"RESOURCE MATERIAL ON MATHEMATICS LABORATORY APPROACH TO TEACHING OF MATHEMATICS AT THE UPPER PRIMARY LEVEL (ANDHRA PRADESH AND PUDUCHERRY)"

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CONTENT LIST

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MATHEMATICS LEARNING THROUGH ACTIVITIES

Mathematics Laboratory

Introduction

Initiation to mathematics and mathematical concepts at the Primary and Secondary Stages through physical activities with the help of observation and doing is desirable. Mathematical abstraction and rigour can wait till the thinking faculty develops. Learning mathematics by seeing and doing using material objects available around makes the process enjoyable and facilitates understanding the mathematical concepts and results. A feel of the mathematical ideas is imperative and should precede proof in mathematics. Conjecture and verification, inductive thinking are the stepping stones leading to heights of rigour, abstraction and realizing mathematics as a discipline.

In science laboratories, scientific principles are learnt through verifications by experiments. Likewise a mathematical laboratory must provide a hands-on experience and exposure to mathematics. Thus the purpose of mathematical laboratory is to provide a forum for seeing mathematical facts to believe them. Accordingly, we need to plan the activities, keeping in mind the mental ability of the learner, his/her age and background.

Stages of Planning :

Firstly, identify the concepts and results topicwise and sequentially. The sequence is determined by the logical links connecting the concepts in the long chain of mathematical development.

Secondly, one or more activity has to be designed with a flexible format. Identifying different techniques to be used suitable to the activity is the next thing.

Objectives of the activity and the methodology with procedural details, identifying the materials need for the construction of the model to be used, etc. must follow.

Reinforcement is to be ensured with suggested exercises and assignments.

Thus a format for an activity could be

1. **Topic - concept / result**
2. **objective**
3. **Activity**
   - a) **Pre-knowledge required**
   - b) **Material needed**
   - c) **Construction / Designing of the activity**
   - d) **Methodology - how to conduct the activity**
4. **Conclusion**
5. **Additional Assignments**
6. **References**

Lists of Different activities and Construction of Models (material)

1. Diagrams/ Pictures/ Graphs - with suggested animation.
2. Paper folding experiments
3. cut and paste activities
4. Material models for plane and space related ideas (i.e. plane figures and solids) - using strings - needle like objects, pin, wooden/ paper contents.
5. Geoboard : Take a large square. Fix nails vertically and horizontally separated by unit distances. Passing a rubber band same nails so that the bank is tight, we get different types of polygonal figures.
6. Parallel lines board

Activity 1

Title - Equation Balance

Objective:

(i) To show that if we add or subtract the same number to both sides of equality, it still holds.

(ii) To understand the procedure of solving equations.

Material needed: A balance some weights (say 1, 2, 4, 5, 10, ... etc) and some stones.

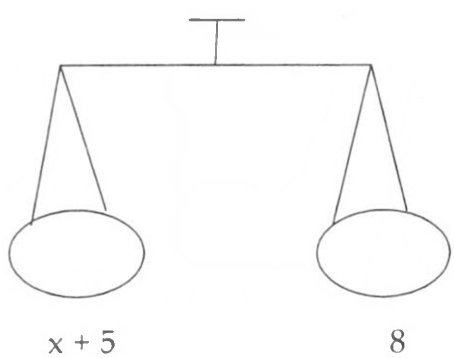
Description An equation is like a weighing balance. Doing a mathematical operation on an equation is like adding or removing weights from the pans of the weighting balance.

Consider the equation,

$$x + 5 = 8$$

Put a stone of weight x (unknown weight) and 5 grams to the left side pan (LHS) and put 8 grams to the right side pan (RHS).

Check that the beam of the balance is horizontal. If not horizontal, odd or subtract some weights from the RHS and keep the balance horizontal.



In order to find the value of x in the equation we have subtract 5 from both sides.

Remove 1 gram from each pan of the balance (both LHS and RHS). You can see that the beam is horizontal.

$$x + 5 - 1 = 8 - 1$$

Remove one more gram from each sick still the beam is horizontal

$$x + 5 - 2 = 8 - 2$$

Continue the procedure till you get

$$x + 5 - 5 = 8 - 5$$

Now also the beam is horizontal. But there is the stone of unknown weight is LHS and there are only 3 grams on the RHS pan.

$$x + 5 - 5 = 8 - 5$$

$$\text{i.e. } x = 8 - 5$$

$$= 3$$

This is the solution of the equation.

Verify that the weight of the given stone is 3 grams by weighing in an electric balance.

During this procedure, show the children that,

(1) If we add or subtract the same weights from both the pans, the beam will be still horizontal. In other wards, if we add or subtract the same number to both sides of on equality, it still holds.

(2) Repeat this procedure and show the children the steps involved in solving equations.

For Example:

$x - 7 = 15$

Add 7 to both sides

$x - 7 + 7 = 15 + 7$

$x = 15 + 7$

$= 22$

Therefore $x = 22$ is the answer.

Verify this value by substituting $x = 22$ in the original equation (both LHS and RHS should be equal).

(3) For solving equations of the type,

$$3x + 4 = 19$$

You will have keep three stones of equal weights and 4 grams in LHS 19 grams on the RHS and make the beam horizontal.

$$3x + 4 = 19$$

Add – 4 to both sides

$$3x + 4 - 4 = 19 - 4$$

$$3x = 15$$

Divide both sides by 3

$$\frac{3x}{3} = \frac{15}{3}$$

Therefore verify the answer by putting x = 5 in the original equation.

Precautions:

Any stone + 5 grams in the LHS will not balance (horizontal) with 8 grams in the RHS. The teacher has to keep the appropriate stone between showing it to the children.

Open Questions:

1) How do you show the solutions for the equation 4 (x + 3) = 16 using balance?
2) How do you explain 3 (x – 5) = 21?

Activity 2

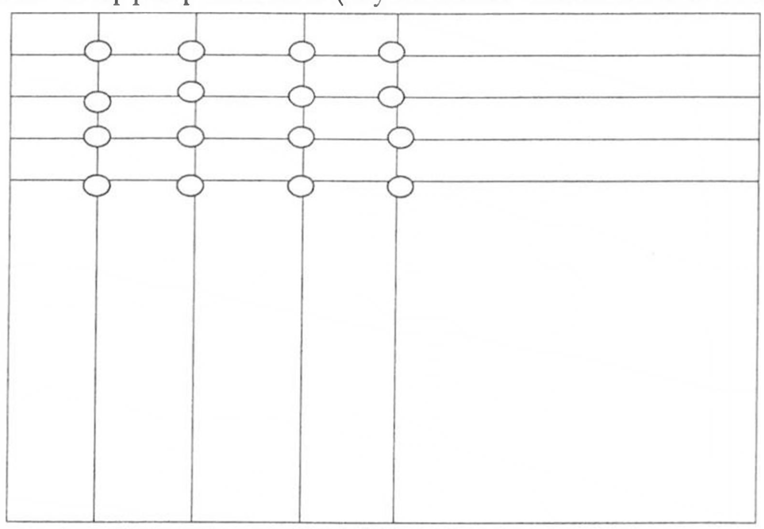
Title - Expansion of the identity $(a + b)^2$.

Objective:

To show that the expansion of the identity $(a+b)^2$ is equal to $a^2 + b^2 + 2ab$.

Materials needed:

A beads plate of appropriate size (say 20 beads & horizontal and 209 vertical)



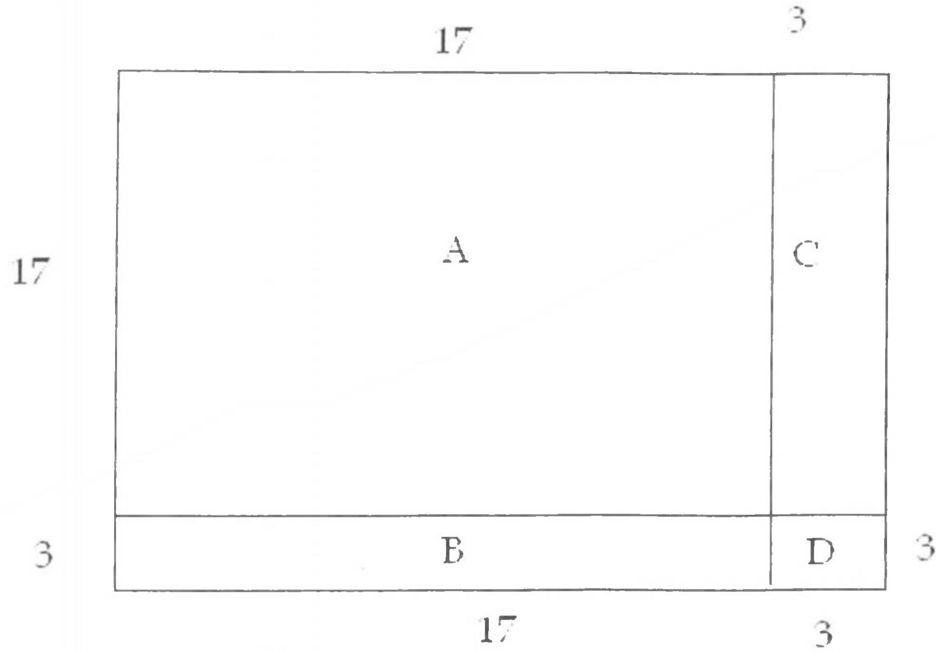
20 x 20 grid

and two white wooden scales to fix in the board as separators.

Description:

Let a = 17 and b = 3 (here take a and b such that a + b = 20 for convenience)

Fix a white scale horizontally after 10 beads, another white scale vertically after 10 beads as shown



Now the beads plate is divided into four parts as shown in the previous figure. One can see that

A contains 17 x 17 beads
B contains 3 x 17 beads
C contains 17 x 3 beads
D contains 3 x 3 beads

It can be seen the area of the complete board is equal to 20 x 20 beads.

$$
\begin{aligned}
(a + b)^2 = \text{total area} &= \text{Area A} + \text{Area B} + \text{Area C} + \text{Area D} \\
&= (17 \times 17) + (3 \times 17) + (17 \times 3) + (3 \times 3) \\
&= (a \times a) + (b \times a) + (a \times b) + (b \times b) \\
&= a^2 + ba + ab + b^2 \\
&= a^2 + 2ab + b^2
\end{aligned}
$$

Again this procedure can be repeated by changing the values of a and b.

<u>Note:</u> The above experiment is only a verification and not a proof of the result. The above result can also be proved by the simple multiplication.

$$
\begin{aligned}
(a + b)^2 &= (a + b)(a + b) \\
&= a(a + b) + b(a + b) \\
&= a^2 + ab + ba + b^2 \\
&= a^2 + 2ab + b^2
\end{aligned}
$$

<u>Open Questions:</u>

(1) Explain the procedure of verification when $a = 7$ and $b = 12$

(2) Gove a general proof without taking a beads plate, but taking only $(a + b)^2$ as the area.

