RESOURCE MATERIAL FOR S
OF MATHEMATICAL LABORATORY IN HIGH SCHOOLS OF TAMIL NADU

Programme Coordinator

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2008-09

Foreword

School Mathematics Education is always fraught with difficulties presumably due to its abstract nature and being taught in a way delinked from learners' everyday experiences. In the early stages of learning Mathematics, the child begins to experience the difficulties in comprehending the fundamentals which in the later years pose greater challenge if not attended and finally the adult learner may develop a fear of Mathematics. In the recent past, many strategies have been attempted by Mathematics Educators to teach mathematics in child friendly manner. Activity based learning is gaining ground in providing joys of learning mathematics. The laboratory based approach of teaching mathematics can provide students with opportunities to understand and discover the beauty, importance and relevance of mathematics as a discipline.

The objective of any laboratory is to perform experiments and same is true of mathematics. An experiment in mathematics is an exercise or project to highlight some known concepts based on well-known theorems in mathematics. It should throw new light on some subtle aspect of topic studied. It should facilitate the students to come out with some new discovery. It can also focus on some interesting applications of mathematics in real life situations. By constructing low cost models, the students can be made to work on projects or activities which can highlight the practical use of mathematics. Mathematical modeling has found profound applications in science and technology. With the teacher as the facilitator, the students can be led to define the problem in mathematical terms. In general, mathematics laboratory helps to sustain students' interest in mathematics. At the same time, teacher also gains from his/her involvement with mathematics laboratory. The teachers' role is to facilitate learning and use innovative methods to help students to discover mathematics on their own. This is far more challenging than conventional classroom teaching. Through the activities of the lab, the teachers can explore and design new teaching methods. A serious teacher of mathematics may even think of doing research concerning pedagogical issues in mathematical learning through mathematics lab.

The team of mathematics educators coordinated by Dr.V.S. Prasad of DESM, RIE, Mysore have developed an exemplar resource material for setting up mathematics lab, to cater to the need of teaching mathematics at secondary level. Although the request for the resource material came from Tamil Nadu Government, the material is designed considering also requirements of other States of the region. The team has selected exemplar topics of mathematics at secondary level and suggested interesting activities to develop fundamental concepts. It is hoped that learning by doing in mathematics will motivate the learners and lead them to some original discoveries. The painstaking efforts of the resource persons led by the coordinator Dr.V.S. Prasad can only be realized when the resource material is put into practice and enriched further.

G T Bhandage
Principal, RIE, Mysore

Preface

Joyful learning in Mathematics through appreciation of its intrinsic worth poses a challenge to practicing teachers of Mathematics and parents. Children need to engage themselves in manipulating the abstract concepts in a simple manner through activities. Mathematics Laboratory is considered as an established medium of giving meaning and interest to the subject. It perfects and clarifies the skills and ideas that are being taught in the classroom and so it enhancing teaching learning process in the subject from the elementary level. This not only makes study of mathematics more meaningful but at the same also correlates the problems to pupils' daily life experiences. It provides an opportunity for individualized instructions. Schools which have Mathematics Laboratory have found that the resulting stimulus has amply justified the experiment. With this background, Government of Tamil Nadu has requested to take this programme and accordingly it is formulated.

In the beginning, two days inhouse meeting was conducted with resource persons on 19th and 20th June 2008. In that meeting various content areas from secondary level are identified and some model materials are prepared as a guideline for key resource persons.

Next, a five-day workshop was conducted from 18th to 22nd August 2008. 33 Key Resource Persons from Tamil Nadu are participated. Based on the guidelines given they prepared the resource materials for the Mathematics Laboratory under the supervision of all resource persons.

Finally a review meeting was conducted to edit the draft manual on 3rd to 5th December 2008. After careful observations, they select 40 model activities for the manual.

I would like to submit my wholehearted gratitude to resource persons Dr.N.B. Badrinarayan, Prof.N.M. Rao, Prof.D. Basavayya, Prof.B.S.P. Raju, Prof.B.S. Upadhyaya and Shri B.C. Basti for their constant support and involvement in the preparation of the manual.

I am grateful to our beloved Principal Prof.G.T.Bhandage for his continuous encouragement and inspiration throughout the program. Also I am thankful to Prof.B.S. Raghavendra, Dean of Instruction and Prof.B.S. Upadhyaya, Head of Dept. of Extension Education and the staff of DEE for their support and help.

I am thankful to the staff of CPU Section for their secretarial help and cooperation throughout the programme.

Last but not the least, I am thankful to all the participants for their valuable participation, preparation of materials and continuous interactions. I hope that this material will help them to increase the quality of classroom transaction and facilitate joyful learning in teaching Mathematics.

Programme Coordinator

Workshop to prepare a Manual for Setting up of Mathematics Laboratory in High Schools of Tamil Nadu

18 – 22 August 2008

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7. **Dr V S Prasad** - *Coordinator*
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List of Activities

MATHEMATICS LEARNING THROUGH ACTIVITIES

Mathematics Laboratory

Introduction

Initiation to mathematics and mathematical concepts at the Primary and Secondary Stages through physical activities with the help of observation and doing is desirable. Mathematical abstraction and rigour can wait till the thinking faculty develops. Learning mathematics by seeing and doing using material objects available around makes the process enjoyable and facilitates understanding the mathematical concepts and results. A feel of the mathematical ideas is imperative and should precede proof in mathematics. Conjecture and verification, inductive thinking are the stepping stones leading to heights of rigour, abstraction and realizing mathematics as a discipline.

In science laboratories, scientific principles are learnt through verifications by experiments. Likewise a mathematical laboratory must provide a hands-on experience and exposure to mathematics. Thus the purpose of mathematical laboratory is to provide a forum for seeing mathematical facts to believe them. Accordingly, we need to plan the activities, keeping in mind the mental ability of the learner, his/her age and background.

Stages of Planning :

Firstly, identify the concepts and results topicwise and sequentially. The sequence is determined by the logical links connecting the concepts in the long chain of mathematical development.

Secondly, one or more activity has to be designed with a flexible format. Identifying different techniques to be used suitable to the activity is the next thing.

Objectives of the activity and the methodology with procedural details, identifying the materials need for the construction of the model to be used, etc. must follow.

Reinforcement is to be ensured with suggested exercises and assignments.

Thus a format for an activity could be

1. **Topic – concept / result**
2. **objective**
3. **Activity**
   a) **Pre-knowledge required**
   b) **Material needed**
   c) **Construction / Designing of the activity**
   d) **Methodology - how to conduct the activity**
4. **Conclusion**
5. **Additional Assignments**
6. **References**

Lists of Different activities and Construction of Models (material)

1. Diagrams/ Pictures/ Graphs – with suggested animation.
2. Paper folding experiments
3. cut and paste activities
4. Material models for plane and space related ideas (i.e. plane figures and solids) – using strings – needle like objects, pin, wooden/ paper contents.
5. Geoboard : Take a large square. Fix nails vertically and horizontally separated by unit distances. Passing a rubber band same nails so that the bank is tight, we get different types of polygonal figures.
6. Parallel lines board

ALGEBRA

Activity - 1

Topic : Quadratic expressions and their formations.

Objective : Geometrical interpretation of linear and quadratic expressions in single variable.

Pre-knowledge :

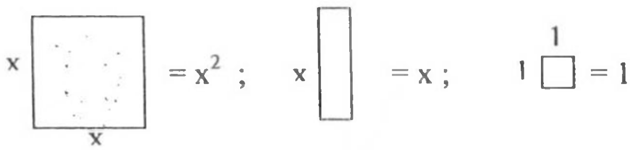
i) If a and b are the sides of a rectangle, then a × b (= ab) is the area of the rectangle.

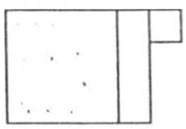
ii) In particular, x^2 = the area of a square of sides.

Material Required : White paper sheets

Activity (through paper cut outs)

a)



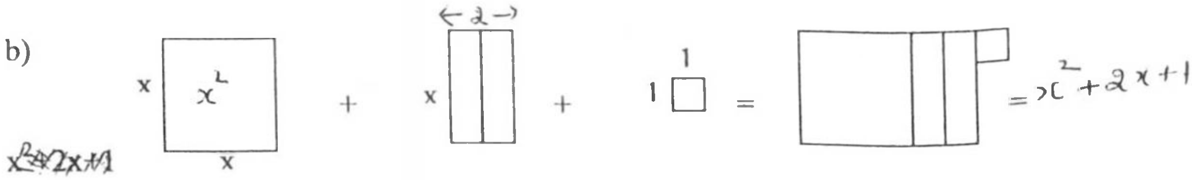


$= x^2 + x + 1$

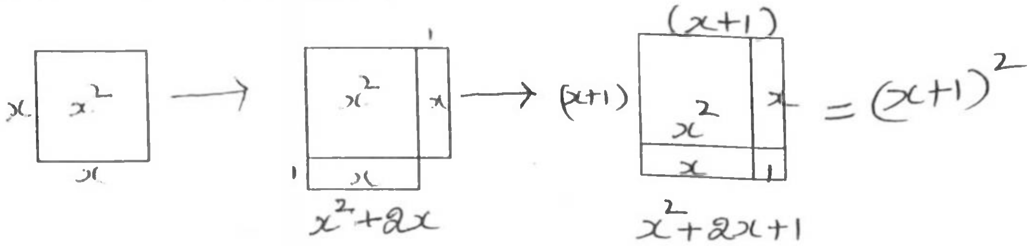
Meaning of addition (+) is Adding Areas

Meaning of subtraction (–) is Removing area

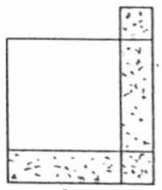
b)



