribution table and find the approximate average score.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 14 | 7 | 5 | 10 | 7 | 12 | 9 | 1 | 6 | 13 |
| 12 | 11 | 3 | 19 | 18 | 14 | 13 | 17 | 15 | 10 |
| 17 | 4 | 13 | 19 | 12 | 11 | 16 | 8 | 10 | 13 |

The explanation of the term 'tally' and 'tallying' is given in full detail in the same section.

**UNIT TEST**

Time : 45 mts
Marks: 25

1. Fill in the blanks 1x6 = 6

(a) To represent the temperature of a patient ________ graph is most appropriate

(b) The mid-point of the class interval having the highest frequency is called ________

(c) If the class interval is 71-75, the mid-point will be ________.

(d) ________ graph drawn is within a circle.

(e) The season rainfall of a town for successive four years is represented by ________.

(f) Size of class interval 71-77 is ________.

2. 2x2 = 4

(a) Find the approximate average score from the following frequency distribution table.

| C.I. | Frequency |
|---|---|
| 0-4 | 5 |
| 5-9 | 20 |
| 10-14 | 14 |
| 15-19 | 25 |

(b) Draw a pictograph to represent the sale of mangoes for the following table.

| Year | Number of mangoes sold |
|---|---|
| 1997 | 5 lakhs |
| 1998 | 2.5 lakhs |
| 1999 | 3 lakhs |
| 2000 | 10 lakhs |

3. 3x5 = 5

(a) The ages of 30 students (to the nearest year) are given below.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 9 | 12 | 14 | 9 | 8 | 12 | 12 | 19 | 13 | 14 |
| 13 | 14 | 18 | 16 | 13 | 14 | 11 | 17 | 13 | 16 |
| 17 | 15 | 17 | 13 | 9 | 13 | 9 | 10 | 20 | 10 |

Prepare a frequency distribution table and find the approximate average age.

(b) Draw a bar diagram using the following data:

| Year | Population of India in millions |
|---|---|
| 1951 | 360 |
| 1961 | 440 |
| 1971 | 550 |
| 1981 | 690 |
| 1991 | 840 |

(c) Draw a pie-chart (sector graph) to represent the following data.

| Name of the magazine | Circulation (Number of copies) |
|---|---|
| Sudha | 500 |
| Tharanga | 300 |
| Reader's Digest | 100 |
| Karmaveera | 300 |

**Suggested Enrichment Activities and Questions for Teachers**

1. Search for the different types of graphs in the newspapers, magazines and elsewhere. Make a list of what the graphs represent. Then try to investigate the reasons for the types of the graphs used for the different types of facts.
2. When you visit exhibitions, search for graphical representations of data. Design suitable problems for these graphical representations.
3. Can a data represented by line graph be represented by bar graph ? Give reasons for your answer.
4. Can a data represented by bar graph be represented by pie-chart ? Give reasons for your answer.
5. Draw charts for appropriate topics and use them to explain the concepts.

# CHAPTER XVI

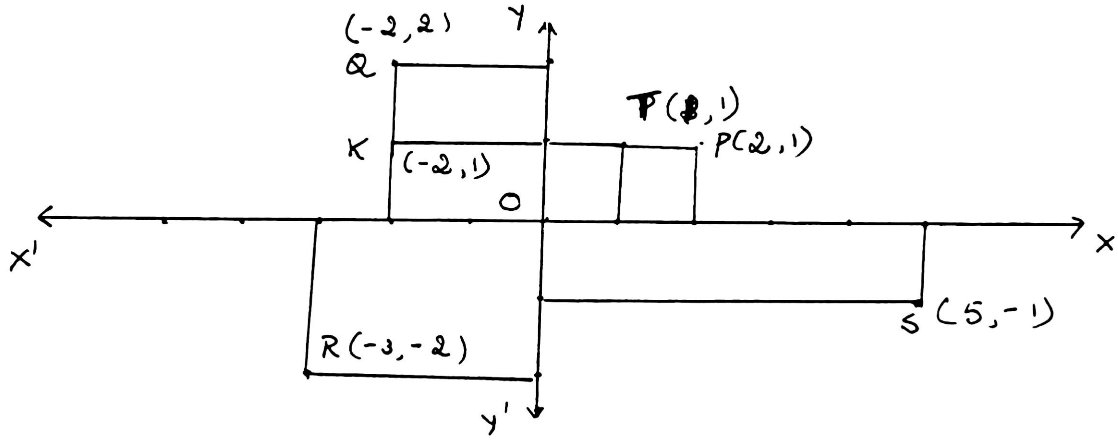
# COORDINATE GEOMETRY

## I. Preview

In standard VI you have already studied about

(a) coordinates- x coordinate and y coordinate of a point.