Activity 3

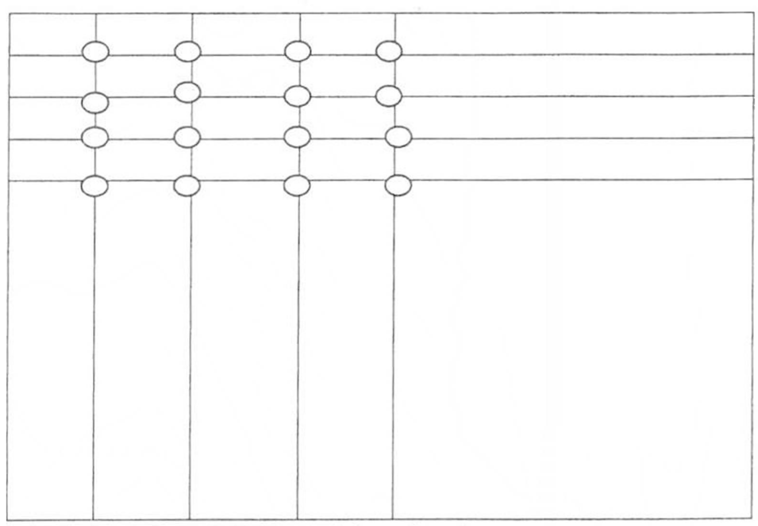
Title - Expansion of $(a - b)^2$

Objective:

To show that the expansion of the expression $(a - b)^2$ is equal to $a^2 - 2ab + b^2$

Materials needed:

A beads plate of size says 20 x 20 grids and two wooden scales for fixing in the board as separators.

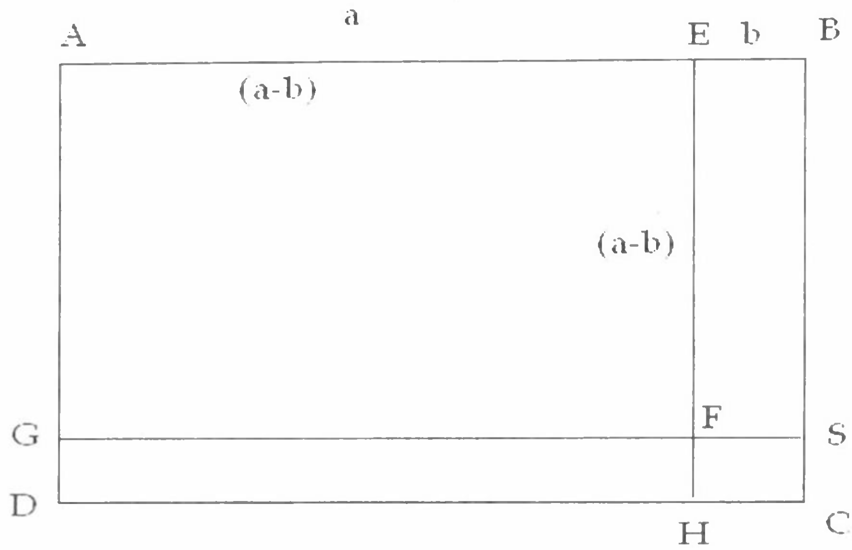


20 x 20 beads plate

Procedure:

Let a = 20 and b = 5

Fix a white scale at b = 5 beads on one side row wise and column wise as shown.



(a = 20 & b = 5)

Area of the square ABCD	=	a^2 (say 20 x 20)
Area of the rectangle EBCH	=	ab
Area of the rectangle GDCS	=	ab
Area of the square FHCS	=	b^2
Area of the square AEFG	=	$(a - b)(a - b)$
	=	$(a - b)^2$

One can see from the diagram that

Area of AEFG	=	Area of ABCD - Area of EBCH + Area of FHCS - Area of GDCS
$(a - b)^2$	=	$a^2 - ab + b^2 - ab$
Therefore $(a - b)^2$	=	$a^2 - 2ab + b^2$

This result can be verified by counting the number of beads contained in the respective areas.

<u>Open questions:</u>

1) Explain the procedure of verification of the identify when a = 17 and b = 4
2) Take the strips of areas a^2, ab, ba, b^2 and verify the result by joining them together.

Activity 4

Title - Expansion of the expression $a^2 - b^2$

Objective:

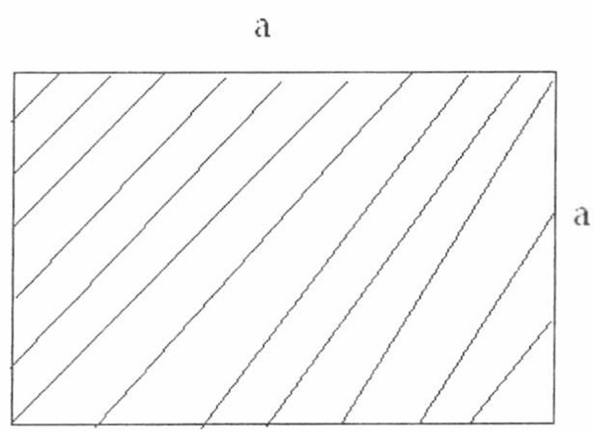
To show that $a^2 - b^2$ is equal to $(a - b)(a + b)$ [To find its physical significance]

Materials needed:

Chart papers, scales, scissors.

Procedure:

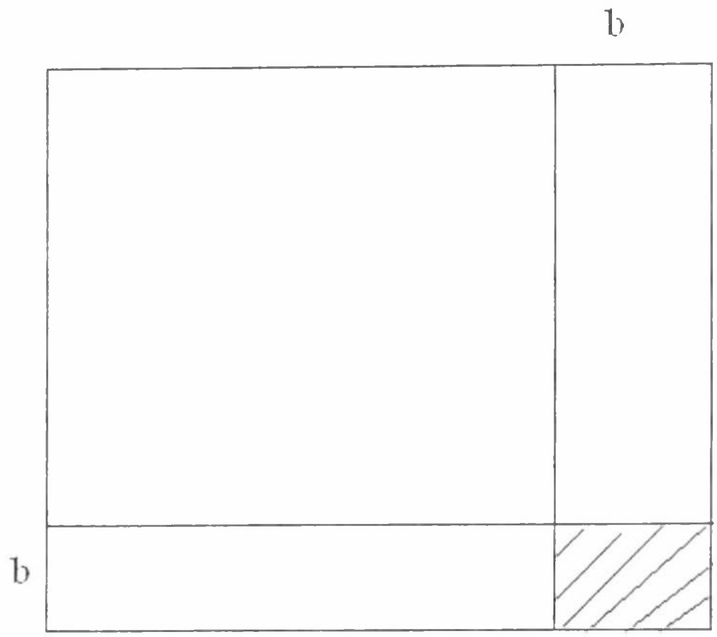
Take a square paper (coloured) of side length a (say a = 20)



Its area = a^2 sq units.

Draw a strip of width **b** and length **a** in one side horizontally.

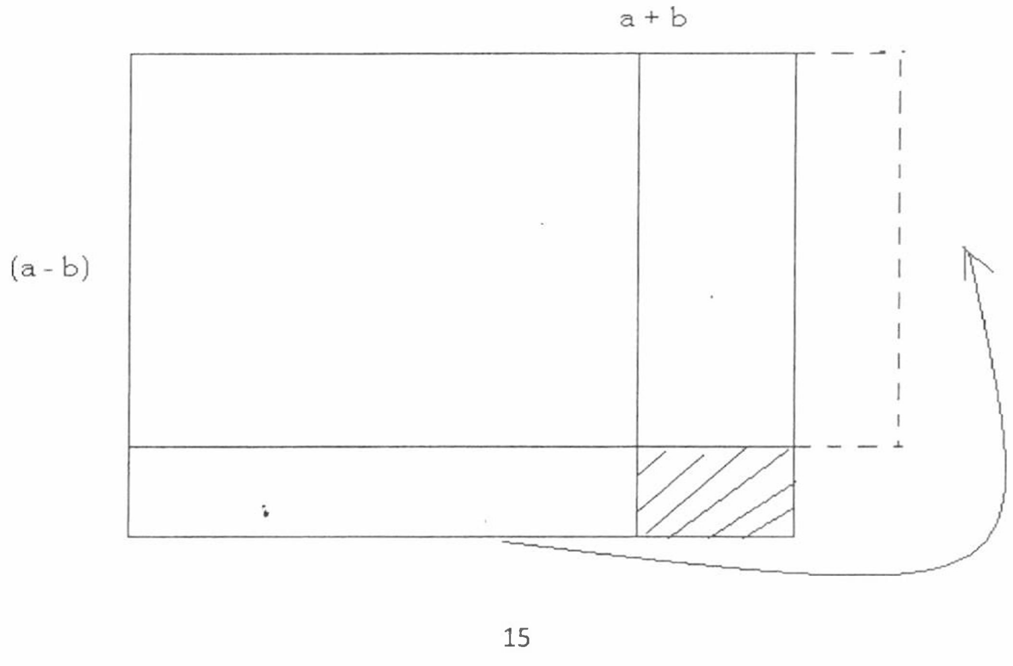
Draw another strip of width **b** and length **a** units in another side vertically as shown below.



Total area = a^2
Shaded area = b^2

There Unshaded area = $a^2 - b^2$

Now to find the unshaded area, remove one strip of width **b** and length a (horizontal) and keep it by the side of the other strip (vertical) as shown (by dotted lines)



Now we get a rectangle of length a + b and width (a – b). Its area being equal to $a^2 - b^2$.

So we have shown that $a^2 - b^2 = (a + b)(a - b)$

Open questions:

1. Verify this result by the actual calculations of the rectangles (by measurement)
2. Repeat the experiment by changing the values of a and b
3. Try to get a similar result for $a^3 - b^3$
4. Show a relation between $(a + b)^2$ and $(a - b)^2$ in the similar way.

Activity 5

Title – What is the next number?

Objective:

A Mathematical puzzle to find the next numbers in the sequence 100, 121, 144, 202, 244, ____, ___, ____?

Description:

It is neither in A.P or G.P, because the differences between the terms are 21, 23, 58, - - - etc. Therefore we look for a different system (i.e. different bases).

Look at,

100 in base 10 is equal to 100 in base 10
121 in base 9 is equal to 81 + 18 + 1 = 100 = 100 in base 10
144 in base 8 is equal to 64 + 32 + 4 = 100 in base 10
202 in base 7 is equal to 98 + 0 + 2 = 100 in base 10
244 in base 6 is equal to 72 + 24 + 4 = 100 in base 10

Hence it is clear that 100 in base 10 is expressed a

100	121	144	202	244	____,	____.	_____
(base 10)	(base 9)	(base 8)	(base 7)	(base 6)	?	?	?

Hence the next number in the sequence can be found by converting 100 in base 10 to base 5 system.

$100 = 4 \times 5^2 + 0 \times 5^1 + 0 \times 5^0$

Hence 100 base 10 is equal to 400 in base 5 therefore the next number in the sequence is 400.

Open questions:

1) Find the next three numbers in the above sequence.
2) There are only 9 terms in this finite sequence why not more?
3) Can you express 2 in terms of base 1?

Activity 6

Purpose : To explain the process of finding the volume of certain solids like cube, cuboid, cylinder

Objective:

1. To explain the concept of volume of a solid
2. To explain the concept of unit cube
3. To define the right circular cylinder
4. To derive the formulae to find the volumes of cube, cuboid, right circular cylinder.