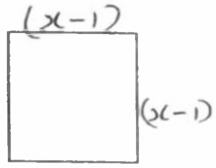
Verification of $x^2 + 2x + 1 = (x + 1)^2$



c) Verification of $x^2 - 2x + 1 = (x - 1)^2$



Removing the shaded region



$x^2 + 1 \longrightarrow x^2 + 1 - 2x = (x-1)^2$

Activity – 2

Topic: Sum of a) $1 + 2 + 3 + ...+ n = \frac{n(n+1)}{2}$

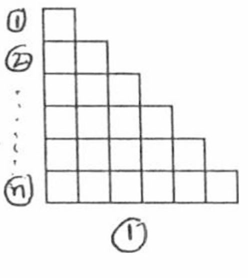
b) $1 + 3 + 5 + ...$ n terms $= n^2$

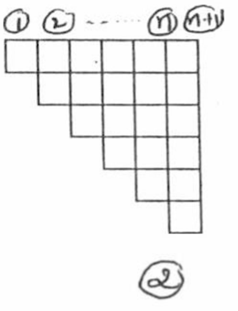
Objective : Understand the formulas (a) and (b).

Materials Required : White paper sheets

Activities:

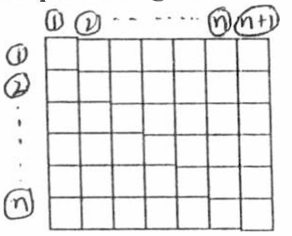
a) i) Construct two identical pieces as shown





ii) The area of each piece is $1 + 2 + 3 + ...+ n$

iii) Put the two pieces together as shown below :

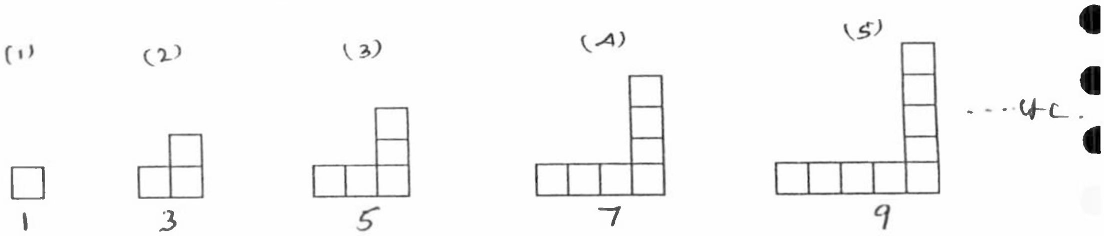


iv) The area of the two pieces put together = Area of the rectangle

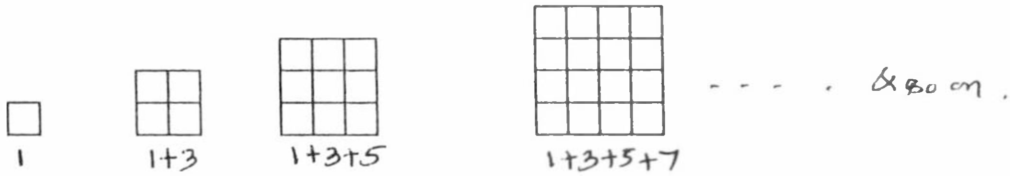
$\therefore \quad 2(1 + 2 + ...+ n) = n(n + 1)$

$\therefore \quad (1 + 2 + ...+ n) = \frac{n(n+1)}{2}$

b) i) Prepare cut outs in the shapes given below (formed by unit squares)



ii) Join the pieces in a sequence as below :



iii) Observe that each time we get a square.

Accordingly, $1 = 1^2$; $1 + 3 = 2^2$; $1 + 3 + 5 = 3^2$; $1 + 3 + 5 + 7 = 4^2$ and so on.

iv) The pattern suggests that in the nth step (n can be any positive integer),

$$1 + 3 + 5 + \ldots\ldots n \text{ terms} = n^2$$

Activity – 3

Topic : **Algebraic Identities**

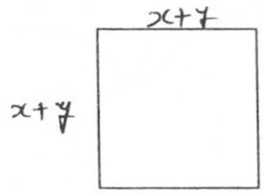
Objective : To understand the algebraic identity $(x + y)^2 = x^2 + 2xy + y^2$

Pre-knowledge : Knowledge of algebraic expressions and areas of square and rectangle.

Material Needed : Chart, scale, pencil and cutter.

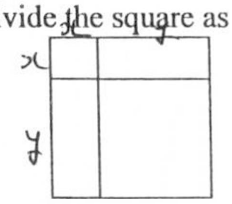
Prepare a square of length $(x + y)$ using a white paper.

i)



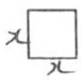
Area of this figure is $(x + y)^2$

ii) Divide the square as shown :

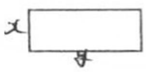


Then cut it.

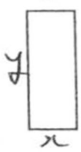
iii) Find the area of each part.



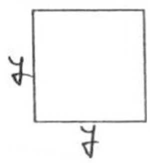
Area is x^2



Area is xy



Area is xy



Area is y^2

iv) Add the areas of all parts.
$x^2 + xy + xy + y^2 = x^2 + 2xy + y^2$

v) Hence, $(x + y)^2 = x^2 + 2xy + y^2$

Exercise :

1. Design an activity for

   i) $(x + 5)^2 = x^2 + 10x + 25$

   ii) $(7 + a)^2 = 49 + 14a + a^2$

2. Design an activity for $x^2 + y^2 = (x + y)^2 - 2xy$.

Activity - 4

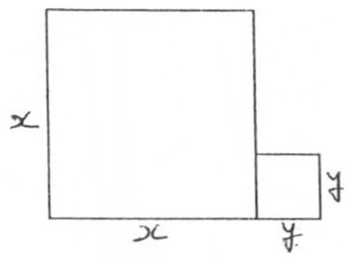
Topic : Algebraic Identities

Objective : To understand the algebraic identity $(x\text{-}y)^2 = x^2 - 2xy + y^2$

Pre-knowledge : Area of square is a^2 when side is a. Area of rectangle is lb when length is l and breadth is b.

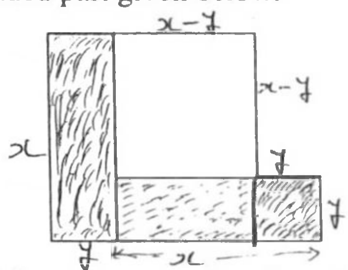
Material Needed : Chart, scale, pencil and cutter.

i) Prepare a chart as given below.



ii) Area of the above figure is $x^2 + y^2$.

iii) We remove shaded part given below.



We have removed the areas xy and yx from the area $x^2 + y^2$. The left out shape is a square of side length $(x - y)$. Hence its area is $(x - y)^2$. Also the remaining area after removing the shaded part is $x^2 + y^2 - (xy + yx) = x^2 + y^2 - 2xy$.
Hence $(x - y)^2 = x^2 + y^2 - 2\,xy$.

Exercise : Can you give an activity for $x^2 + y^2 = (x - y)^2 + 2xy$.

Activity – 5

Topic : Algebraic Identities

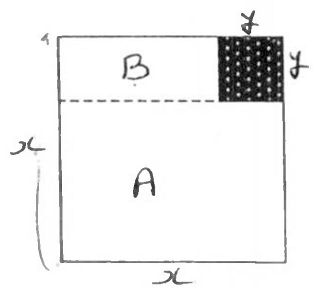
Objective : To understand the algebraic identity
$x^2 - y^2 = (x + y)(x - y)$

Pre-knowledge : Area of square is a^2 when side is a.
Area of rectangle is lb when sides ~~is lb~~. are l and b .

Material Needed : Chart, scale, cutter.

Activities :

i) Take a paper square of side length x. Its area is x^2. Let it be in white colour. Take another paper square of side length y. Its area is y^2. Let it be of black colour. Overlap the smaller square inside the bigger square as shown below.



If we remove the shaded area, then we get the two rectangles A and B.

Area of A = $(x - y)\,x$
Area of B = $y\,(x - y)$
$\therefore$ Area of A + area of B = $(x - y)\,x + y\,(x - y)$
$= (x - y\,(x + y)$
$\therefore$ The area of unshaded figure = $x^2 - y^2$
= Area of A + Area of B
$= (x + y)(x - y)$

Conclusion : Hence, we get
$x^2 - y^2 = (x + y)(x - y)$

Exercise : i) Prepare an activity for $(x + 5)(x - 5)$.