(b) Coordinate axes - x-axis and y-axis are denoted in the figure by mutually perpendicular straight lines xox' and yoy' at a point O called origin.




(c) Location (or coordinates of a point) is denoted by the ordered pair (a,b) where a and b denote the x- and y-coordinates of the point. An ordered pair (a,b) means that generally (a,b) is not equal to (b,a), i.e. the interchange of the variables a and b in (a,b) is not allowed. If (a,b) = (b,a) then a = b, If a $\neq$ b, then (a,b) $\neq$ (b,a). Here the order of the occurrence of the variables and a and b in (a,b) matter. For example, (2,-3) and (-3,2) represent different points. In the figure, P, Q, R, S are the points whose coordinates (2,1), (-2,2), (-3,-2) and (5,-1) respectively. The coordinates of the origin is always (0,0).

In chapter XI (Equations) you have also studied about (a) the linear equations of one variable of type $2x + 5 = 7$ or $-2x - 5y = 8$ and (b) the linear equations of two variables of the type $2x + 3y = 7$ or $2x - 4y = 10$.

**Review Questions**

1. Plot the following points on the graph paper — A(+5,-3), B(+4,-6), C(+2,+4), D(+2,+4).
2. Plot the following points on a graph paper and join these points in each case by a straight line.
   (i) (3,5), (0,4), (6,6)
   (ii) (0,0), (1,1), (2,2), (3,3)
   (iii) (1,-1), (2,-2), (-2,2), (3,-3)
3. Solve the simple equations
   (i) $2x + 5y = 7$
   (ii) $4x - 3 = 5$
4. Solve the simultaneous linear equations
   (i) $2x + 3y = 5$, $3x + 2y = 5$
   (ii) $5x + 3y = 7$, $5x - 2y = 12$

## II. Introduction

The invention of coordinate geometry is one of the greatest landmarks in the history of mathematics. The concept of number line was known to mathematicians even before Newton and DesCartes. But it took the genius of a mathematician like Rene DesCartes who used this concept of number line in formulation the subject of coordinate geometry, where the problems of pure plane geometry are translated into the language of algebra through the equations in two variables. For example, in coordinate geometry a straight line is represented by the linear equation $y = mx + c$ and a circle is represented by the equation $x^2 + y^2 + 2gx + 2fy + c = 0$.

Once we are acquainted with the coordinate geometry of 2-dimensions wherein only equations with two variables are used, it is possible to build successfully the coordinate geometry of 2,3 and higher dimensions by suitably increasing the number of variables in the equations. Coordinate geometry of higher dimensions are used in mechanics, relativity theory and social sciences like economics.

**III. Concepts and Notations**

The following concepts and terms have already been listed in the preview with examples:

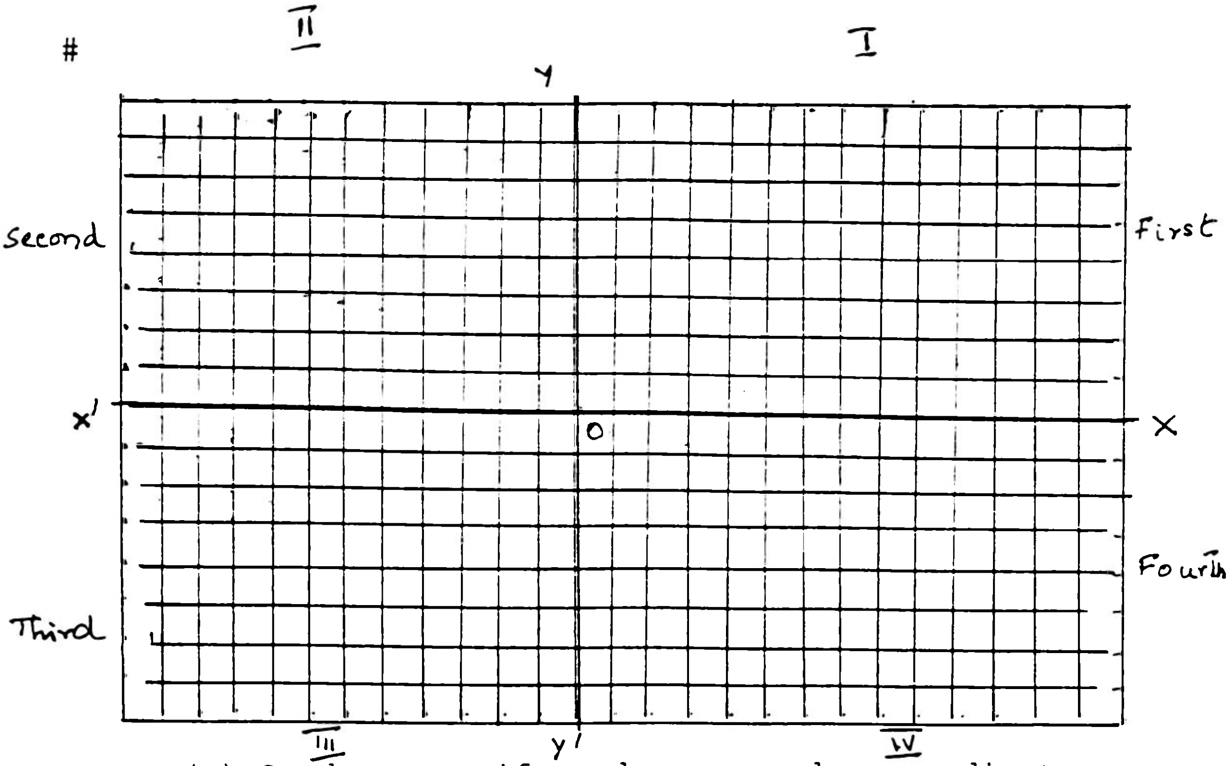
(a) Coordinates of a point - x- and y-coordinates, (b) coordinate axes - x- and y-axes, (c) location of a point.

Now we list the concepts to be studied in the present chapter.

(d) Graph sheet - Graph sheet is a sheet of a paper where the sheet is divided into equal small squares by two sets of parallel and straight lines*. The lines of one set is perpendicular to the lines of the other. A sample of graph sheet is given below.

(* adjascent parallel lines are separated by unit distance)

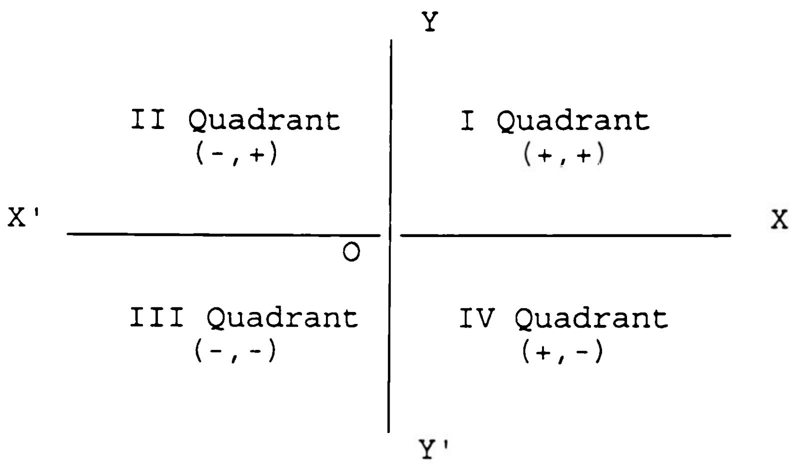
#




(e) Quadrants - if we draw x- and y-coordinate axes on the graph sheet, the sheet will be divided by the axes in four parts. These parts of the graph sheet are called four quadrants - first quadrant, second quadrant, third quadrant and fourth quadrant. Signs of coordinates of a point (x,y) with reference to the origin O and the axes x and y in four quadrant are as below.

| Quadrants | Sign of coordinates x | Sign of coordinates y | Signs of the coordinate x and y of the point (x,y) |
|---|---|---|---|
| I quadrant | + | + | (+,+) |
| II quadrant | - | + | (-,+) |
| III quadrant | - | - | (-,-) |
| IV quadrant | + | - | (+,-) |

In a graph sheet, the quadrants are situated as below.