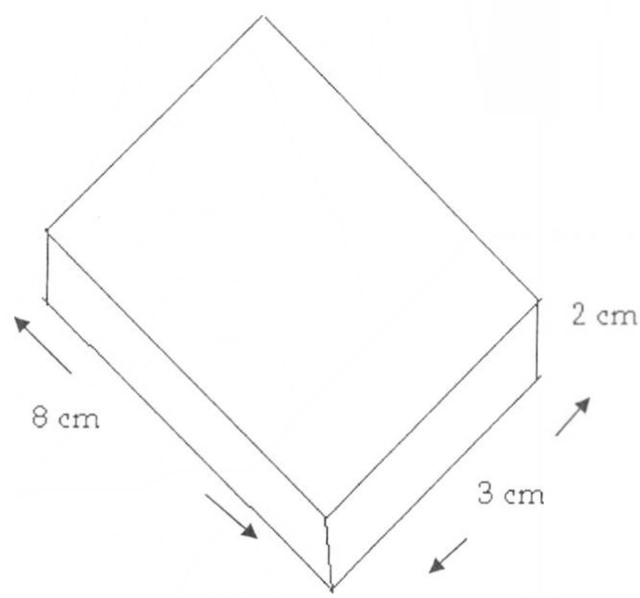
Materials Required: Solids like cube, cuboid, right circular cylinder, ordinary circular cylinder, polyhedron cylinder

Activities:

- Explain that the amount of space occupied by a solid object is known as its volume.
- Ask the students to recall the area of a region and to try them to extend that concept to find the volume.
- Define the unit cube by showing that object as the object which is in the shape of a cube having 1 unit side
- Demonstrate that the volume of any solid is equal to the number of unit cubes required to cover the entre space occupied by that solid
- Ask the students to find the volume of a cube with side 3 units and also let them demonstrate it with unit cubes.
- Guide the students to derive the formula for volume of a cube as l^3 (where l is the length of the side)
- Consider cuboid solid made of unit cubes
- Ask the students to divide the above cuboid into unit cubes. The number of unit cubes so obtained is known as its volume.
- Ask the students to relate the volume with the sides and height of the cuboid and to derive the formula as l b h.

- Explain the process of dividing the cuboid into smaller cuboids with unit height and the volume of each smaller cubiod is l b and hence the total volume of the cylinder as l b h.
- Show right circular cylinder tin and ask the students to find the amount of water required to fill that tin.
- Guide the students for this they need to find the volume of the cylinder
- Ask the students to divide the cylinder into smaller cylinder of unit length.
- Guide the students to know that the volume of each smaller cylinder is equal to its base area and it is equal to $\pi r^2 + \pi r^2 + \ldots\ldots$ h times = $\pi r^2 h$.
- Make the students to note the formula for calculations of the volume of a right circular cylinder is equal to $\pi r^2 h$.
- Guide the students to solve the following problems

i) Find the volume of the following Cuboid



ii) Water is pouring into a cubiodal reservoir at the rate of 60 liters per minute. If the volume of reservoir is 108 m^3, find the number of hours it will take to fill the reservoir.

iii) A milk tank is in the form of cylinder whose radius is 1.5m and length is 7m. Find the quantity of milk in liters that can be stores in the tank?

Activity 7

Purpose: To explain the process of calculating total surface area and lateral surface area of some solids

Objective:

1. To define the faces of a solid
2. To identify the faces of a given solid
3. To define the total surface area and lateral surface area of a solid.
4. To derive the formula for total surface area and lateral surface area of the solids like cub, cuboid, cylinder

Materials Required: Solid objects like cube, cuboid, cylinder, some paper sheets, scissors

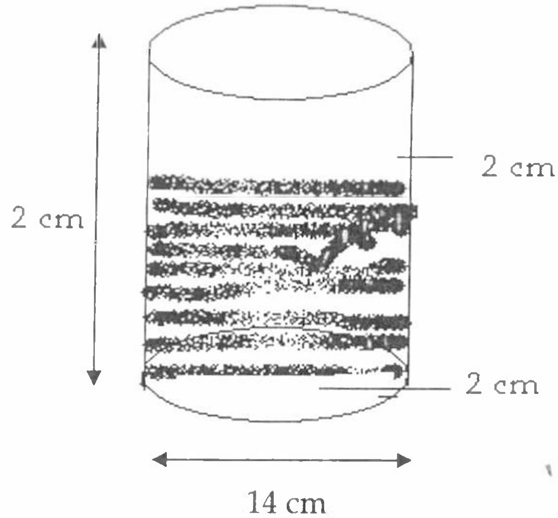
Activities:

- Show to the students some plane figures and some solid objects and ask them to identify the solids
- Ask to give the characteristics of a solid
- Ask the students to identify the faces of those solids
- Ask the students to identify the faces of a circular cylinder
- Ask the students to recall the properties of circular cylinder - Two parallel circular faces and other circular face
- Explain that the right circular cylinder is the cylinder where the line joining the centers of two circular faces is perpendicular to the base.
- Explain the concept of surface area by showing objects like cube, cuboid, cylinder
- Guide the students to find the surface area of a cuboid as 2 (lb + bh + hl).
- Ask the students to derive the formula for the surface area of a cube using the formula for surface area of a cuboid.
- Explain some of the daily life situations where we need to calculate surface area. For example, the paper required to paste around a cubical box.
- Explain the bases of some solids like cube, cuboid, cylinder
- Explain the lateral surface area and define it.
- Explain the difference between the total surface area and the lateral surface area of a solid.

- Guide the students to derive the formula for lateral surface area of a cuboid as $2h\ (l + b)$ and of a cube as $4\ a^2$
- Show some solids which are in right circular cylinder shape
- Explain that you need to cover the faces of this cylinder by a paper. How much paper I need to do this?
- Guide the students to identify the necessity of calculating the surface area of the cylinder for the above purpose.
- Ask the students to find the surface area of the cylinder as equal to the area of the bases + area of the lateral face.
- Ask the students to recall the areas bases as $\pi\ r^2 + \pi\ r^2$ (with radius r) and area of lateral surface as $2\ \pi rh$.
- Ask the students to find the total surface area as equal to $\pi\ r^2 + \pi\ r^2 + 2\ \pi\ r\ h = 2\ \pi\ r\ (r + h)$ where h is the height of the cylinder and r is the radius of the circular base.
- Ask the students to think about the use of this formula to find surface area of any cylinder.
- Demonstrate the derivation of the formula by considering the right circular cylinder as the set of right circular disks of unit height and the lateral area of each disk as $2\ \pi\ r$ and hence lateral surface area of the entire cylinder as $2\pi rh$ (where h is the height of the cylinder).
- Guide the students to solve the following problems

i) Find the total surface area of a cuboid where length, width and height are 20, 15 and 10 cm respectively

ii) A company packages its milk powder in cylindrical container whose base has a diameter of 14cm and height 20cm. Company places a label around the surface of the container. It the label placed 2 cm from top and bottom, what is the area of the label.



- Motivate the students to do other type problems.

Activity 8

Purpose: To explain the process of measuring the area of a closed region.

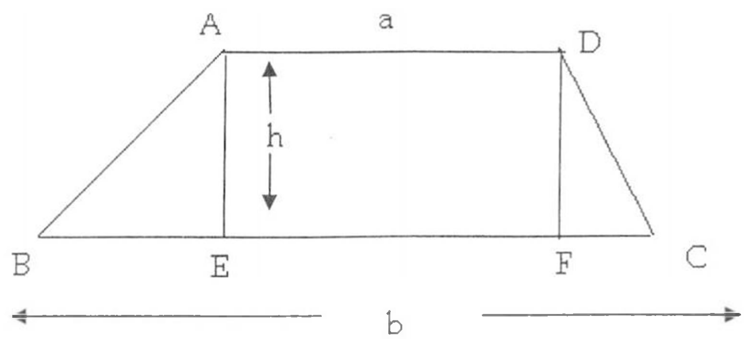
Objective:

1. To explain the concept of area of a region
2. To derive the formula to find the area of square, rectangle, parallelogram, trapezium rhombus, polygons.

Materials Required: Paper cuttings of the figures like square, rectangle, parallelogram, trapezium rhombus, polygons. Graph sheet, pencil, scissors.

Activities:

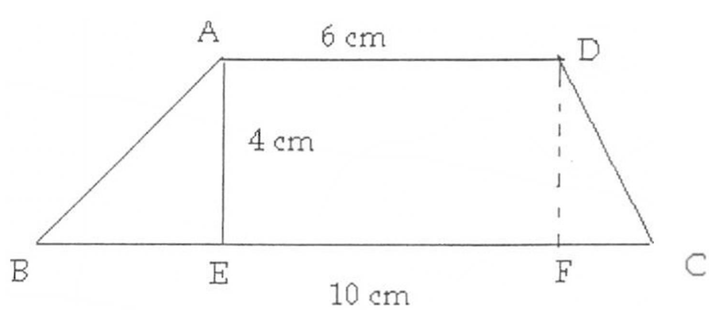
- Ask the students to compare the sizes of two class rooms when you say a particular room is larger than the other?
- Ask the question 'Is it possible to compare the rooms when they are not of the same shape like square for rectangular etc?'
- Ask the students: 'How you compare a rectangular room with a square room?'
- Explain that the size of a room is expressed in term of the space it occupies.
- Define square unit as the measure of any space which is in the form of a square having side 1 unit.
- Explain that the size of the room means the number of sq. units required to occupy the entire ground space in the room. Other words, the size of any such regions is measured in terms of sq. units and the number of those sq units is called as the area of that region.
- Demonstrate the above aspect by drawing a region on a graph paper and counting the number of unit squares in the region.
- Explain that the regions are compared based on their regions
- Ask the students to measure the area of a square with side 10 cm.
- Guide the students to measure the area of a rectangle with length 12 units and bread the 8 units.
- Ask the students to use graph paper and find the areas of such closed regions.
- Ask the students to explore any other simple method to find such areas.
- Explain the use of formula to find the areas of different regions.
- Ask the students to draw a trapezium and divide it into a rectangle and triangles as shown below:



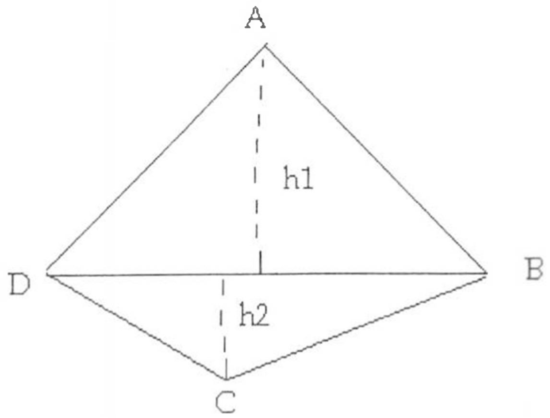
❖ Ask the students to find out the areas of these triangles and the rectangle. Guide them to arrive the areas as $\frac{1}{2}$ BE x h, $\frac{1}{2}$ FC x h and A D x h. The total area $= h\left[\frac{BE + FC}{2} + AD\right] = h\left[\frac{b - a}{2} + a\right] = h\left[\frac{a + b}{2}\right]$

❖ Guide the students to identify the above total area is nothing but the area of that trapezium with 'a' and 'b' as parallel side and 'h' is the distance between these parallel sides. So explain the formula for the area of the trapezium and equal to $= h\left[\frac{a + b}{2}\right]$

❖ Ask the students to find the area of the following trapezium using the formula and drawing the trapezium on graph paper.



❖ Guide the students to derive the formula to find the area of any quadrilateral is $\frac{1}{2}d\ (h_1 + h_2)$. Where d is one of the diagonal and h_1 and h_2 are the distances of the opposite vertices from this diagonal as shown in the figure.

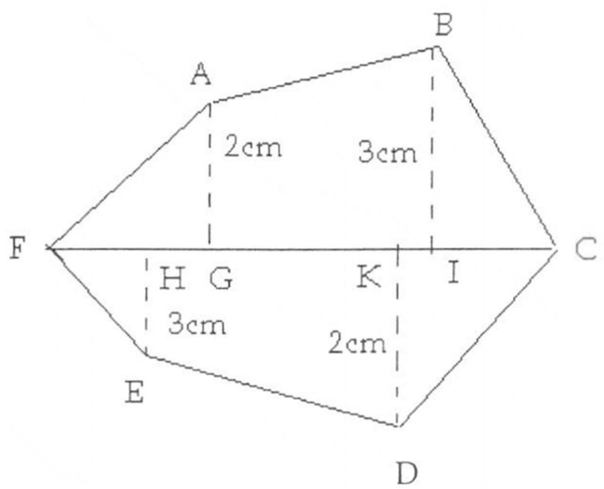


- Encourage the students to derive as to find area of any quadrilateral by dividing it into two triangles.
- Guide the students to derive the different formula

Area of a square	=	a^2
Area of a rectangle	=	$a\ b$
Area of a parallelogram	=	$\frac{1}{2}$ Base x height
Area of a Trapezium		$= \frac{1}{2} h \left[\frac{a+b}{2}\right]$

Guide the students to find the method of finding the area of any given polygon by dividing it into quadrilaterals and triangles. If the polygon is a regular polygon we can derive a formula for its area as $\frac{na}{2}h$ when n is the number sides, a is the length of the side and h is the distance of the centre of the polygon from any side.