Activity – 6

Topic : Algebraic Identities

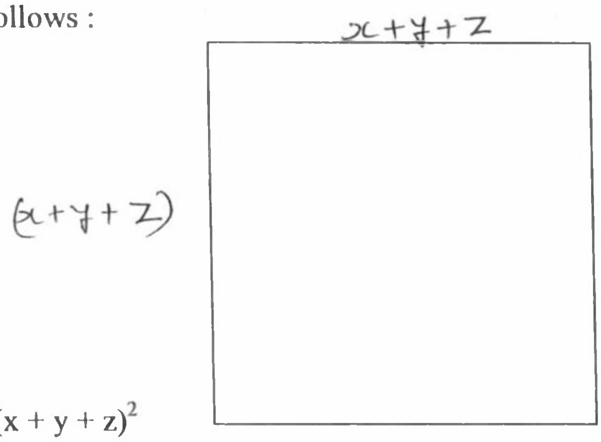
Objective : To understand the algebraic identities $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

Pre-knowledge : Area of square is a^2 when side is a. Area of rectangle is lb when length is l and breadth is b.

Material Needed : Chart, scale, pencil and cutter.

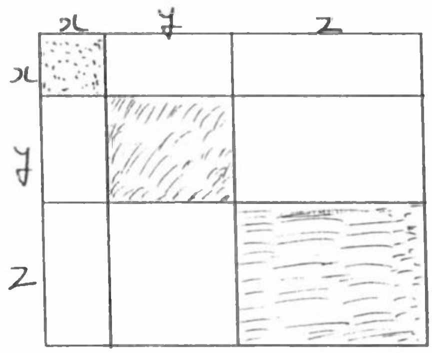
Activities : (Through cut out chart).

i) Prepare a chart as follows :

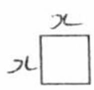


Area of the square is $(x + y + z)^2$

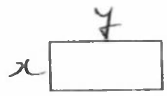
ii) Divide the square like



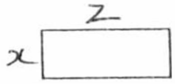
Cut it.
We get,



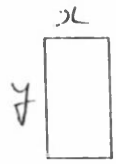
Area is x^2



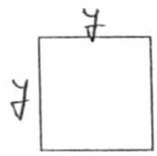
Area is xy.



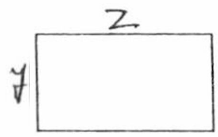
Area is xz.



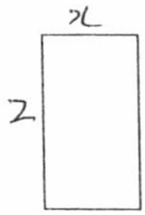
Area is xy.



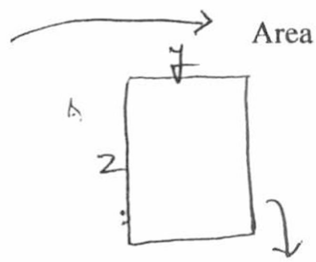
Area is y^2



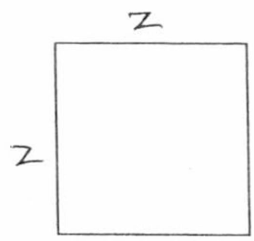
Area is yz.



Area is xz.



Area is yz.



Area is z^2.

Add the above areas, we get the expression for the area as
$x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
which is also equal to $(x + y + z)^2$
Hence, we get $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

Exercise : Prepare the activity for $(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$.

Activity – 7

Topic : Geometric Progression

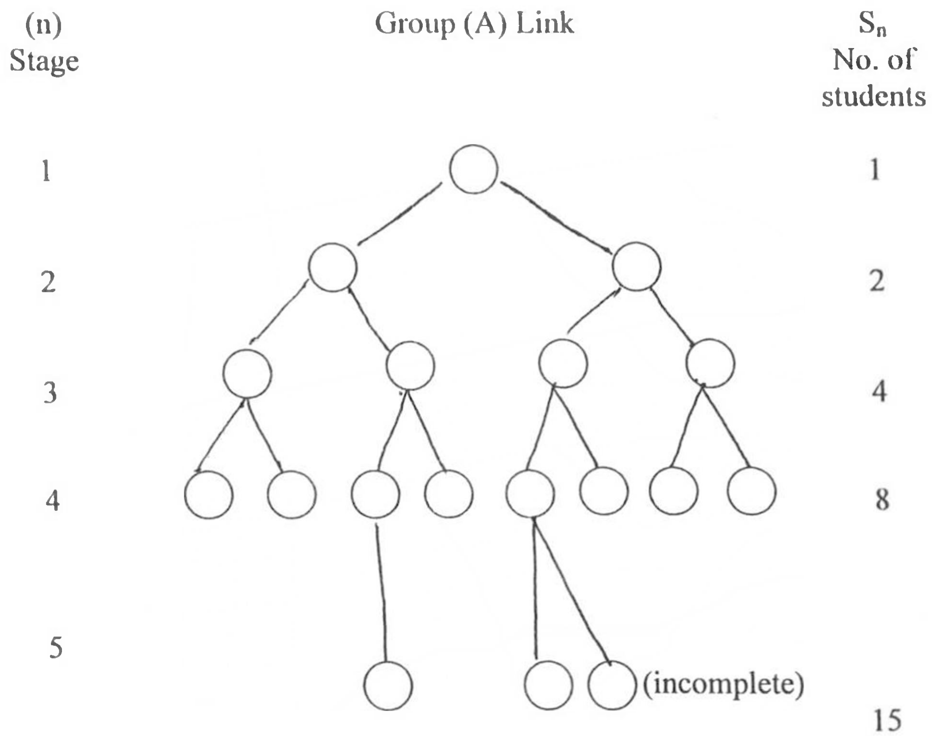
Objective : Students understand the sum of n terms in G.P.

Pre-knowledge : Now only we are introducing the G.P. through a game.

Material Needed : Using classroom and students

Activities : Through a game "Increase"

Divide the classroom into two groups and select each person from each group say A and B.
The students A have to touch 2 students (we can choose the number 2 or 3 etc.) And the next 2 students have to touch 2 students each. This will continue upto a certain time. Then consolidate the game as per the stage as follows:



The above figure, stage 4 is incomplete. So we must calculate the number of students upto stage 3. Count the total number of students in group 1 (i.e. 15).
The same process should be implemented to group 2 also. Finally the group which has the minimum number of students wins the game.

Conclusion Here we are introducing the G.P., general form and common ratio and sum of n terms in G.P.

GEOMETRY

Activity – 8

Topic : Concurrency properties of plane figures – a) triangle, b) parallelogram, c) circle

Objective : To understand that

(i) in a triangle, a) medians, b) altitudes and c) angle bisectors are concurrent.

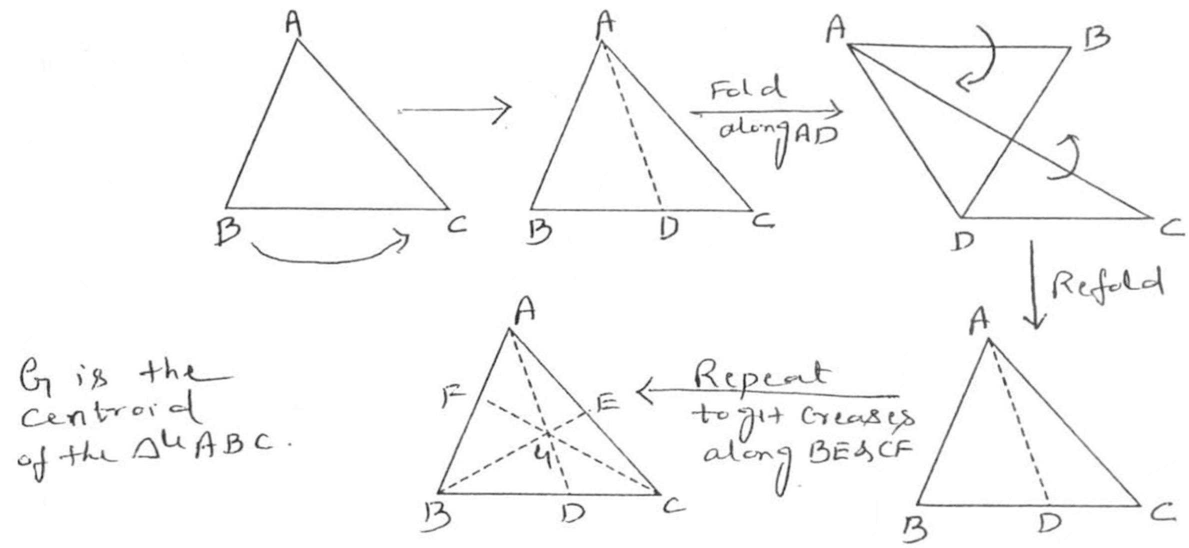
(ii) In a parallelogram, the diagonals bisect each other.

(iii) In a circle, the perpendicular bisectors of chords are concurrent at the center.