Points on the x-axes and y-axes are not situated in any quadrant. For all points of the x-axis, the y-coordinate is zero. So in general, the point (a,0) is a point on the x-axis. Similarly, for all points of y-axis,the x-coordinate is zero. So the point (O,a) is a point on y-axis.

**f. Graph of the equation y = mx + c**

In coordinate geometry, every equation in x and y represents a line or a curve on a graph sheet (as mentioned earlier). In the equation y = mx + c, x and y are variables, and, m and c are constants. The numerical values of the constants m and c are known and they are fixed (do not change) for all the points of the line y = mx + c. The variables x and y can have any value. For the different combinations of values of x and y, here (x,y) will be different points on the straight line y = mx + c. The graph of the equation y = mx + c is the straight line representing the equation y = mx + c.

g. Satisfy

This term is best explained by the meanings of this term in the sentences where it is used in the mathematical context. Consider the two sentences

(i) 2 satisfies the equation $2x + 2 = 6$.

(ii) Coordinates of a point A satisfy the equation $x+y = 5$.

In the first case (i) the meaning is that 2 when substituted for x in the equation $2x + 2 = 6$, L.H.S. and R.H.S. become equal. Similarly in (ii) it means that substituting the values of the coordinates of the point A in $x + y = 5$ will make L.H.S. = R.H.S.

**h. Linear equation**

An important reason for $y = mx + c$ being called a linear equation is that it represents a straight line. The word 'linear' is an adjective derived from the word 'line'.

**IV. Teaching Strategies**

The subject of coordinate geometry basically depends upon the number line. So in the beginning the teacher should emphasise to the student that every point on the number line corresponds to a unique number (positive, negative or zero) and vice versa, every number (positive, negative and zero) corresponds to a unique point on the number line. This can be done by the teacher without using the term 'real'.he can, for example, take different rational numbers or numbers with fractional part (in decimal) and ask the students to plot them on the number line, or, he can take a point on the number line and ask the students identify the corresponding number. In fact, while identifying a point on the graph sheet or plotting a point P(a,b) on the graph sheet this is the principle involved. The teacher should purposefully avoid the uses of words 'real' and 'irrational'.

Then the teacher can ask the students to plot the points on the graph sheet and ask in which quadrants these points are situated.

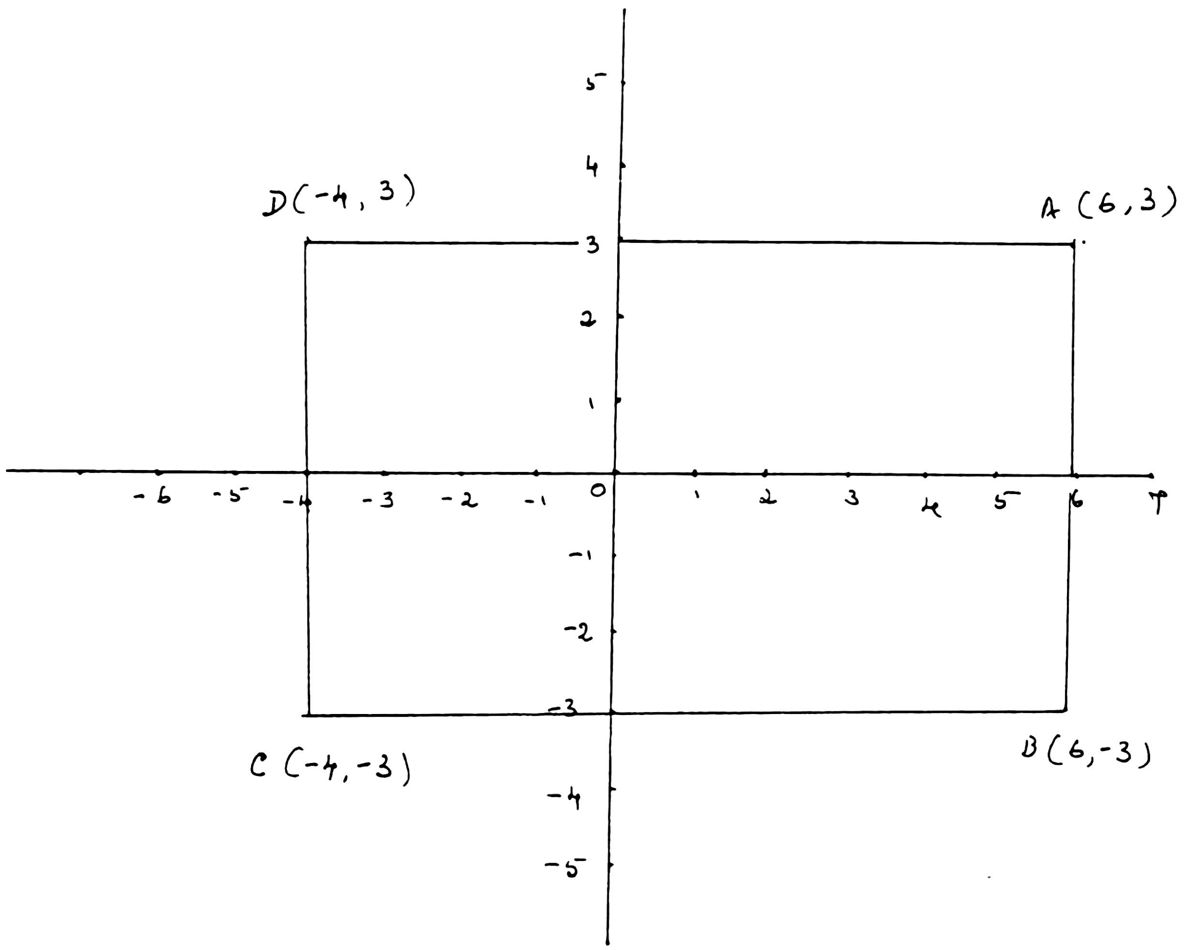
He can also give the try activity given in this chapter of the text book to the students. Also, he can devise more similar activities.