- Ask the students to find the area of the following polygon



Also

FC = 8cm
FI = 7
FK = 6
FG = 4
FH = 2

- Ask the students to workout the following problem

The area of a trapezium shaped field is 480 m^2, the distance between two parallel sides is 15m and one of the parallel side is 20m. Find the other parallel side.

Activity 9

Purpose: To explain the characteristics of a polyhedrons.

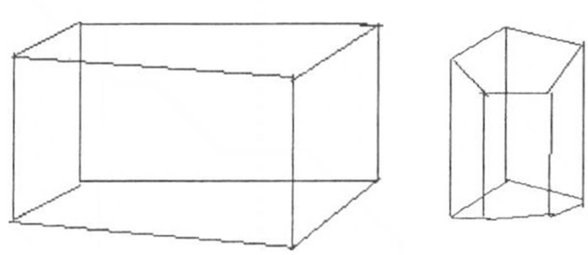
Objective:

1. To define polyhedron, convex polyhedron
2. To define faces vertices and edges of a polyhedron
3. To define regular polyhedron
4. To distinguish the different types of polyhedrons
5. To define cube, cuboid, parallelepiped, prism and pyramid
6. To explain and derive the Euler's formula related to polyhedrons.

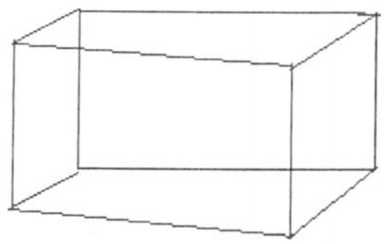
Materials Required: Objects like sphere, cone, cylinder, prism, polyhedrons, cuboids, parallelepiped, pyramid, cube, cuboid. If possible flexible to change the shapes

Activities:

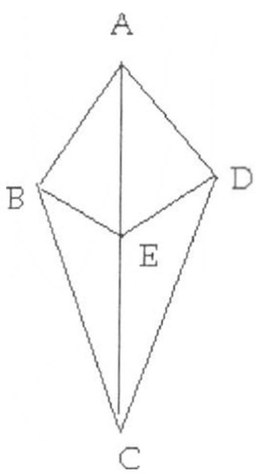
- Show different solid objects like sphere, cone, cylinder, prism, polyhedrons, cuboids, parallelepiped, pyramid.
- Ask the students to distinguish the objects with edges/sides and not having edges.
- Define the polyhedron and its faces and edges. All solids made up of polygone regions are known as polyhedrons. The polygone regions in a polyhedron are called its faces. The intersections of these faces (which are in line segment] are called its edges.
- Ask the students to identify the polyhedrons in the above set of solids.
- Define convex polyhedron and regular polyhedron. If all the faces of a polyhedron are convex polygons then that polyhedron is called as convex polyhedron. A polyhedron is said to be regular if its faces are made up of regular polygon and the same number of faces meet at each vertex.
- Give the following example for convex polyhedron and regular polyhedron.



Convex polyhedrons



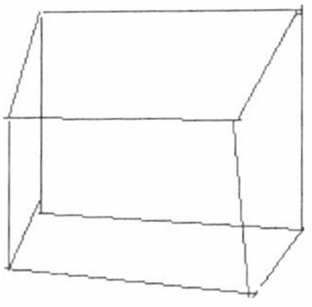
Regular Polihedron



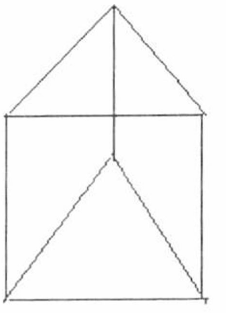
Convex polyhedron but not regular

(All sides are congruent but the verities are not formed with the same number of faces.

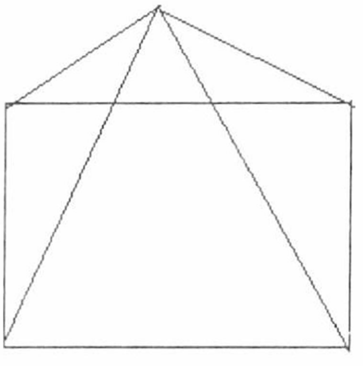
- Ask the students to identify the regular polyhedron in the above mentioned set of solids - cube, cuboid, parallelepiped, pyramid, prism etc.
- Define prism and pyramid. Prison is a polyhedron whose base and top are congruent polygons and whose other faces (lateral faces) are parallelograms in shape. Pyramid is a polyhedron whose base is a polygon and whose lateral faces are triangles with a common vertex. A prison as pyramid is named after its base, like rectangular, triangular pyramid etc.
- Give the following examples for prism and pyramid



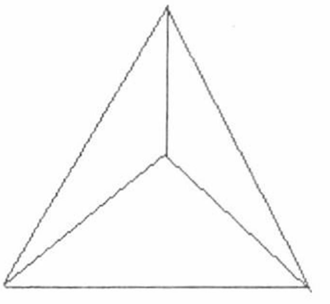
Rectangular Prisons



Triangular Prison



Rectangular pyramid



Triangular pyramid

- Ask the students to prepare a table pertaining to faces (F), vertices (V) and edges (E) in case of some polyhedrons like cuboid, prism with square base, prism with hexagon base, triangular pyramid, pyramid with square base etc.
- Ask the students to prepare a chart continuing the values of F + V and E + 2 regarding the above mentioned polyhedrons
- Ask the students to compare the values of F + V and E + 2 and to inter that they are equal.
- Formulate the Euler's formula.

Activity 10

Purpose: To explain the properties of Polygons and quadrilaterals as a special case.

Objective:

1. To explain the concept of closed curve, simple closed curve
2. To define a polygon convex polygon, quadrilateral, square, rectangle, parallelogram, rhombus, kite trapezium
3. To explain the concept of interior and exterior angles of a polygon, sides of a polygon vertices of a polygon diagonals of a polygon, regular polygon
4. To derive the formula for the sum of interior angles of a polygon as $[n-2]\ 180^0$.
5. To derive the formula for the sum of exterior angles of a polygon as 360^0
6. To explain that the lengths of opposite sides of a parallelogram are equal
7. To explain that the opposite angles of a parallelogram are equal
8. To explain that the adjacent angles of a parallelogram are supplementary
9. To show that the diagonals of a parallelogram are of equal length.
10. To show that the diagonals of parallelogram bisect each other.
11. To show that the square, rectangle and rhombus are special cases of a parallelogram
12. To demonstrate that the diagonals of a rhombus bisect each other and they are perpendicular.
13. To guide the students to construct quadrilateral giving the following measures.
    i) Four sides one diagonal
    ii) Two adjacent sides and three angles
    iii) Two diagonals and three sides
    iv) Three sides and two included angles
    v) Other properties like square, rectangle, trapezium kite etc with additional measures.

Material Required: Paper cuttings of different plane figures such as triangles, squares, rectangles, parallelograms, kits, trapezium, polygons, rhombus. If possible in the form of metal sheets.

Activities:

- Show different types of curves (pre-drawn) (including straight lines)
- Ask the students to identify the curves and lines

- ❖ Ask the students to distinguish different types of curves like where starting point and ending point are same and not the same
- ❖ Ask the students to identify the curves where the starting point and ending point are not the same and also to distinguish them like one point common (interbeeting as in Fig. (a)]

Fig (a)

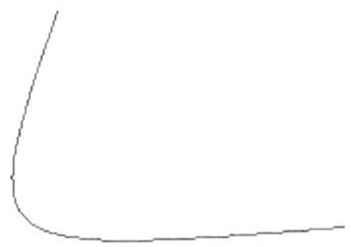


Fig (b)

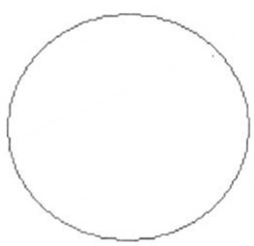


Fig (c)

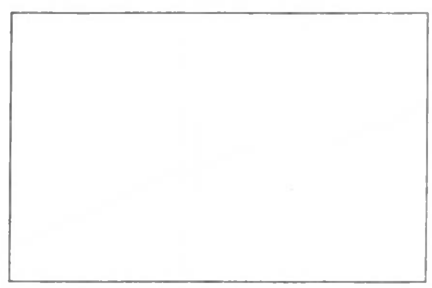


Fig (d)

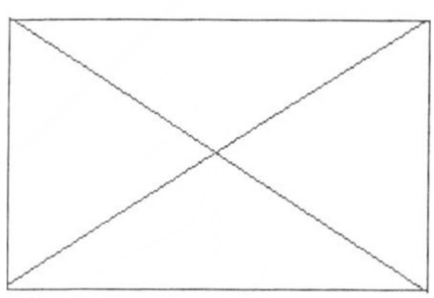
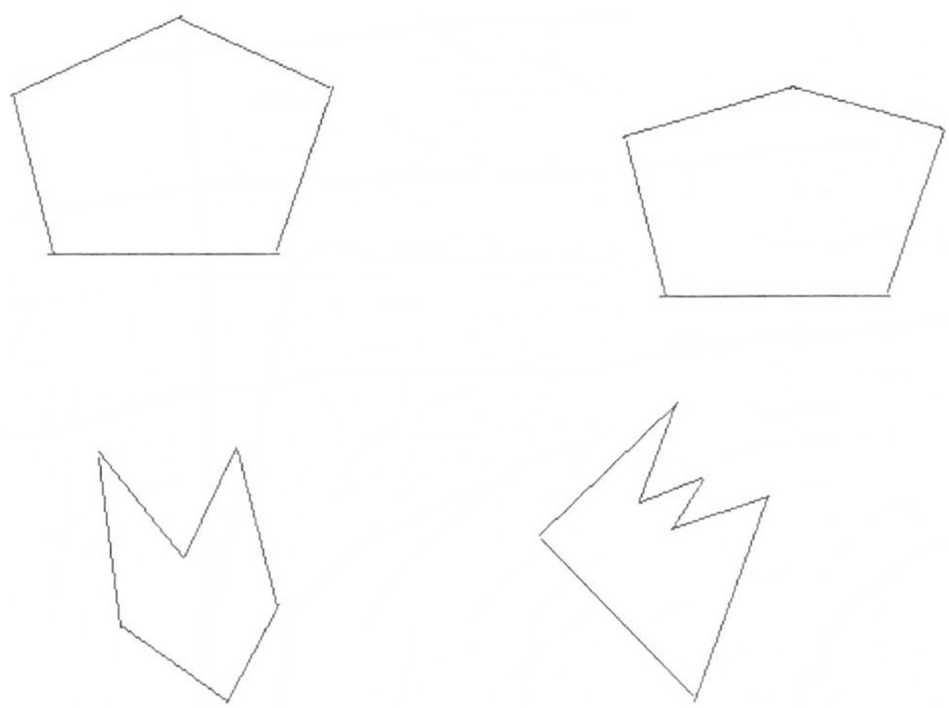


Fig (e)

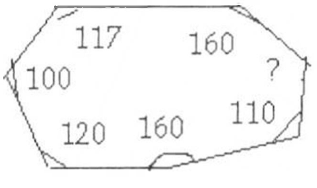
- ❖ Name the closed and not closed curves in the given set of curves
- ❖ Name the simple closed curves (Fig (c), Fig (d))
- ❖ Ask the students to draw closed curves with lines and curves.