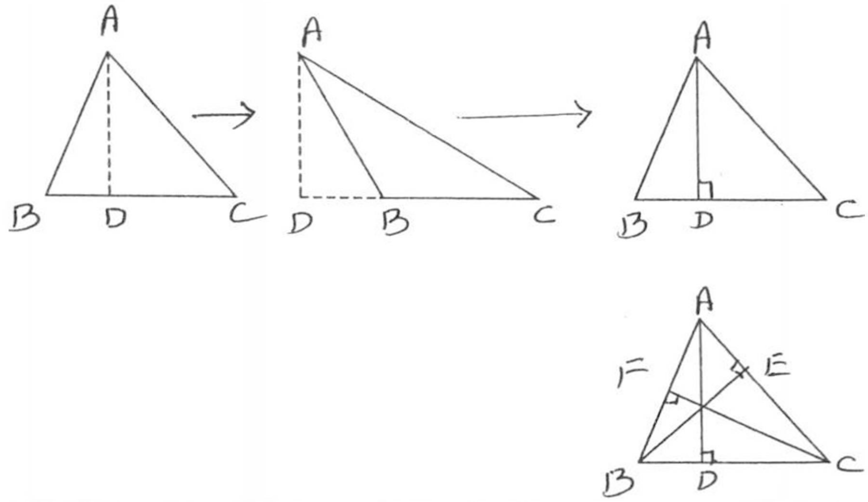
Previous Knowledge: Knowledge of plane figures like triangles, parallelogram and circle.

Material needed : Triangular and rectangular white paper sheets.

I. Cut out a triangular piece- ABC. Mark the midpoint of each sides as D,E,F. For this, bring B and C to coincidence along BC. The crease cuts BC at the midpoint and so on. Then fold along AD, BE and CF in succession.

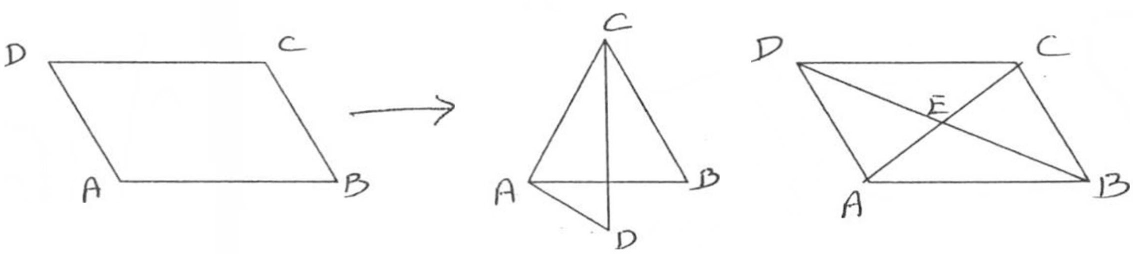


II. Cut out a triangle ABC. Fold the triangle bringing the segments of BC to coincidence so as to get the crease AD perpendicular to BC. AD is an altitude. Likewise, get the altitudes BE and CF. All these creases along the altitudes pass through a point O.

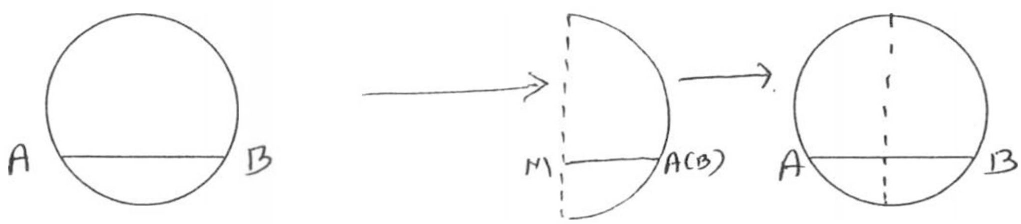


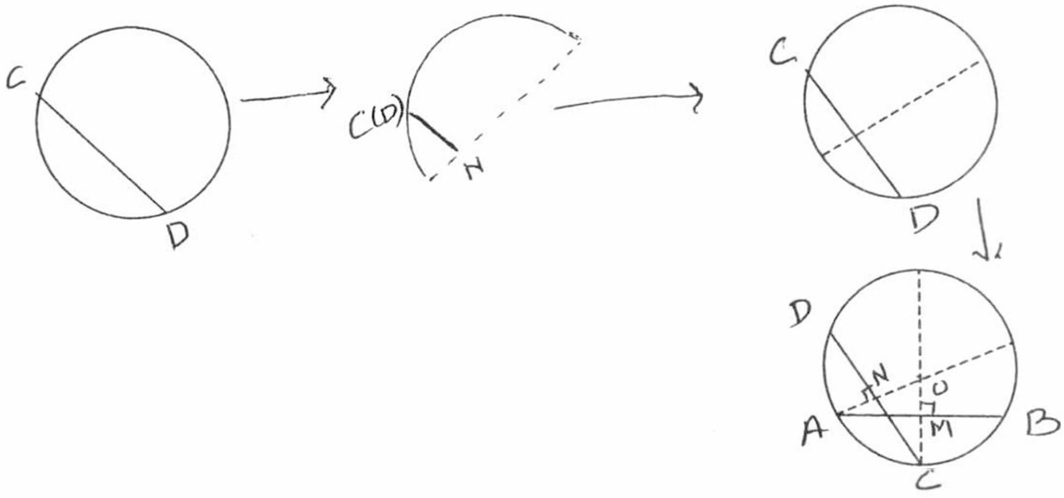
Ex : Design a paperfolding activity for the concurrency of angle bisectors of a triangle.

III. Cut out a parallelogram ABCD. Fold along AC. Fold along BD. The creases along AC and BD are diagonals. They intersect at E. Verify that E is the midpoint of each diagonal – by first bringing A and C to coincidence so that the crease passes through E. And next, by bringing B and D to coincidence to see that the crease passes through E again.



IV. Cut out a circle. Fold it along a chord AB. Fold it perpendicular to AB by bringing A and B to coincidence. The crease so got is the perpendicular bisector of AB. Similarly, have another chord CD and its perpendicular bisector. The creases corresponding to the perpendicular bisectors of the chords are concurrent at the center of the circle – verify that the point so got is the center of the circle.




C
D
C(D)
N
C
D
D
N
O
A
M
B
C

Activity – 9

Topic : Geometry – Pythagoras Theorem

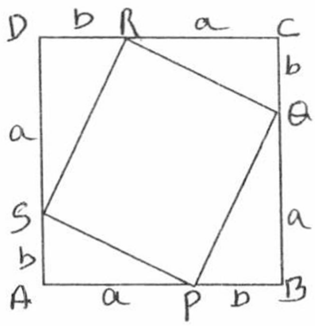
Objective : Verification of Pythagoras Theorem

Pre-knowledge: Knowledge of right angled triangle

Materials Needed: White paper sheets

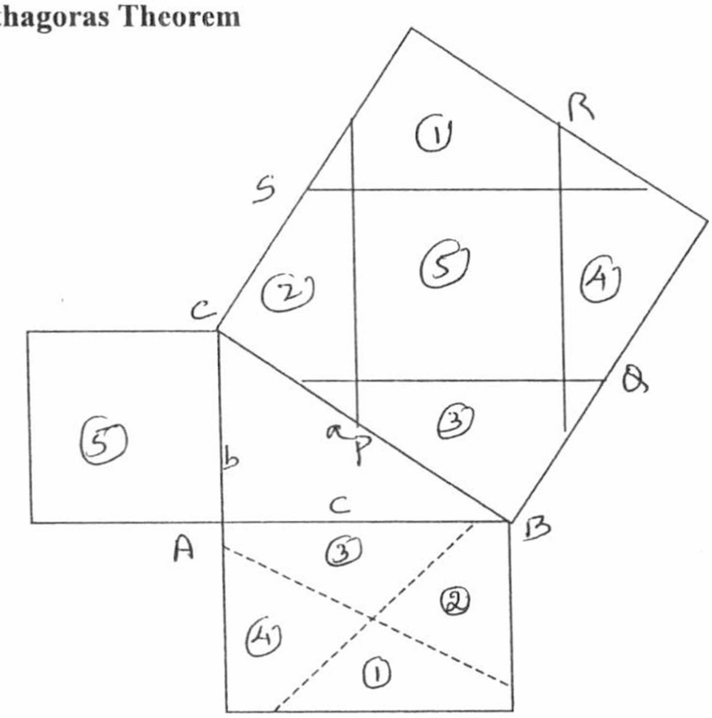
Activity : In the following figure, we find

$(a + b)^2 = 4\left(\frac{1}{2}ab\right) + c^2 \quad \Rightarrow \quad a^2 + b^2 + 2ab = 2ab + c^2 \quad \Rightarrow \quad c^2 = a^2 + b^2$



$\Rightarrow (a + b)^2 = 4\,(\frac{1}{2}ab)^2 + c^2$

Verification of Pythagoras Theorem



Construction : Construct a right angled triangle ABC. BC is the hypotenuse of the triangle so that $\hat{A} = 90^{\circ}$. Mark the mid points of the sides of the square on the hypotenuse BC say P, Q, R, S. P and R are midpoints of opposite sides parallel to BC, while Q and S are midpoints of the other parallel sides. Construct square on the other sides of the triangle ABC. Through P and R, draw parallels to AC. Through Q and S, draw parallel to AB. The lines so drawn divide the square on BC into 5 regions of which four are identical quadrilaterals (1), (2), (3) and (4) and a square (5). Square (5) is translated to the square on AC. The four quadrilaterals are fitted into the square on AB. Thus the pieces (1), (2), (3) and (4) together have the area of the square on AB and the area of the square (5) is equal to that of the square on AC.