Though the terms 'abscissa' and 'ordinate' are used in a problem of the text, they have not been defined in this chapter or in earlier texts. So the teacher should make it clear to the students that x- and y- coordinates of a point are called its abscissa and ordinate respectively. For example the abscissa and ordinate of the point (3,-6) are 3 and -6 respectively, i.e. for the point (3,-6), its abscissa is 3 and its ordinate is -6.

Now we will solve some problems to illustrate how to draw a figure on a graph sheet as well as to show how to draw the graph (which is a straight line) of a linear equation in 2 variables x and y.

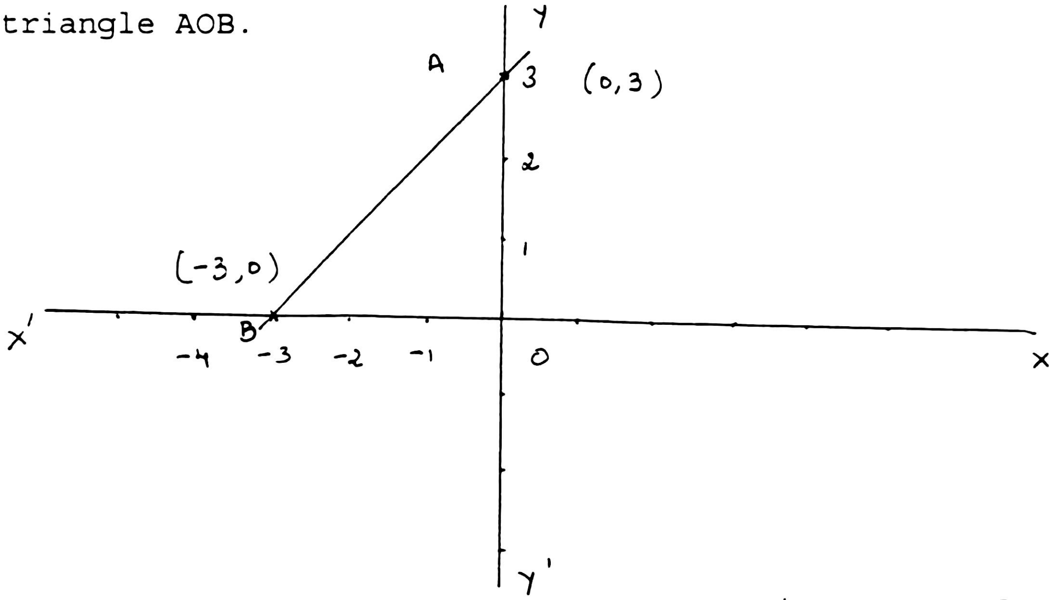
**Problems**

1. On a graph sheet, draw the rectangle ABCD whose vertices are A(6,3), B(6,-3), C(-4,-3) and D(-4,+3).




Note: Q.No. 6 of exercise 16.1 is wrong. The problem solved here is the correct version of it.