- Ask the students to identify simple closed curves with lines
- Name the simple closed curves with lines as polygons - may contain 3 or 4 or more than 4 lines
- Discuss the interior angels and exterior angles of a polygon by using polygon model (a paper cutting). Also number of sides.
- Ask the students the draw some polygon.
- Ask the students to measure the interior angles of their drawn polygon
- Show some pre-drawn polygon like the following



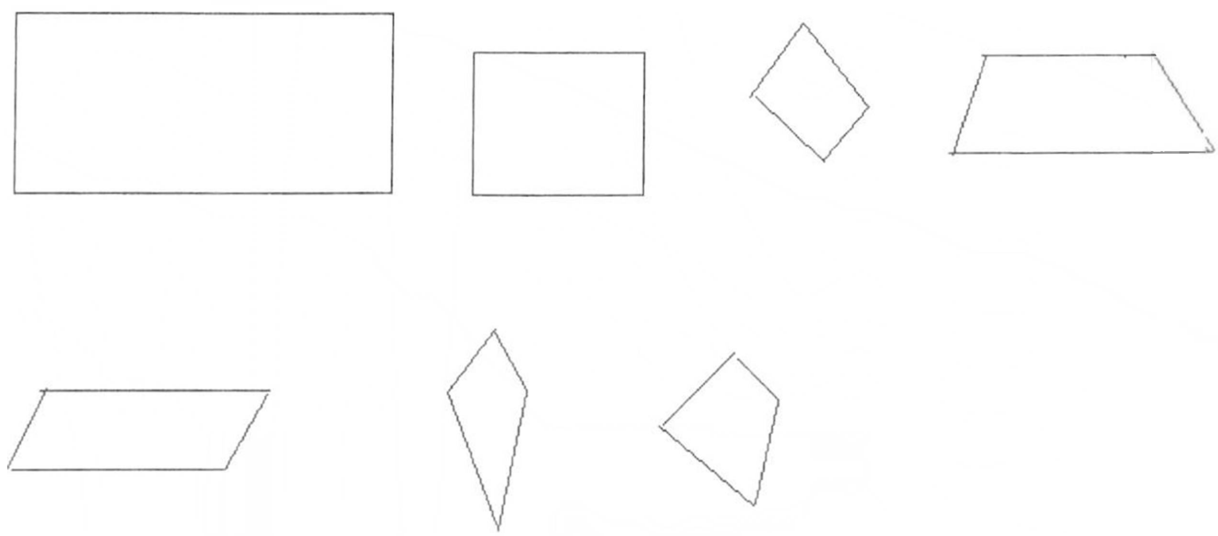
- Ask the students to identify the polygons where any interior angle is greater than 180^0
- Name the polygons where all the interior angles are less than 180^0 as complex polygons and all other polygons as concave polygons
- Indicate that you consider only convex polygons
- Ask the students to find the sum of the interior angles of their drawn polygon
- Ask the students to draw the more polygons and to find the sum of the interior angles.
- Ask the students to express the calculated sum of the interior angles as multiple of 180^0

- Ask the students to draw a conclusion about relation between the sum of the interior angles and the number of sides
- See the students of obtain the conclusion that the sum of the interiors angles of polygon as equal to (n - 2) 180^0 where n is the number of sides
- Show different polygons with equal sides with equal angles and also with different angles.
- Ask the students to identify the polygons with equal angles and equal sides. For example a square, Hexagon.
- Define the regular polygon as the polygon with equal angles and equal sides.
- Define the vertices of a polygon
- Ask the students to consider any one of their drawn polygon and to join any two non-adjacent vertices.
- Define the diagonals of a polygon
- Ask the students to draw all the diagonals and count the number of such diagonals
- Consider the following activities to derive the formula for the sum of the interior angles of a polygon.
- Ask the students to consider any polygon and to divide that into non-over lapping triangles. Also ask then to measure the sum of the angles of all such triangles
- Guide the students to notice that the sum of the angles of such obtained triangles is equal to the sum of the interior angles of the given polygon.
- Ask the students to discover the relationship between the number of triangles in which the polygon is divided and the interior angles of the polygon.
- Guide them to observe that the number of such triangles is equal to (n - 2) where n is the number of sides of a polygon.
- Ask the students to find the unknown angle in the following polygon

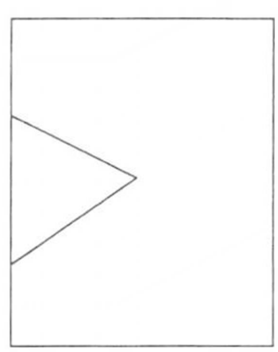


- Ask the students to identify the exterior angles of their drawn polygon
- Guide the students to notice that the sum of any exterior angle and the corresponding interior angle is 180^0

- Ask the students to find the sum of the exterior angles of their polygon
- See the students to obtain sum of the exterior triangles of any polygon as 360^0
- Explain that the quadrilateral is a special case of a polygon.
- Show different quadrilaterals like the following



- Ask the students to identify the quadrilaterals with one pair of parallel sides
- Define such quadrilaterals as trapeziums
- Ask the students to identify the quadrilaterals with two pairs of equal adjacent sides.
- Name such quadrilaterals as kites
- Explain the method of drawing/preparing a kite using the following procedure
    - i) Take a thick white sheet
    - ii) Fold the paper along the central line
    - iii) Draw two lines segments of different lengths as in the figure
    - iv) Cut along the line segments and open up
    - v) We can have the shape of a kite

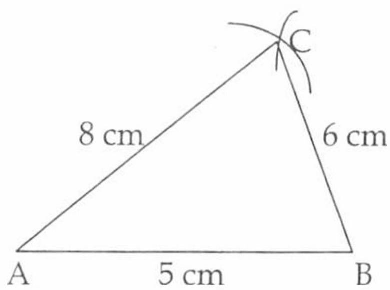


- Explain the meaning of the kite
- Now ask the students to identify the quadrilaterals with two pairs of parallel lines
- Ask the students to measure the angles and sides of such quadrilaterals
- Ask the students to discriminate the quadrilaterals with different angles among them.
- Name such quadrilaterals with two pairs of parallel lines and unequal angles as parallelograms
- Explain the vertices, sides and diagonals of a parallelogram.
- Ask the students to notice that the opposite sides of a parallelogram are of equal length. This can be verified by paper folding activity.
- Ask the students to verify that the opposite angles of a parallelogram are of equal measure. This can be verified by paper folding activity.
- Ask the students to notice or prove that the adjacent angles in a parallelogram are supplementary. This can be verified by paper folding/cutting activity.
- Define the diagonals of a quadrilateral
- Ask the students to measure the length of the two diagonals of a parallelogram
- Ask the students to notice that the diagonals of a parallelogram are of equal length
- Ask the students to observe that the two diagonals intersect at their mid points. This should be explained with the help of paper folding activity
- Ask the students to consider the kites with all sides are equal and angles may not be equal
- Define the rhombus as the quadrilateral with all sides are equal and all angles are not equal. Explain that the rhombus is a particular case of a parallelogram (where all sides are equal).
- Ask the students to observe the nature of the diagonals in a rhombus.
- Explain the students that the diagonals of a rhombus bisect perpendicularly. This should be verified with paper folding activity.
- Define rectangle as a parallelogram where all angles are equal
- Define square as a parallelogram where all sides and angles are equal
- Ask the students to construct a quadrilateral giving the following measures.

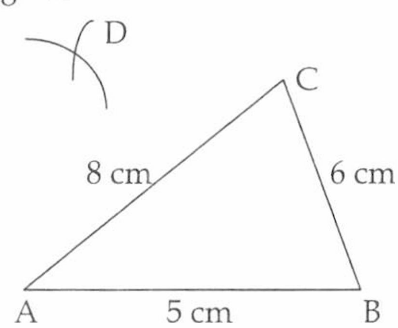
In a quadrilateral ABCD where AB = 5cm BC = 6cm CD = 5cm, AD = 4cm and AC = 8cm

- Guide them to follow the steps given below:

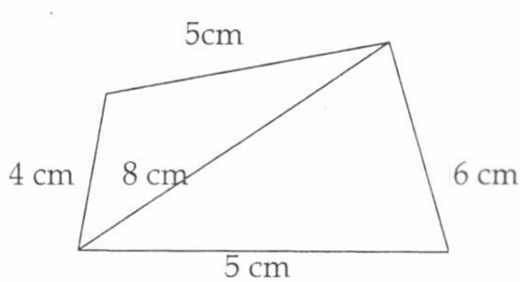
  Step 1: First construct the triangle ABC



Step 2 : Take C as centre and draw an arc with radius 5 cm. Also take A as centre and draw an arc with 4 cm. call the intersection of the above arcs as D as shown the Figure.



Step 3 : Join A, D and C, D as shown in the figure to obtain the required quadrilateral.



- Guide the students to construct the quadrilaterals as above with the following measures.

i) PQ = 4cm, QR = 3 cm, PS = 3cm
PR = 5cm, QS = 4 cm
(Three sides and two diagonals)

ii) AB = 4cm, BC = 6 cm, $\angle A = 90^0$
$\angle C = 110^0$, $\angle D = 85^0$
(Two adjacent sides and three angles)

iii) AB = 3cm, BC = 4cm, CD = 5cm
$\angle B = 75^0$, $\angle C = 120^0$
(Three sides and two included angles)

iv) A rhombus whose diagonals in 5cm and 6cm long.

- Ask the students to construct a quadrilateral with given four sides like
  AB = 5cm, BC = 7cm, CD = 8cm, DA = 9cm

- Explain the student that in the cases like above we cannot construct a unique quadrilateral, but we can construct a quadrilateral.

Motivation: Explain the students that the knowledge of polygons, quadrilaterals etc is very useful to prepare a model for a given field to calculate the area of the field.

Review: Recall the concepts discussed in the class by asking questions to the students or to draw the corresponding figures.

Activity 11

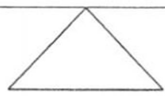
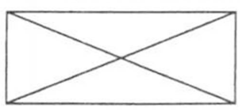
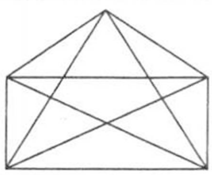
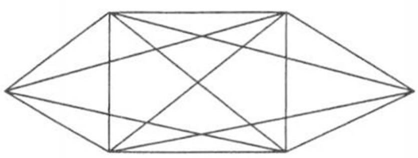
Title – How many matches?

<u>Objective:</u> To count the number of cricket matches to be played between 10 teams if every team has to play with every other team once only.

<u>Material needed:</u> Stick of appropriate lengths to prepare the polygons of 3-sides, 4-sides, ………………… 10-sides and sticks to draw the diagonals.

<u>Description</u>

Ask the students to prepare the polygons of 3, 4, 5, …………….. 10 sides and draw their diagonals as shown.

No. of sides	No. of diagonals D	D + No. of sides
3	 0	0 + 3 = 3
4	 2	2 +4 = 6
5	 5	5 + 5 = 10
6 7 8 9 10	 ? . . .	 35 + 10 = 45

One can see from the diagram and the diagonals that of there are 10 teams, each team has to play with 9 other teams. Therefore the total number of matches to be played is given by,

$$T = 9 \times \frac{10}{2} \quad \text{(Each match is committed twice Hence division by 2)}$$

$$= 9 \times 5$$

$$= 45$$

$$= 35 + 10$$

= The number of diagonals + Number of sides

Open Questions:

1. How many diagonals can be drawn to a square, pentagon, hexagon?
2. How many diagonal can be drawn to a regular polygon of 10 sides?
3. What is the number of diagonals for a regular polygon on n-sides?