$\therefore$ Square on AB + sq. on AC = sq. on AC

i.e. $c^2 + b^2 = a^2$ or $a^2 = b^2 + c^2$

Activity – 10

Topic : **Geometry – Area – properties**

Objectives : Using Geo-board to verify

i) area of a triangle = ½ (base) × (height)

ii) A diagonal of a parallelogram bisects the area

iii) Area of the parallelogram does not change if the base and height are constant or the area of a parallelogram on the same base between two parallels is constant.

Previous Knowledge : Knowledge of Triangle and Parallelogram.

Materials Needed : Geoboard

Activities:

i) Base = 4 (= BC)

height = 2 (= AD)

No. of square in ΔABC

= 2 full + 4 half squares

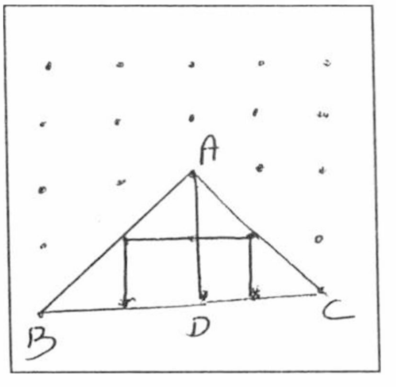
= 2 + 2 = 4

Area = 4

Formulawise, area = ½ BC× AD

= ½ × 4× 2 = 4

Thus the formula is verified.

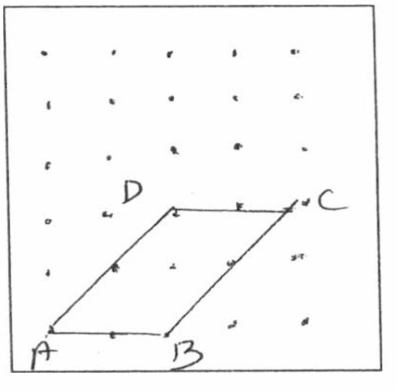


ii) Area ABD = 3 full square + 3 half

Area BCD = 3 full square + 3 half

∴ Area Δ ABD = Area ΔBCD

∴ The diagonal BD of the parallelogram ABCD bisect the area



iii) Ex : Verify this property using Geo-board.

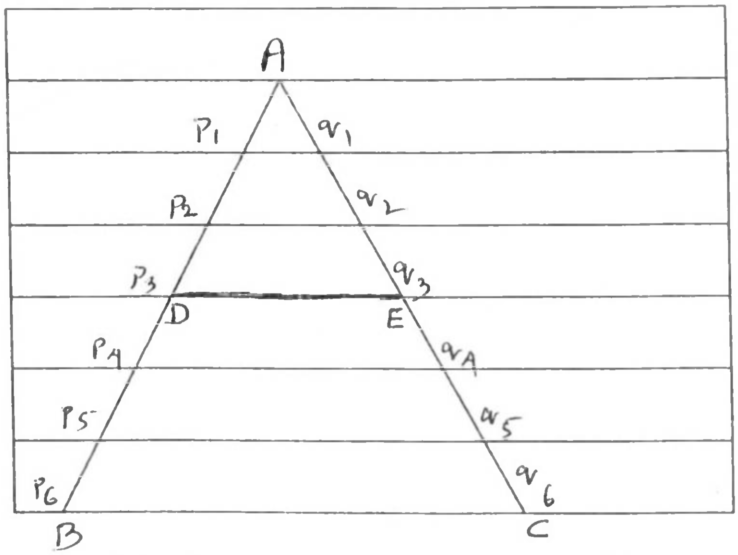
Activity – 11

Topic : Basic Proportionality Theorem in Triangles

Objective: To verify Basic Proportionality Theorem (Thales)

Pre-knowledge : Knowledge of parallel lines.

Materials Needed : A parallel line board consists of a number of parallel lines separated by the same (or unit) distances.



Fix one vertex A on a parallel and two vertices B and C on the different parallel. Join the vertices. ΔABC is got. Each side AB and AC is divided into equal segments and same number of segments. Any parallel DE to BC divides AB and AC in the same ratio.

Activity – 12

Topic : Triangles

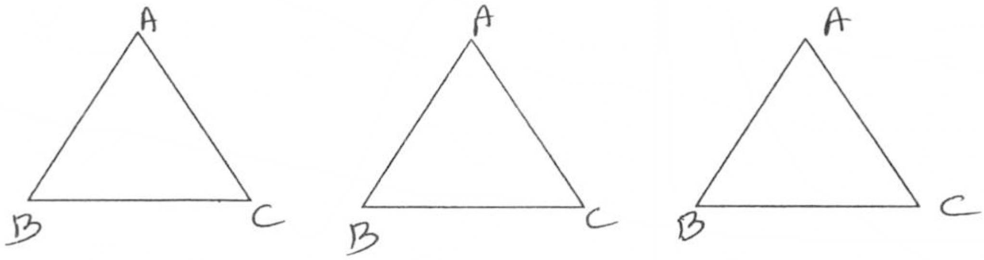
Objective : To know that the sum of the angles of a triangle is 180^{0}.

Pre-knowledge : Types of triangles.

Material Needed : 3 different colour chart papers, cutter.

Activities : Cut and paste activities.

1. Cut out three triangular pieces ABC of same size.
2. Paste the three triangular pieces in such a way that $\angle A$, $\angle B$, $\angle C$ from the three triangles concur at a point.



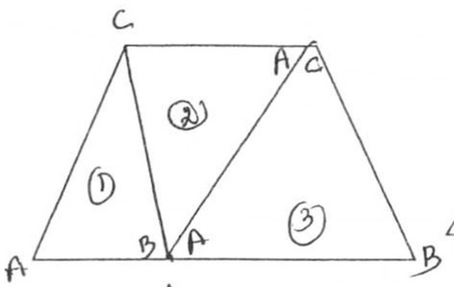


fig-1

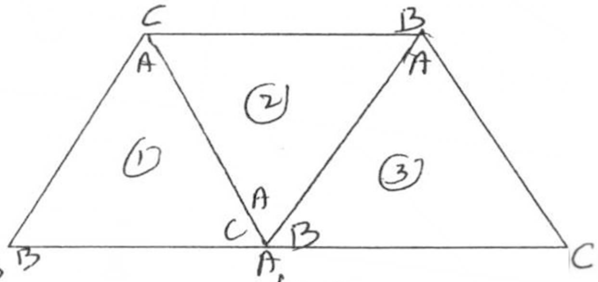


fig 2

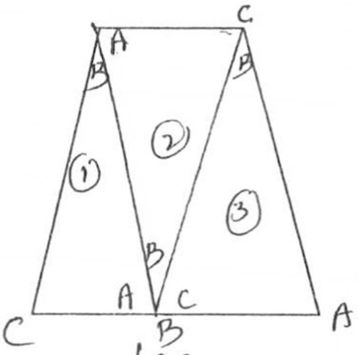


fig 3

Observation : In each of the cases when the edges containing the angles $\angle A$, $\angle B$, $\angle C$ forms a straight line.

Conclusion	:	The sum of the angles of a triangle is 180^{o}.
Follow up Activity	:	1. Take a single triangle and perform the activity by folding activity. Explain. 2. Take a right angled, obtuse angled and an acute angled triangle and verify the result. 3. How do you generate a logical proof by using the above activity?

Activity – 13

Topic : Properties of a Parallelogram

Objective : To understand that in a parallelogram,

i) the diagonal divides it into two congruent triangles.
ii) Opposite sides are equal.
iii) Opposite angles are equal.

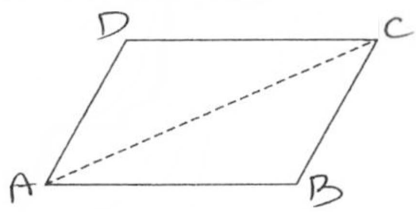
Pre-knowledge : Knowledge of terms associated with quadrilaterals, congruency of triangles.

Material Needed : Chart paper and Geometry Box.

Activities :

Paper cutting :

- Cut out a parallelogram ABCD from a sheet of paper.
- Join the diagonal AC.
- Cut it along the diagonal AC to get two triangles ΔABC and ΔADC.
- Place one triangle over the other so that they overlap. We can see that ΔABC coincides with the ΔCDA.



Observation :

- The triangles are overlapping. The triangles are congruent. $\Delta ABC \cong \Delta CDA$.
- $AB = DC$ and $AD = BC$.
- $\angle B = \angle D$.

Conclusion

- The diagonal of a parallelogram divide it into two congruent triangles.
- In a parallelogram, opposite sides are equal.
- In a parallelogram opposite angles are equal.

Follow up activity : Perform the above activity in case of the following quadrilaterals and record your observations.

i) Rectangle
ii) Rhombus
iii) Square
iv) Trapezium
v) Kite

Activity – 14

Topic : Properties of a parallelogram

Objective : To verify that the diagonals of a parallelogram bisect each other.

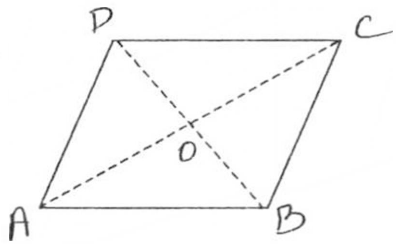
Pre-knowledge : Congruency of triangles.

Material Needed : Chart paper, geometry box.