2. Draw the straight line AB on the graph, given the coordinates of A(0,3) and B(-3,0), and name the type of triangle AOB.




Here OA = 3 units, OB = 3 units and $\angle AOB = 90^{\circ}$. So triangle AOB is a right angled isosceles triangle.

**Graph of a linear equation**

Drawing the graph of the linear equation y = mx + c is best explained below.

**Problem**: Draw the graph of the linear equation 10x - 5y = 20

**Solution**: First, bring this equation to the form y = mx + c

$10x - 5y = 20$

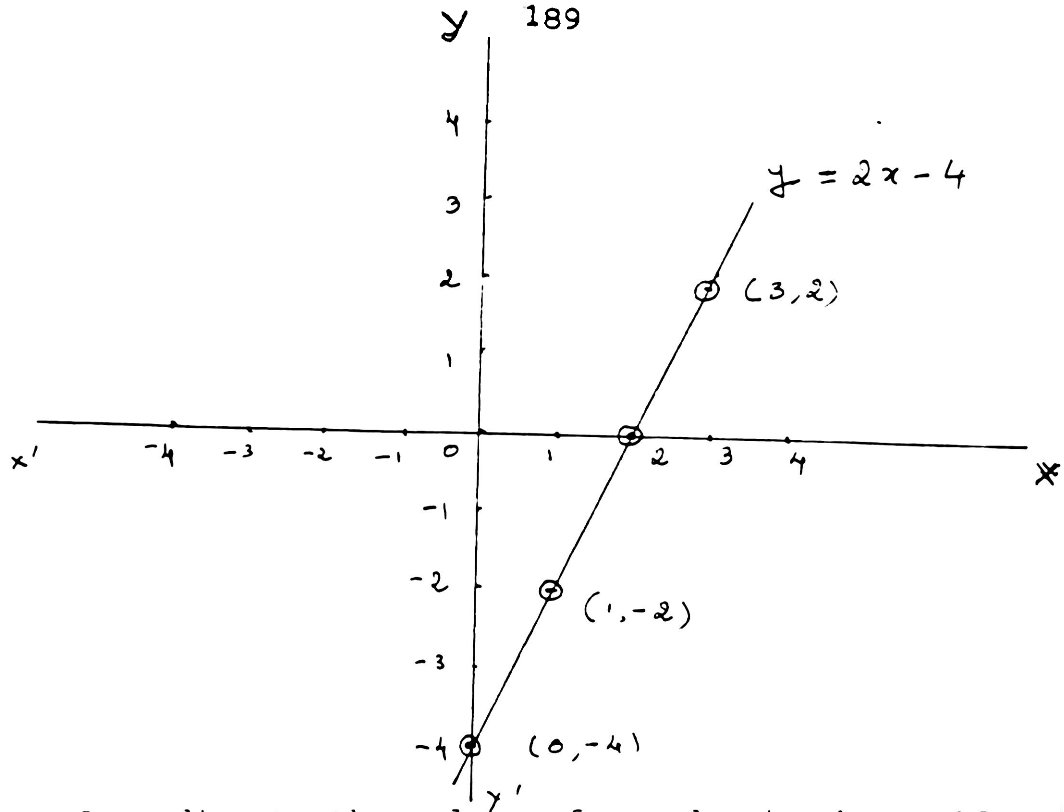
$-5y = 20 - 10x$ by transposing 10x to R.H.S.

$y = 2x - 4$

We will draw a table (list) of corresponding values of at least two points of the graph satisfying this equation, i.e. y = 2x - 4.

| x | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| y | -4 | -2 | 0 | 2 |

#




According to the values of x and y in the table four points on the graph are (0,-4), (1,-2), (2,0), (3,2), we can draw a line through these four points and this line will be the graph of the given equation.