Activity 12

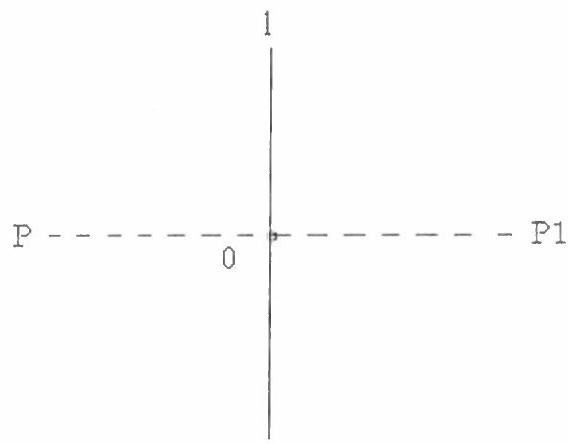
Title - Invariants under symmetry.

Objective: To find the properties which remain invariant under mirror reflections (symmetry)

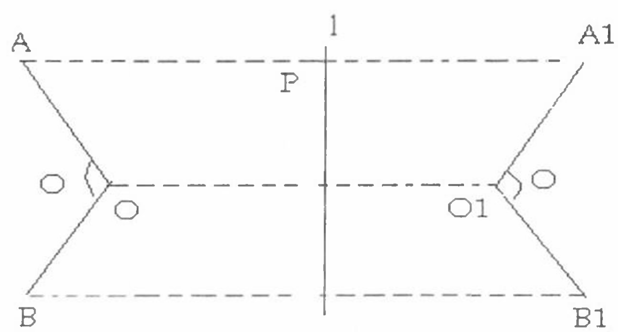
Material needed: A plain mirror, drawing sheets, sticks to form angles and triangles, measuring tapes.

Procedure

l is the line of symmetry for the points P and P^1. They are the mirror reflections of each other. The perpendicular distances PO and P^1O are equal .



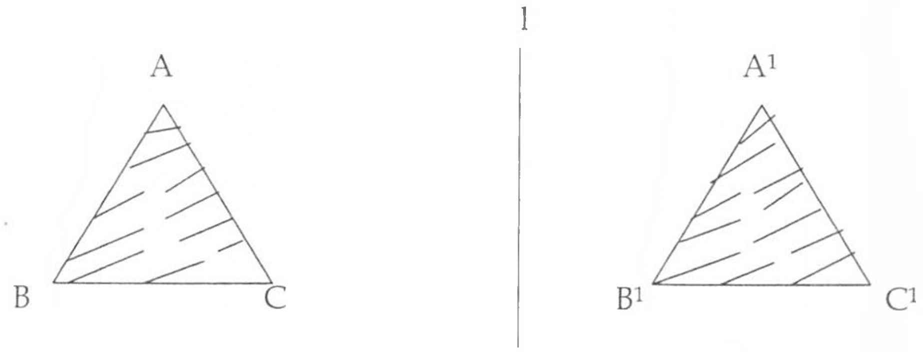
(i) Angles remain invariant



Make an angle AOB and keep it in one side of the mirror. Find its reflection A 'OB'.

Draw the point P^1 on the chart paper by making the perpendicular distances $AP = PA^1$. Similarly the points O^1 and B^1 may be marked. One can see that $A^1O^1B^1$ is the reflection of AOB. By folding the paper along l, it can be verified that $AOB = A^1O^1B^1$ (that is the measure of the angle does not change)

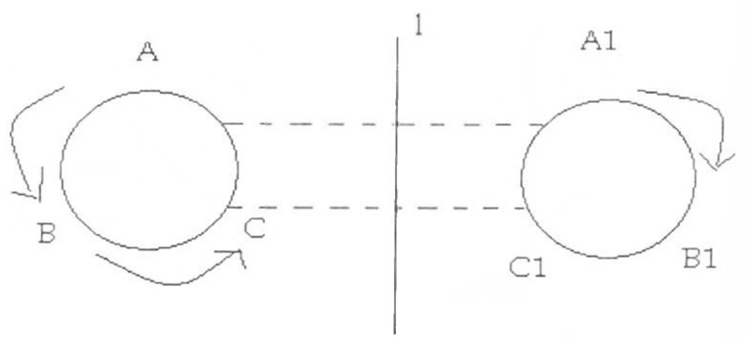
(ii) Size and shape remains invariant:



Mark a scalene triangle ABC on side of the line l.

Find (draw) its image $A^1B^1C^1$ along l, we can see that the triangle ABC and $A^1B^1C^1$ invariant under mirror refection (symmetry).

(iii) The orientation is reversed:



Draw a circle ABC on one side of the line l. Find its image $A^1B^1C^1$ by considering the perpendicular distances from the line l to the corresponding points i.e. AA^1, CC^1 and BB^1 are bisected equally by l. One can see the orientation of ABC is anticlockwise. While, the orientation of its image $A^1B^1C^1$ is clockwise. Orientation is reversed.

<u>Open Questions:</u>

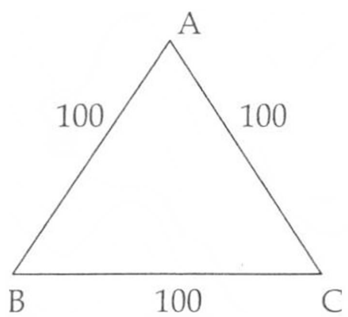
1. Show that the line l is the perpendicular bisector of the line PP^1
2. List all other properties which remain invariant (by looking at the mirror)

Activity 13

Title - Horses run

<u>**Objective:**</u> To find the path of horses run and to find the total distance travelled.

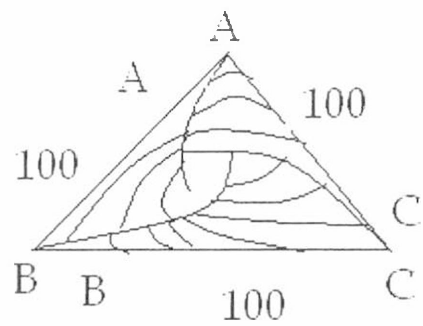
<u>**The Problem :**</u> Three horses are standing in a triangular field at A, B and C which is exactly 100 meters on each side. One hose stands at each corner and simultaneously all three are set off running. Each horse runs after the horse in the adjacent corner on his left, thus following a curved cowse, which terminates in the middle of the field. The horses run at the same speed. Find the path of running. Also find the distance travelled by each horse.



<u>Procedure:</u>

The horses are at the vertices A, B and C of the equilateral triangle. By the time A travels to A_1, the horse at B has already reached B. Hence the direction of AB has changed to A_1 B_1 similarly, it happens to other horses also. The curved path fo the horses can be formed by drawing the A_1B_1, B_1C_1, C_1A_1, etc. It V is the velocity of speed of the horses, then they $\frac{100}{v}$time to reach the centre.

Since the equilateral triangle ABC reduce to the O with the same speed. The distances that the horses travel during this time is equal to v x $\frac{100}{v}$ which is equal to 100 meters.



Open Questions:

1. It there are four horses on the four vertices of a square ABCD and if they run as above, then what is the path? Where do the four horses meet? What is the time taken by them?

Activity 14

Examples for Guided Discovery Approach

Example – 1

A class is asked to find this sum:

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+ \dots\dots + \frac{1}{99.100}$$

The Task appears to be impossible. The teacher suggests that one problem solving strategy is to consider a small part of the problem at a time. Thus the class is led to consider the first term, the first two terms, the first three terms and so on

$$\frac{1}{1.2}=\frac{1}{2}$$

$$\frac{1}{1.2}=\frac{1}{2.3}=\frac{1}{2}+\frac{1}{6}=\frac{2}{3}$$

$$\frac{1}{1.2}=\frac{1}{2.3}+\frac{1}{3.4}=\frac{1}{2}+\frac{1}{6}+\frac{1}{12}=\frac{2}{3}+\frac{1}{12}=\frac{3}{4}$$

At this point the teacher asks the class to guess the sum of the first four terms, pointing out the pattern if necessary. Hopefully there will be members of the class who will guess that it will be $\frac{4}{5}$. This answer is confirmed by actual computation. Finally, the class should be ready to guess that the answer to the given problem is $\frac{99}{100}$. Of course, it is important to point out that this is just a conjecture and not a proof.

We can prove this sum of $\frac{99}{100}$ by elementary methods. First we need to recognize these relationships. Which also lend themselves to discovery approach?

$$\frac{1}{1.2}=\frac{1}{2}=\frac{1}{1}-\frac{1}{2}$$

$$\frac{1}{2.3} = \frac{1}{6} = \frac{1}{2} - \frac{1}{3}$$

$$\frac{1}{3.4} = \frac{1}{12} = \frac{1}{3} - \frac{1}{4}$$

...... so as

Then write the given series as follows:

$$\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots\ldots + \left(\frac{1}{98} - \frac{1}{99}\right) + \left(\frac{1}{99} - \frac{1}{100}\right)$$

Finally, note that every term except the first and last subtract out giving this sum.

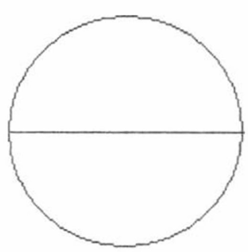
$$1 - \frac{1}{100} = \frac{99}{100}$$

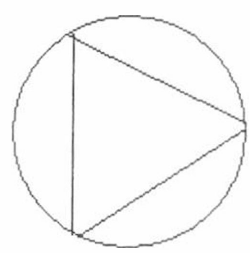
<u>Example - 2</u>

It is important that students recognize that a conjecture is notes proof, and that without a proof there is no guarantee that a pattern that fails after a certain point. One of the most dramatic ones concerns the maximum number of regions, into which a circle can be divided by connecting points on the circle. Consider the following apparent pattern.

2 points, 2 regions

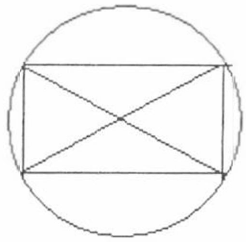
3 points, 4 regions,

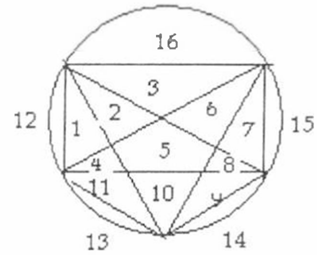




4 points, 8 regions

5 points, 16 regions





In summary

Number of points	Number of region
2	2
3	4
4	8
5	16

What is your conjecture about the number of regions expected for the six points connected in all possible ways?

Test your conjecture on another circle. The apparent answer is 32. However, much to everyone's surprise, the maximum number of regions possible with six points proves to be only 31!

Activity 15

Title : Magic numbers

Example - 1

A student is asked to go to the board with the teacher's back turned so as not to see the work, the student is given these instructions.

"Write a two digit number between 50 to 100" (in fact between 24 to 100)
"Add 76 to this number"
"Cross out the digit in the hundreds place"
"Add the crossed out number to the remaining two digit number."
"Subtract this result from the original number"

These are the steps if a student were to begin with the number 83:

Original number	:	83
Add 76	:	+76
		159
Cross out and add	:	159 : 59 + 1 = 60
Subtract from original number	:	83
		-60
		23

The interesting thing about this trick is that the final outcome will always be 23, regard less of the number selected by the study. Provided that the steps are following.