Activities :

Paper cutting :

- Cut out a parallelogram ABCD from a sheet of paper.
- Draw the diagonals AC and BD to intersect at O.
- Cut the ΔAOB from the parallelogram.
- Place the triangular piece AOB over the triangle COD.



Observation :

- The Triangles are overlapping i.e. they are congruent. $\Delta AOB \cong \Delta COD$.
- OA = OC and OB = OD.
- O is the midpoint of AC and O is the mid point of BD.

Conclusion : Diagonals of a parallelogram bisect each other.

Follow up activity :

1. Suggest another activity to prove the above result by paper folding method.
2. Perform the activity for a square and rhombus.
3. What can you deduce from that activity?

Activity – 15

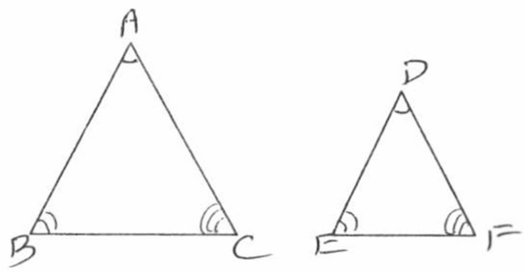
Topic : Similarity of Triangles

Objective : To understand the similarity of triangles.

Pre-knowledge : Concept of similarity in simple plane figures.

Materials Needed : White sheet pieces of triangular shape.

Activity No.1 :

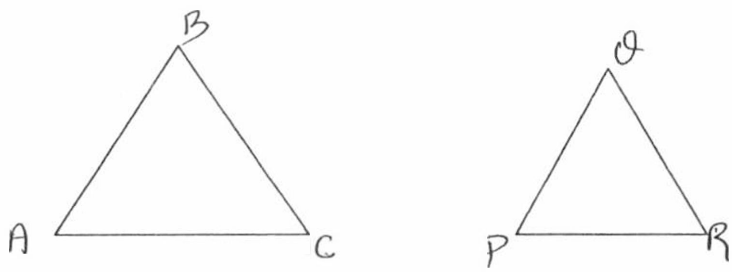


If ΔABC and ΔDEF are similar, then

- A corresponds to D
- B corresponds to E
- C corresponds to F

Symbolically, we write the similarities of these two triangles as,
$\Delta ABC \sim \Delta DEF$ and
read it as
Triangle ABC is similar to triangle DEF.
The symbol ‘|||’ stands for “is similar to”.
Recall that we have used for the symbol “≅” for is ‘congruent’.

Activity 2:
Construct two triangles ABC and PQR as shown below.



In triangle ABC and triangle PQR,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$$

$$\frac{10}{5} = \frac{6}{3} = \frac{8}{4} = \frac{2}{1} = 2$$

Here corresponding sides are in the same ratio

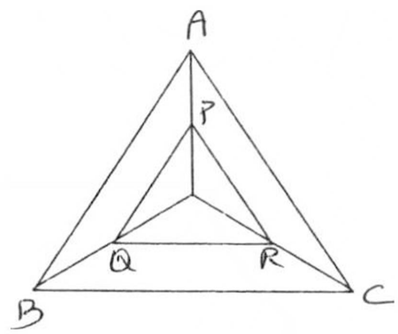
$\therefore$ ΔABC ||| ΔPQR

Exercise : Draw two triangles ABC and DEF such that AB = 3cm, BC = 6 cm, AC = 8 cm, DE = 4.5 cm, EF = 9 cm and FD = 12 cm and check the similarity by comparing the corresponding angles.

Activity 3 :

ΔABC ||| ΔPQR

Construction : Draw ΔABC and take one point inside the triangle and join the point with vertex of each side and then take midpoints PQR and join we get another triangle PQR then ΔABC ||| ΔPQR.



Activity 4 :

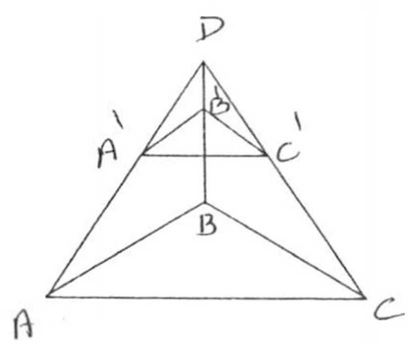
ΔABC ||| ΔA′B′C′.

Construction:

Draw ΔABC in big triangle ADC and join BD to get one pyramid.

Then take A′B′C′ on the triangle ADC and join to get ΔA′B′C′.

$\therefore$ ΔABC ||| Δ A′B′C’.



Activity 5 :

i) $\Delta ADG \;|||\; \Delta GCF$

ii) $\Delta ADG \;|||\; \Delta FEB$

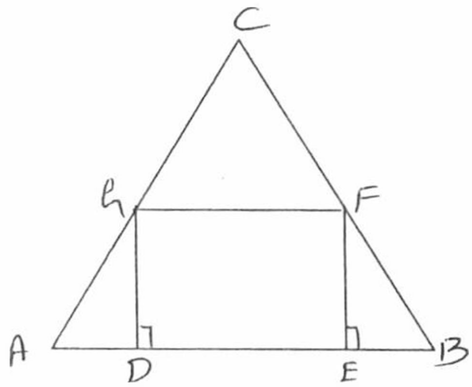
Construction:

$m\angle GDA = m\angle GCB = 90^o$

$\therefore$ $\Delta ADG \;|||\; \Delta GCF$.

$\Delta ADG \;|||\; \Delta FEB$, ΔGCF.

$\therefore$ $\Delta ADG \;|||\; \Delta FEB$.



Activity 6 :

In ΔABC, the mid point of BC, CA, AB is D, E, F.

To join DEF to get ΔDEF.

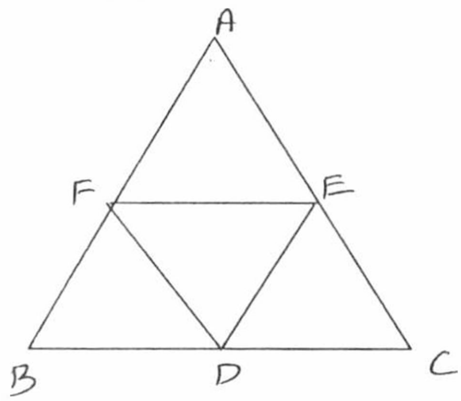
D, E are the midpoints of BC, CA

Or DE = ½ AB.

Similarly, EF = ½ BC, FD = ½ CA

$\therefore \quad \frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA} = \frac{1}{2}$ or

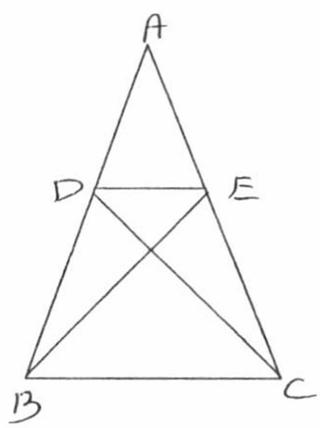
$\Delta DEF \;|||\; \Delta ABC$.



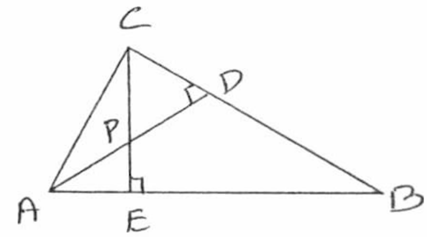
Exercise :

1. In Fig. If $\Delta ABE \equiv \Delta ACD$

Show that $\Delta ADE \;|||\; \Delta ABC$.



2. In figure altitudes AD and CE of ΔABC intersect each other at the point P. Show that
   i) $\Delta AEP \;|||\; \Delta CDP$
   ii) $\Delta ABD \;|||\; \Delta CBE$
   iii) $\Delta AEP \;|||\; \Delta ADB$
   iv) $\Delta PDC \;|||\; \Delta BEC$



Activity – 16

Topic : Geometry

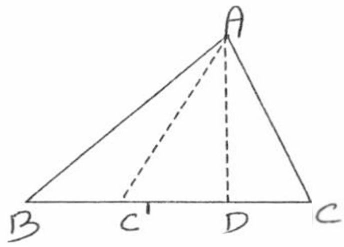
Objective : To verify that in a triangle, the angle opposite to the bigger side is bigger than the angle opposite to smaller side.

Pre-knowledge : Knowledge of angles of a triangle.

Material Needed : A triangular piece of paper.

Construction :

Mark a triangle ABC with AB > AC.



1. Fold the triangle ABD so that the crease passes through 'A' and perpendicular to BC. Let the crease intersect BC at D.
2. AC now takes the position AC′.
3. $A\hat{C}C' = A\hat{C'}C > A\hat{B}C$ as $A\hat{C'}C$ is the exterior angle of triangle ABC′.
4. $A\hat{C}B > A\hat{B}C$.

Suggested Activities : Do the above activity with different types of triangles and verify the result.

Activity – 17

Topic : Geometry

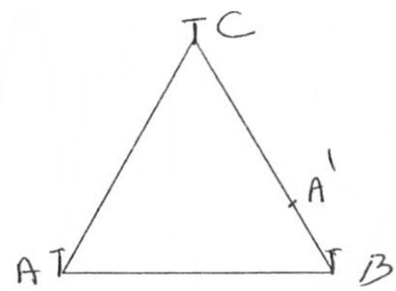
Objective : To see the validity of triangular inequality i.e. sum of any two sides of a triangle is greater than the third side and the difference of any two sides of a triangle is less than the third side.