Note that here the given equation 10x - 5y = 20 has been brought to the form y = mx + c, i.e. y = 2x - 4.

**Solving two simultaneous equations by graphical method**

Two simultaneous linear equations in x and y will represent in general two different straight lines, as a linear equation in x and y represents a unique line. All the points on a single line will satisfy the linear equation representing that line. If the two straight lines intersect at a point P, then the values of the coordinates of P will satisfy both the linear equations. This is because the point P lies on both the straight lines. Thus the common values of x and y for the two simultaneous equations in x and y are given by the values of the coordinates of P which is the point of intersection of the two lines.

The above concept will be more clear by the example given below.

**Problem:** Solve graphically and verify the answers

$$3x - y = 7$$

$$x + y = 5$$

**Solution:** First of all, we have to bring the equations in the form $y = mx + c$.

Given equations are

$$3x - y = 7 \qquad \ldots\ldots (1)$$

$$x + y = 5 \qquad \ldots\ldots (2)$$

(i) $3x - y = 7$

$-y = 7 - 3x$

$y = 3x - 7 \qquad \ldots\ldots (3)$

The corresponding values of x and y for (3) are given in the following table

| x | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| y | -7 | -4 | -1 | 2 |

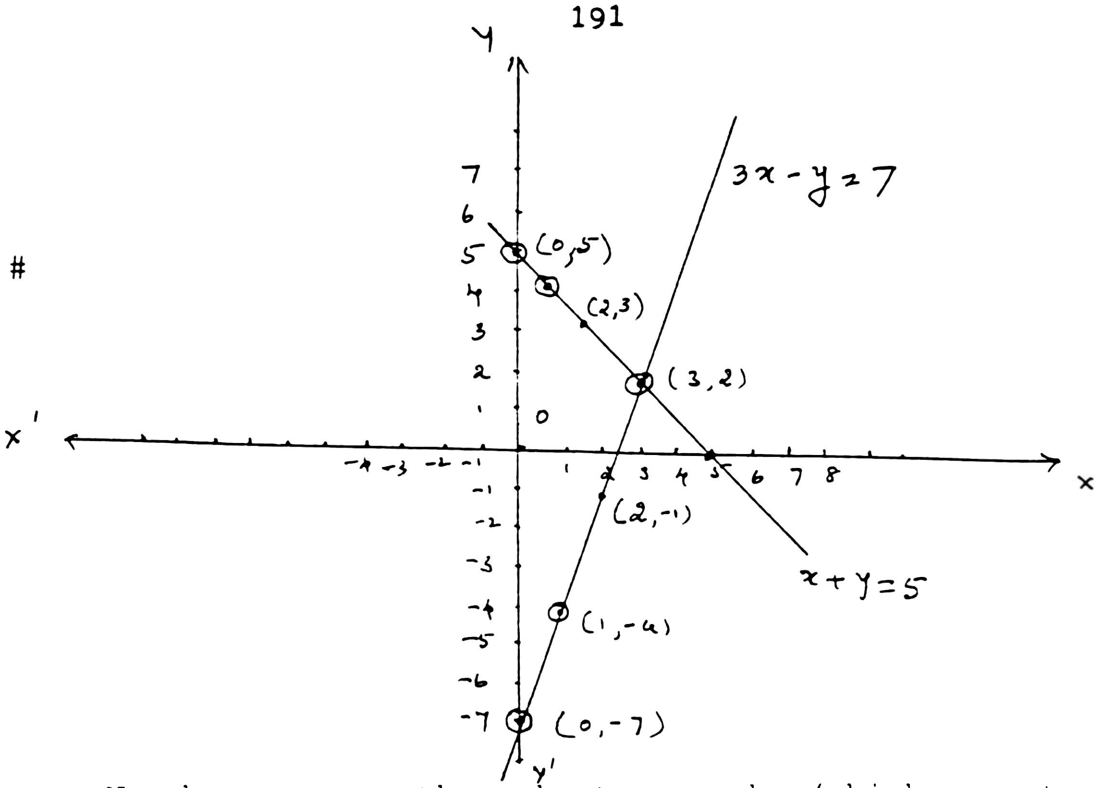
(ii) $x + y = 5$

$y = -x + 5 \qquad \ldots\ldots (4)$

The corresponding values of x and y are for (4) are given in the following table.

| x | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| y | 5 | 4 | 3 | 2 |

We plot the points of the two lines represented by (3) and (4) in the same graph sheet and identify the point of interaction of the lines.