Example - 2:

Have each student in the class write a four digit number; using four different digits. Then form three additional cyclic numbers by moving the digit in the thousands place to the hundreds, tens and units place.

Here is an example, using as the initial number 8234:

8234
4823
3482
2348

Have students find the same of their four numbers.
(For the example above, the sum is 18,887)

Find the sum of the digits in the original number.
$(8 + 2 + 3 + 4 = 17)$

Divide the sum of the numbers by the sum of the digits.
$(18{,}887 \div 17 = 1111)$

Surprisingly, regardless of the original number chosen, the sum will always be 1111!

Activity 16

Title : The Mangoes problem

One night the king couldn't sleep, so he went down into the Royal Kitchen, where he found a bowl full of mangoes. Being hungry, he took 1/6 of the mangoes.

Later that same right, the Queen was hungry and couldn't sleep. She too, found the mangoes and took 1/5 of what the king had left.

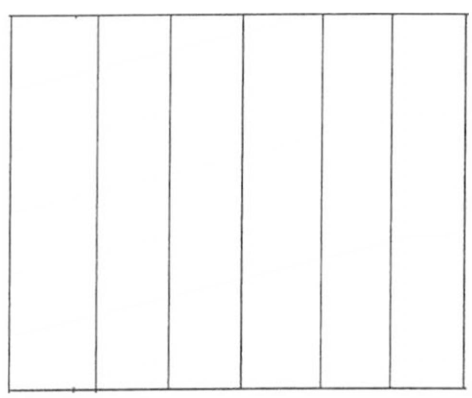
Still later, the first prince awoke, went to the kitchen, and ate ¼ of the remaining mangoes. Even later, his brother, the second prince, ate 1/3 of what was then left. Finally the third price ate ½ of what was left, leaving only three mangoes for the servants.

Question: How many mangoes were originally in the bowl?

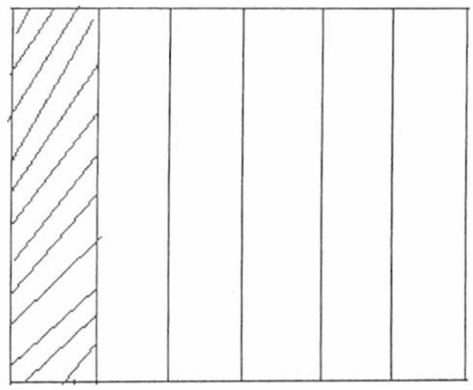
Mangoes Solution:

Draw a picture or diagram

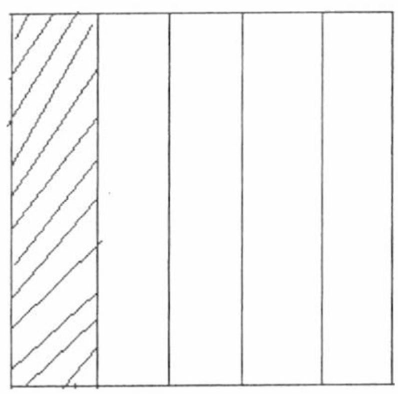
Entire rectangle represents original number of mangoes



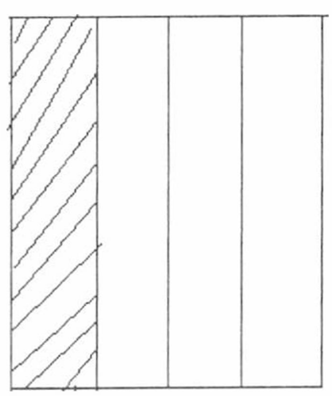
King removes 1/6 of Mangoes



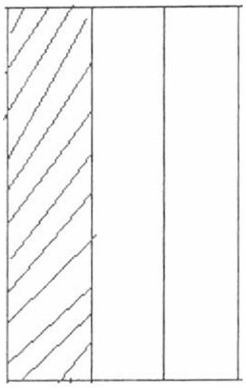
Queen removes 1/5 of remainder



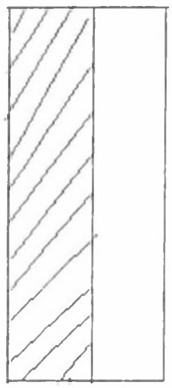
First Prince removes ¼ of remainder



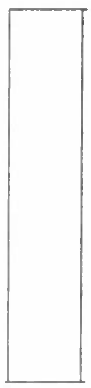
Second Prince removes 1/3 of remainder



Third Prince removes ½ of remainder



3 mangoes left



Therefore the answer 18 $3 \times 6 = 18$

Algebraic method:

Let x be the number of mangoes in the bowl before any are removed.

1. Since the king removed $(1/6)x$, then $x - 1/6\ x$ mangoes are left after his removes. Thus $(5/6)x$ mangoes are left.
2. The queen removes one fifth of $(5/6)x$. So $(5/6)x - (1/5)\ (5/6)x$ or $(4/6)x$ mangoes are left after her removal.
3. The first prince removed one fourth of $(4/6)x$. So $(4/6)x - (1/4)\ (4/6)x$ or $(3/6)x$ mangoes are left after the first prince's removal.
4. The second Prince removed one third of $(3/6)x$. So $(3/6)x - (1/3)\ (3/6)x$ or $(2/6)x$ mangoes are left.
5. Finally the third Prince removed one half of $(2/6)x$, leaving 3 mangoes. So $(2/6)x - (1/2)(2/6)x = (1/6)x = 3$
   Solving $(1/6)x = 3$ resulting $x = 18$

Extension Problems:

To challenge students to more generalizations and recognize patters in Mathematics, the mangoes problem can be extended. Teachers can use these extensions to generate many variations of the root problem for their own classroom instruction.

Extension - 1:

Suppose ten people take the "remaining" mangoes, just like in the original problem, that is the first person takes one-tenth of the mangoes in the bowl, the second takes one-ninth of the remaining mangoes, the third takes one-eight of the remaining ones, and so on. Until only three are left. How many were in the original bowl?

Extension - 2:

Work a number of mangos like problems starting with ten people, then nine people then may be only seven people. Figure out how many mangoes were originally involved in each problem. Then make a generalization that would enable us to told how many mangoes were in the original bowl if three were left and we knew how many people removed mangoes.

Activity 17

· Title: Sailors and Coconuts Problem

Three sailors were marooned on a deserted island that was also inhabited by a band of monkeys. The sailors worked all day to collect coconuts but were too tired that night to count them. They agreed to divide them equally the next morning.

During the night, one sailor woke up and decided to get his share. He found that he could make three equal piles, with one coconut left over, which he threw to the monkeys. There upon, he had his own share and left the remaining in a single pile.

Later that night, the second sailor awoke and, likewise decided to get his share of coconuts. He also was able to make three equal piles, with one coconut left over, which he threw to the monkeys.

Some what later, the third Sailor awoke and did exactly the same thing with the remaining coconuts.

In the morning, all three Sailors noticed that the pile was considerably smaller, but each thought that he knew why and said nothing. When they then divided the remaining Coconuts equally, each sailor received seven and one was left over, which they throw to the monkeys.

Questions: How many Coconuts were in the original pile?

Answer: 79

Open Questions

1. Suppose it is given that each get some equal part at the end, then is it possible to find the answer?
2. Find out an algebraic method to find the answer.

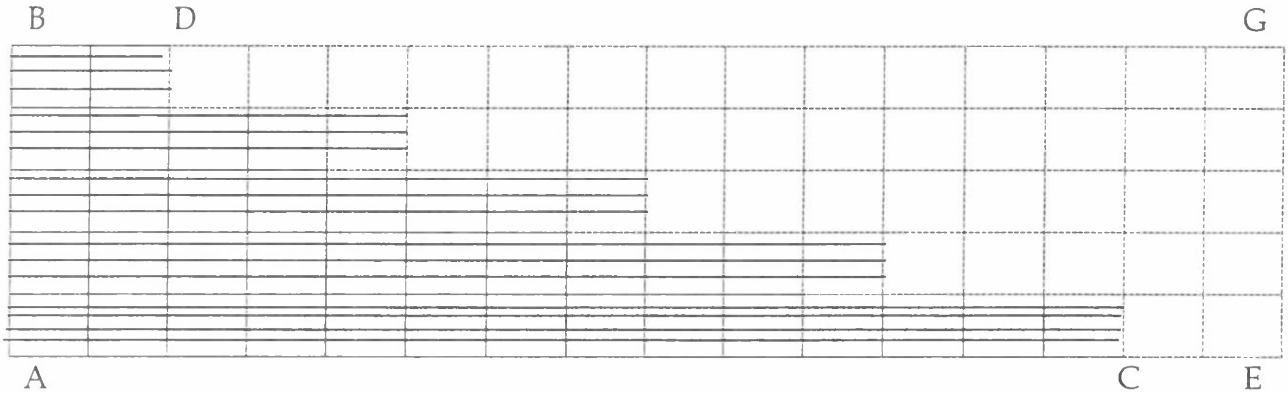
Activity 18

Title: Finding sum of A.P through squared paper

Activity: To find the sum of the terms of an arithmetic progression through squared papers.

Let us consider the following A.P with five terms:

$$2, 5, 8, 11, 14$$



In the above diagram the part ABDC depicts the given progression. In order the determine the sum of its terms, find out the diagram to complete the rectangle ABGE. We then have two equal figures ABDC and DGEC. The area of each describes the sum of the terms of our progression. Hence, the double sum of the progression is equal to the area of the rectangle ABGE or (AC + CE) AB.

But AC + CE gives the sum of the first and fifth terms of the progression and AB indicates the number of terms in the progression. Therefore the double sum is equal to;

25 = (the sum of the extreme terms). (The number of terms)

or

$$S = \frac{(first\ term - last\ term).(number\ of\ terms)}{2}$$

Which is the formula for finding the sum of terms in an arithmetic progression.

Fun with Algebra

Title: The Equation Does the Thinking

If you have ever doubted that on equation can sometimes be cleverer than you yourself, workout the following problem!

Problem : The father is 32 years old and his son is 5. How many years will pass before the father is 10 times older then the son?

If we denote the sought for years by x, then x years later the father will be 32 + x and the son will be 5 + x. And since the father must be 10 times older then his son, we have the equation,

$$32 + x = 10\,(5 + x)$$

Which, when solved, yields x = -2

"In minus 2 years' of course simply means two years before. When we set up the equation, we did not give thought to the fact that the age of the father will never be 10 times that of his son "in the future": that ratio could only be "in the past". The equation, this time was a bit wiser than we were and remainded us of our faulty thinking!

What kind of Rectangle?

The sides of a rectangle are positive numbers. What must their length be for the perimeter of the rectangle to be numerically equal to its area?

To find the solution, let us denote the sides of the rectangle by x and y. We set up the equation

$$2x + 2y = xy$$

And hence

$$x = \frac{2y}{y-2}$$

Since x and y must be positive, so also must the number y-2, or y must be greater then 2.

Now look at

$$x = \frac{2y}{y-2} = \frac{2(y-2)+4}{y-2} = 2 + \frac{4}{y-2}$$

Since x must be a positive number, the expression $\frac{4}{y-2}$ must be a positive number too. But when y > 2, this is only possible if y is equal to 3, 4, or 6. The corresponding values of x are then 6, 4, 3.

To summarize: The sought for figure is either a rectangle with sides 3 and 6 or a square with side 4.

<u>Title: Two two-digit numbers</u>

The numbers 12 and 63 are having some specialties. Their product does not change if the digits are interchanged.

That is $12 \times 63 = 756 = 21 \times 36$

It is required to find out whether there are any other pairs of two digit numbers having the same property.