Pre-knowledge : Knowledge of triangle

Material Needed : Drawing board and elastic strings with pins.

Construction :

1. Take a drawing board.
2. Mark two points A and B on it so that when the board is put in vertical position, AB is horizontal. Fix nails at A and B.



3. Mark another point C where C does not lie on the line AB and fix a nail at C.
4. Take an elastic string. Tie one end with A. Pass the string around the nail at C. Running along AC and bring it to B. The part of the string from A to B around C has length equal to AC + CB. Tie the other end to A. The part ACB looks like a garland shape and so we have
   $AC + CB > AB$

Also if $BC > AC$, bring the part AC of the string ACB to the position along BC. Keeping the string around C, but removing the end at A. Now the string takes the position BCA′ (where $CA = CA'$).

Then $BC - AC = BC - A'C = A'B < AB$

$\therefore\ BC - AC < AB$

Activity – 18

Topic : Geometry

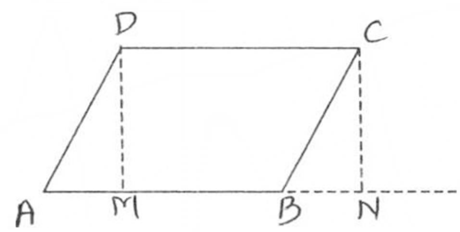
Objective : To show that parallelograms on the same base with same height have equal area.

Pre-knowledge : Knowledge of parallelogram.

Material Needed : Parallelogram shaped paper

Construction :

Cut out a parallelogram shaped card.



Remove the right angled triangle piece ADM from the parallelogram. Join the piece so that AD coincides with BC and the triangle piece does not overlap the other piece. The piece ADM now is in the position BCN.

We observe that
Area of ABCD = Area of MNCD

Any parallelogram on AB as base with the same height as ABCD has the area of the rectangle.

Activity – 19

Topic : Geometry

Objective : To verify concurrency theorem by activity.

Pre-knowledge : In a triangle

a) the medians
b) the altitudes
c) the angle bisectors and
d) perpendicular bisectors

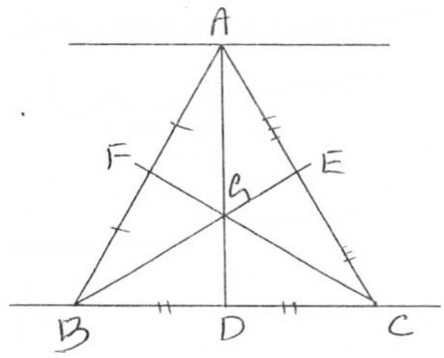
are all concurrent. Their points of concurrence are

1. the centroid (G)
2. the orthocenter (O)
3. the incentre (I) and
4. the circumference (S) of the triangle.

Material Needed : Drawing board and elastic strings.

Construction :

1. A. Physical Medal :
Mount a triangle on a drawing board with sides and vertices which can be verified as follows :

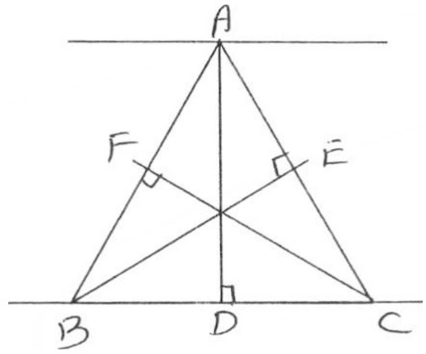


On the drawing board

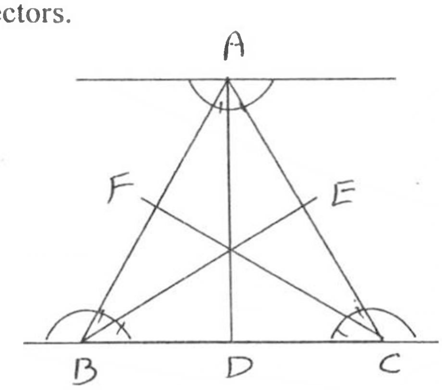
i) make a grove along a straight line
ii) Fix a scale along grove. Any two points B and C are the vertices of triangle ABC.
iii) The position of A can be varied along a parallel to BC.
iv) A is connected to B and C by an elastic string AB and AC with marked midpoints in its natural position. The marked points continue to be midpoints of AB and AC in all its positions.
v) B and C are also connected by an elastic string with midpoint marked in its natural position.
vi) Each vertex is connected to the midpoints of the opposite side.

The strings AD, BE and CF are always concurrent at a point G called the 'centroid' of the triangle.

2. In the above model, bring the strings AD, BE and CF to the positions (perpendicular to the opposite sides) of the altitudes. The strings AD, BE and CF are concurrent at a point O called the orthocentre of the triangle.



3. Fixing protractor at the vertices, bring the strings AD, BE and CF to the position of angle bisectors.



The strings AD, BE and CF are concurrent at a point (I) called the incenter of the triangle.

Exercise :

i) Verify that I is equidistant from the sides.
ii) Can you inscribe a circle in the triangle?

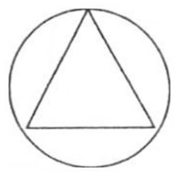
4. Through the midpoint D, E, F of the sides, draw perpendiculars to verify that they are concurrent. This point (S) when the perpendicular bisectors of the sides are concurrent is called the circumcentre of the triangle.

Exercise :

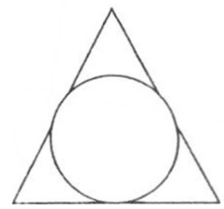
i) Verify that SA,SB, SC are equal.
ii) Can you ascribe a circle to the triangle ?

Note : 1. An explanation about 'inscribe' and 'ascribe'. If two figures P_1 and P_2 are fitted such that one (P_1 say) inside the other (P_2) then P_1 is inscribed in P and in turn P_2 is ascribed in P_1 (lies outside P_1).

(i)



(ii)



In (i) triangle is inscribed in the circle (fitted inside)
In (ii) triangle is ascribed the circle (fitted outside).

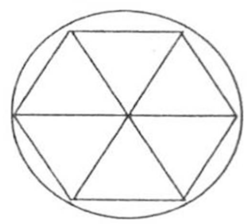
Note 2 : How to (a) inscribe and (b) ascribe a regular polygon in (or out) of a given circle ?

a) To inscribe a regular polygon (of n-sides) in a circle,

i) draw a circle

ii) divide the circle into n equal sector and with angle of each sector equal to $\frac{2\pi}{n}$

iii) Connect (or join) the vertices of the sector in order.

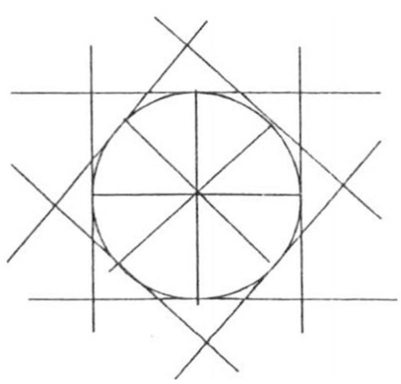


b) In ascribe, a regular polygon (of n-side) in a circle

i) draw the circle

ii) divide the circle into n-equal sectors.

iii) Draw the tangent to the circle at each vertex of the sector. They are perpendicular to the radii. These tangents form the ascribed polygon.



Activity – 20

Topic : Geometry

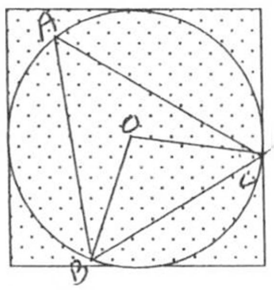
Objective : To verify the interior opposite angle theorem in a triangle.

Pre-knowledge : Concept of interior angle, angles made by an arc at the centre and on the circumference, central angle theorem.

Material Needed : Circle trig geoboard.

Construction :

Step 1 : Form a triangle using Circle Trig Geoboard as shown in the following figure.



Step 2 : Note that you have to verify that $\angle CAB + \angle ABC = \angle ACD$.

Step 3: Use a rubber band and anchor to the peg at the centre of the circle and pegs at A and C so as to form a triangle. Note that the angle made by the arc BD at the centre is equal to twice the angle made arc on the circumcentre of a circle.

Step 4 : Read $\angle AOC$. Now compute the angle $\angle ABC$ as ½ of $\angle AOC$.

Step 5 : By repeating the step 3 for the pegs at A, B and B, C compute the angles $\angle BCA$ and $\angle CAB$. Also obtain the exterior angle, $\angle ACD$ which is equal to $180^{o} - \angle BCA$.

Step 6 : Now compute $\angle CAB + \angle ABC$ and verify this value is equal to $\angle ACD$.