Now here we see that the two graphs (which are straight lines) intersect at the point P whose coordinates are (3,2). So x = 3, y = 2 are the common values for the two given linear equations.

Verifying an answer here means that common values of x and y when substituted for x and y respectively should make L.H.S. = R.H.S. for both equations.

So we do this verification here. Common values here are x = 3, y = 2.

(1) is $3x - y = 7$

$3 \times 3 - 2 = 7$ (substituting for x and y)

$9 - 2 = 7$

$7 = 7$

L.H.S. = R.H.S. for (1)

(2) is $x + y = 5$

$3 + 2 = 5$ (substituting for x and y)

$5 = 5$

L.H.S. = R.H.S. for (2)

So the common values of x and y for (1) and (2) are verified to be correct.

## Word Problems

For solving a word problem graphically,

(i) we have to express the conditions of the problem as two linear equations in x and y.

(ii) Then we have to solve these two equations graphically.

This is illustrated by the following problem.

**Problem**: The sum of the present ages of Sunil and Shrikant is 32. Four years ago Sunil was twice as old as Shrikant. Find their present ages by graphical method.

**Solution**

Let the present age of Sunil = x

and the present age of Shrikant = y

Thus the conditions of the problem lead to the equations

$$x + y = 32$$

$$x - 4 = 2(y-4)$$

(i) $x + y = 32$

$$y = - x + 32 \qquad ..... (1)$$

(ii) $x - 4 = 2(y-4)$

$$2y = x - 4$$

$$y = \frac{x}{2} + 2 \qquad ..... (2)$$

Now draw the graphs of (1) and (2). Here for (2) we see that the value of y will be fractional if that of x is odd. But if x is even number, for (2), y is an integer so for (2) consider the values of x corresponding to x = 0, 2, 4, so the corresponding values of x and y will be as follows.

For (1), table is as follows, the equation being $y = -x + 32$

| x | 0 | 2 | 4 |
|---|---|---|---|
| y | 32 | 30 | 28 |

For (2), where the equation $y = \frac{x}{2} + 2$

| x | 0 | 2 | 4 |
|---|---|---|---|
| y | 2 | 3 | 4 |

Here one has to take on small square on the graph sheet to accommodate the larger values of x and y (like 30, 31, etc.). Now the two graphs (the lines) can be drawn as in the previous problem. Then the point of intersection has to be identified which in this case is (20,12). So according to problem the ages of Sunil and Shrikant are 20 years and 12 years respectively.

## UNIT TEST

Time : 45 mts
Marks: 25

Each question carries 5 marks.

1. Plot the following points on graph sheet A(5,-3),B(4,-6), C(-4,-3), D(2,4). Also mention to which quadrants they belong.

(1x4)+1 = 25

2. Draw on graph sheet the quadrilateral whose vertices are A(2,2), B(-2,2), C(-2,-2), D(2,-2). What type of quadrilateral is this ?

1x4+1 = 5

3. (a) What are the coordinates of the origin ? 1x5 = 1
   (b) WHat is the equation of x-axis ?
   (c) State whether (2,2) lies on the line represented by x+y = 1.
   (d) What are the coordinates of the point on x-axis, four units to the right of x ?
   (e) The point (2,4) -- 2x + y = 8. Find the ________.
4. Solve graphically

$$x + y = 5$$

$$2x - 3y = 0$$

5

5. The sum of two numbers is 3 and their difference is one. Find the numbers by graphical method. 5

Suggested Enrichment Activities and Questions

1. Draw on graph sheet a straight line which is at a distance of 4 units from x-axis. How many straight lines are there with this property ?
2. Draw on graph sheet a straight line no point of which lies on 2nd and 3rd quadrants. How many such lines are there ? What are its relationships with x- and y-axes ?
3. Draw on graph sheet which is inclined equally to both the axes and passes through the origin. How many such lines are there ?
4. Draw on graph sheet 3 straight lines all of which are equally inclined to the x-axis ? What is the relationship between any two of them called ?
5. Select suitable topics amenable to be taught by charts. Prepare charts for them and explain how these charts can be used in classroom or otherwise.