To see this, let us denote the digits of the desired numbers by x and y, z and t. We set up the equation

$xy = 10x + y$ $zt = 10z + t$

& $(10x + y)(10z + t) = (10y + x)(10t + z)$

- ➢ $100xz + 10xt + 10yz + yt = 100yt + 10yz + 10xt + xz$
- ➢ $99xz = 99yt$
- ➢ $xz = yt$

Where x, y, z, t are integers less than 10. To find the solutions we set up pairs of equal products made up of 9 digits:

1 x 4 = 2 x 2
1 x 6 = 2 x 3
1 x 8 = 2 x 4
1 x 9 = 3 x 3
2 x 6 = 3 x 4
2 x 8 = 4 x 4
2 x 9 = 3 x 6
3 x 8 = 4 x 6
4 x 9 = 6 x 6

These are nine equalities. From each one it is possible to set up one or two desired groups of numbers. For example using the equality 1 x 4 = 2 x 2

We find one solution: 12 x 42 = 504 = 21 x 24.

Using 1 x 6 = 2 x 3 we get two solutions:

12 x 63 = 756 = 21 x 36

and 13 x 62 = 806 = 31 x 26

In this manner we obtain the following 14 solutions:

1) 12 x 42 = 21 x 24
2) 12 x 63 = 21 x 36
3) 12 x 84 = 21 x 48
4) 13 x 62 = 31 x 26
5) 13 x 93 = 31 x 39
6) 14 x 82 = 41 x 28
7) 23 x 64 = 32 x 46
8) 23 x 96 = 32 x 69
9) 24 x 63 = 42 x 36
10) 24 x 84 = 42 x 48
11) 26 x 93 = 62 x 39
12) 34 x 86 = 43 x 68
13) 36 x 84 = 63 x 48
14) 46 x 96 = 64 x 69

Title: A Hard Problem!

Using mental arithmetic, solve the following problem at a glance!

$$\frac{10^2 + 11^2 + 12^2 + 13^2 + 14^2}{365} = ?$$

To see the solution, let us observe that the number 10, 11, 12, 13 and 14 have the curious peculiarity that

$$10^2 + 11^2 + 12^2 = 13^2 + 14^2$$

Also since 100 + 121 + 144 = 365, it is easy to work out mentally that the expression given in the above problem is equal to 2.

Is this the only series of five consecutive numbers, the sum of the squares of the first three of which is equal to the some of the squares of the last two?

To see the answer algebraically, let us denote the first of the desired number by x, we get the equation.

$x^2 + (x + 1)^2 + (x + 2)^2 = (x + 3)^2 + (x + 4)^2$

For more convenient we may replace x by x - 1 then the equation takes on a simples aspect:

$(x - 1)^2 + x^2 + (x + 1)^2 = (x + 2)^2 + (x + 3)^2$

Removing brackets and simplifying, we obtain

$x^2 - 10x - 11 = 0$

And hence

$x = 5 \pm \sqrt{25 + 11}$

Therefore $\boxed{x = 11 \text{ and } x = -1}$

Thus there are two sequences of numbers with the required property:

10, 11, 12, 13, 14

and the sequence,

-2, -1, 0, 1, 2,

Which is true, as $(-2)^2 + (-1)^2 + 0^2 = 1^2 + 2^2$

Training on Mathematics Laboratory Approach to Teaching of Mathematics at the Upper Primary level (Andhra Pradesh and Puducherry)
23-01-2012 to 27-01-2012

Programme Coordinator: Dr V S Prasad

Date	9.30 to 11.00		11.15 to 1.00		2.00 to 3.30		3.45 to 5.15
23.1.2012 Monday	Registration & Inauguration	Tea Break	Introduction	Lunch Break	Activities in Geometry (1)	Tea Break	Activities in Algebra (1)
24.1.2012 Tuesday	Activities in Geometry (2)		Activities in Algebra (2)		Activities in Algebra (3)		Lab visit
25.1.2012 Wednesday	Activities in Arithmetic (1)		Activities in Geometry (3)		General Activities		Activities in Geometry (4)
26.1.2012 Thursday	Republic day function & Group discussions		General Activities		Activities in Arithmetic (2)		Activities through games
27.1.2012 Friday	Different Approachs in teaching Mathematics		Activities Algebra (4)		Activities in Arithmetic (3)		Valedictory

Resource Persons:
Prof. G Ravindra, Prof. N M Rao, Prof. D Basavayya, Prof. B SP Raju, Prof. B S Upadhyaya, Sri. B C Basti and Dr. V S Prasad (Coordinator)

FIELDS MEDAL IN MATHEMATICS

The Fields Medals are commonly regarded as mathematics' closest analog to the Nobel Prize (Which does not exist in mathematics), and are awarded every four years by the International Mathematical Union to one or more outstanding researchers. **"Fields Medals"** are more properly known by their official name, ***"International medals for outstanding discoveries in mathematics".***

The Field Medals were first proposed at the 1924 International Congress of Mathematicians in Toronto, where a resolution was adopted stating that at each subsequent conference, two gold medals should be awarded to recognize outstanding mathematical achievement. ***Professor J.C. Fields,*** a Canadian mathematician who was secretary of the International Congress of Mathematicians held in Toronto, later donated funds establishing the medal, which were named in his honor. Consistent with Fields' wish that the awards recognize both existing work and the promise of future achievement, it was agreed to restrict the medals to **mathematicians not over forty** at the year of the Congress. In 1966 it was agreed that, in light of the great expansion of mathematical research, **up to four medals** could be awarded at each Congress.

Each medal carries with it a cash prize of 1500 Canadian dollars. The first two such medals were presented at the Oslo Congress in 1936. After an interruption caused by War, two medals have been presented at each of the Congresses in 1950, 1954, 1958, 1962, 1974 and 2002; four medals at each of the Congresses in 1966, 1970, 1978, 1990, 1994, 1998 and 2006; and three medals at each of the Congresses in 1982 and 1986.

The Fields Medal is made of gold, and shows the head of Archimedes (287-212 BC) together with a quotation attributed to him : *"Transire suum pectus mundoque potiri "* ***("Rise above oneself and grasp the world").*** The reverse side bears the inscription: *"Congregati ex toto orbe mathematici ob scripta insignia tribuere"* ***("the mathematicians assembled here from all over the world pay tribute for outstanding work")***

Why no Nobel prize for Mathematics ?

Nobel prizes were created in the will of the Swedish chemist and inventor of dynamite Alfred Nobel, but Nobel, who was an inventor and industrialist, did not create a prize in mathematics because he was not particularly interested in mathematics or theoretical science. In fact, his will speaks of prizes for those "inventions or discoveries" of greatest practical benefit to mankind. While it is commonly stated that Nobel decided against a Nobel Prize in math because of anger over the romantic attentions of a famous mathematician (often claimed to be Gosta Mittag-Leffler) to a woman in his life, there is no historical evidence to support the story. Furthermore, Nobel was a lifelong bachelor, although he did have a Viennese woman named as his mistress (Lopez-Ortiz).

Note : In the year 1997, film **"GOOD WILL HUNTING",** fictional MIT professor Gerald Lambeau (played by Stallan Skarsgard) is described as having been awarded a Fields medal for his work in combinatorial mathematics.

LIST OF FIELDS MEDAL WINNERS IN MATHEMATICS

Sl.No YEAR **WINNERS**

1) 1936 a) Lars Valerian Ahlfors (Hardvard University)
Sub: Riemann Surfaces of Inverse Functions
b) Jesse Douglas (Massachussetts Institute of Technology)
Sub: Work on the Plateau problem

2) 1950 a) Laurent Schwartz (University of Nancy)
Sub: Theory of Distributions
b) Atle Selberg (Institute of Advanced Study, Princeton
Sub: Elementary proof of prime number theorem

3) 1954 a) Kunihiko kodaira (Princeton University)
Sub: Harmonic integrals & Algebraic varieties
b) Jean-Pierre Serre (University of Paris)
Sub: Cohomology & Sheaf Theory

4) 1958 a) Klaus Friedrich Rot (University of London)
Sub: Analytic Number Theory
b) Rene Thom (University of Strasbourg)
Sub: Cobordism Theory in Differential Topology

5) 1962 a) Lars V.Hormander (University of Stockholm)
Sub: Linear Partial Differential Operators
b) John Willard Minor (Princeton University)
Sub: Differential Topology

6) 1966 a) Michael Francis Atiyah (Oxford University)
Sub: Index Theorem for Elliptic Operators
b) Paul Joseph Cohen (Stanford University)
Sub: Foundations of Mathematics- Forcing
c) Alexander Grothendieck (University of Paris)
Sub: Algebraic Geometry- Schemes
d) Stephen Smale (University of California, Berkelery)

Sub: Dynamic Systems-Structural Stability

7) 1970 a) Alan Baker (Cambridge University)
Sub: Analytic Number Theory- Transcedental Numbers
b) Heisuke Hironaka (Harvard University)
Sub: Algebraic Geometry- Resolution of Singularities
c) Serge P. Novikov (Moscow University)
Sub: Topological Invariance of Pontrjagin class
d) John Griggs Thompson (Cambridge University)
Sub: Finite Simple Groups

8) 1974 a) Enrico Bombieri (University of Pisa)
Sub: Number Theory & Algebraic Geometry
b) David Bryant Mumford (Hardverd University)
Sub: Algebraic Geometry

9) 1978 a) Pierre Rene' Deligne (Institute des Hautes Etudes Scientifiques)
Sub: Weil's Conjecture on Riemann Hypothesis
b) Charles Louis Fefferman (Princeton University)
Sub: Multi Dimensional Complex Analysis
c) Gregori Alexandrovitch Margulis (Moscow University)
Sub: Structure of Lie Groups
d) Daniel G. Quillen (Massachusettes Institute of Technology)
Sub: Serre's Conjecture in Algebraic K-Theory

10) 1982 a) Alain Connes (Institut des Hautes Etudes Scientifiques)
Sub: Operator Algebras & Applications
b) William P. Thurston (Princeton University)
Sub: Low dimensional Manifolds
c) Shing - Tung Yau (Institute for Advanced Study, Princeton)
Sub: Differential Geometry & Partial Differential Equations

11) 1986 a) Simon Donaldson (Oxford University)
Sub: Exotic 4-dimensional Manifolds
b) Gerd Faltings (Princeton University)
Sub: Mordell's Conjecture in Arithmetic Algebraic Geometry
c) Michael Freedman (University of California, San Diego)
Sub: 4-dimensional Poincare conjecture

12) 1990 a) Vladimir Drinfeld (Phys. Inst. Kharkov)
b) Vaughan Jones (University of California, Berkeley)
c) Shigefumi Mori (University of Kyuto?)
d) Edward Witten (Institute for Advanced Study, Princeton)

13) 1994 a) Pierre-Louis Lions (Universite de Paris-Dauphine)
b) Jaen-Christophe Yoccoz (Universite de Paris-Sud, Orsay,France)
c) Jean Bourgain (Institute for Advanced Study, Princeton)
d) Efim Zelmanov (University of Wisconsin)

14) 1998 a) Richard E. Borcherds (Cambridge University)
b) W.Timothy Gowers (Cambridge University)
c) Maxim Kontsevich (IHES Bures-sur-Yvette)
d) Curtis T.McMullen (Hardverd University)

15) 2002 a) Laurent Lafforgue (Institut des Hautes Etudes Scientfiques, Bures-Sur- Yvette, France)
b) Vladimir Voevodsky (Institute for Advanced Study Princeton)

16) 2006 a) Andrei Okounkov (Princeton University)
b) Grigori Perelman (Russia) (declined award)
c) Terence Tao (University of California, Los Angeles)
d) Wendelin Werner (Universite de Paris-Sud, Orsay, France)