Activity – 21

Topic : Three dimensional Geometry

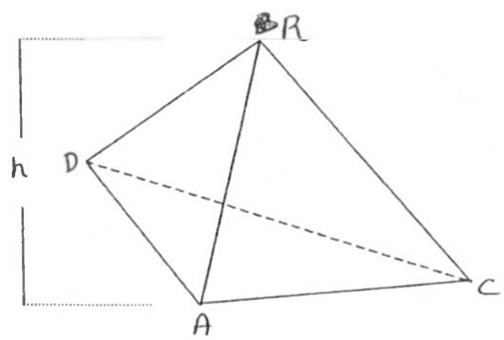
Objective : To find the volume of a Pyramid.

Pre-knowledge : Concept of pyramid, formula to find volume of a right triangle prism.

Material Needed : Solid clay triangular prism, knife, pieces of sting.

Construction :

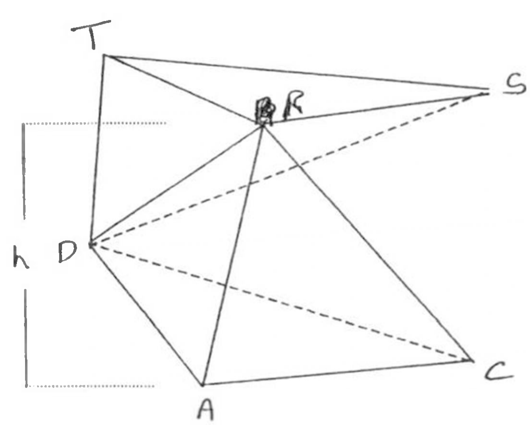
Step 1 : Consider triangular pyramid RACD, area of a base B, altitude h as shown below.



Step 2 : Now you will have to prove that the volume of RACD is one-third of the volume of a prism with the same base and altitude.

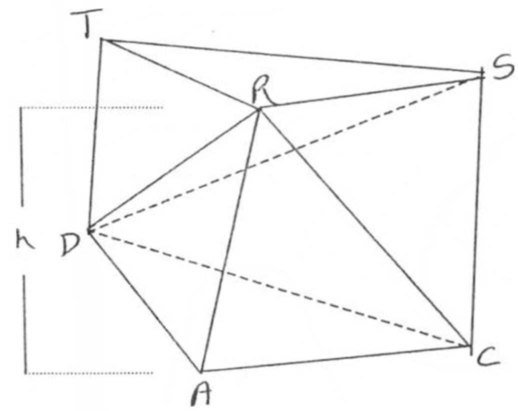
Step 3 : Build a triangular prism with ΔACD as its base by drawing a triangle congruent to ΔACD in a plane parallel to the plane of ΔACD and then joining corresponding vertices as indicated in the following steps.

Step 4 : Draw segment equal and parallel to segment AC. Draw segment RT equal and parallel to segment AD. Then draw segment ST. From this you have plane RST as parallel to plane ACE, $\angle SRT = \angle CAD$ and $\Delta RST \cong \Delta ACD$. Draw segments DS and TD.



Think of a point D as vertex of the pyramid with ΔRST as base. The base of this pyramid is equal in area to the base of the other pyramid, ΔACD and their altitudes are the same, the length h of the perpendicular between planes RST and ACD. Thus their volumes are equal. Letting V_b denote the volume of the one pyramid and V_r the volume of the other $V_b = V_r$.

Step 5 : Draw segment CS. Since segment TS is parallel and equal to segment DC, DCST is a parallelogram.



Segment DS is a diagonal of it, so $\Delta DCS \cong \Delta STD$. Each triangle is the base of a pyramid with vertex R. Since these pyramids have equal bases and the same altitude, the perpendicular from R to the plane DCST, their volumes are equal, $V_g = V_r$. Thus, $V_g = V_r = V_b$.

Step 6 : You may see not only is DCST a parallelogram, but so are ADTR and ACSR. They are the lateral faces of a triangular prism whose bases are ΔACD and ΔRST. This prism is divided into three parts, the three pyramids.
Thus $V_{prism} = V_b + V_r + V_g$. By substitution,
$V_{prism} = 3V_b$. Since $V_{prism} = Bh$, $bh = 3V_b$.

$$\therefore \; V_b = \frac{1}{3}Bh$$

Activity – 22

Topic : Circle – equal chords of a circle subtend equal angles at the circumference.

Objective : To know that if two chords of a circle are equal then they subtend the same angle at the circumference.

Pre-knowledge : Knowledge of the terms associated with a circle.

Activity :

a) **Materials Required** : A cardboard in which a circular grove is made so that a triangle cut-out may be inscribed in the circle to move freely inside the circle.

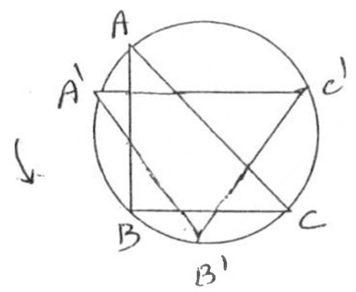
b) **How to demonstrate?**

i) Fix a triangular piece in the circular grove and mark the triangle in one position.

ii) Turn the triangle piece to a different position and mark the triangle in the new position.

iii) Since the same triangle piece changes its position, the length of a side (chord) and the opposite angle (angle subtended by the chord) do not change.

Conclusion: The angles subtended by equal chords at the circumference are equal.



Activity – 23

Topic : Geometry – A property of a circle

Objective : To know that lengths of tangents from a point outside a circle to it are equal.

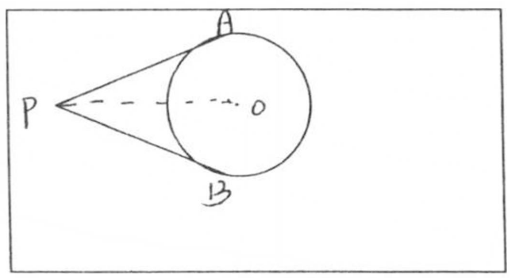
Previous Knowledge : Knowledge of tangents to a circle.

Materials Required : Horizontal Board and a circular disc with graduated scale.

Activity : A fixed circle (circular disk) is mounted on a horizontal board with a number of holes in which nails are driven. A scale (graduated) is hinged at a point P outside the circle and the scale is brought to the positions when it touches (tangent) the circle. The length of the tangents from P is read from the scale. The experiment is repeated changing the point P. For various positions P, the lengths of PA and PB are compared.

Conclusion : PA = PB.

Alternatively, if this figure drawn on a paper indicating the centre O. This can be folded along PO. Then we observe that A and B coincides and hence PA = PB.



Exercise: In the above activity, discuss it when P is outside, on and inside the circle.

Activity – 24

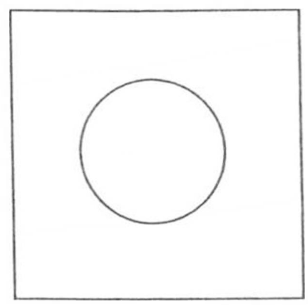
Topic : Circle – Equal chords of a circle are equidistant from the Centre.

Objective : To know that equal chords of a circle are equidistant from the centre by paper folding model.

Pre-knowledge : Knowledge of the terms associated with a circle [diameter, chord].

Material Needed : Chart.

Construction :
1. Draw a circle on the chart.
2. Cut along the circle as separate piece.



Methodology :

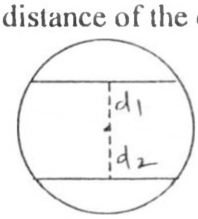
1. Fold the circle in such a way two sides coincide each other [shown in the Fig.A].

Fig A

Fig B

2. Fold the paper on your own. [Shown in Fig. B]

3. Open the fold and measure distance of the creases from the centre.



Conclusion : $d_1 = d_2$
Equal chords of a circle are equidistant from the centre.

Follow up activity : Check that the following are True by paper folding model.

1. Equal chords of a circle subtend equal angles at the centre.
2. If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.
3. The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

Activity – 25

Topic : Circumference of the Circle

Objective : To find the circumference of the circle.

Pre-knowledge : Knowledge of the terms associated with a circle.

Material Needed : A chart paper, scale, pencil, scissors, thread or tape.

Construction :

Methodology :

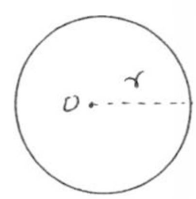
No. of circular discs are given, circumference and diameter of each disc are measured using a measuring tape and ratio of the circumference and diameter of circular discs are measured. The ratio is called π and is approximately equal to 22/7 or 3.14159.

$$\frac{Circumference}{Diameter} = \pi \text{ (constant)}$$

Circumference = $\pi \times$ Diameter

$= \pi \times 2r$

Circumference = $2\pi r$



Conclusion

The circumference of the circle of radius r is given by $2\pi r$.

Activity – 26

Topic : Area of the circle

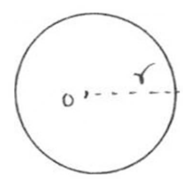
Objective : To know that the area of the circle.

Pre-knowledge : 1. Knowledge of the terms associated with a rectangle.
2. In particular, if a and b are the sides of a rectangle, then a × b is the area of the rectangle.

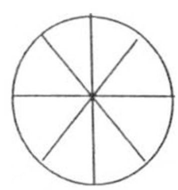
Material Needed : A cardboard with circle and rectangle shape.

Construction :

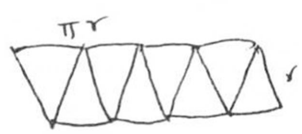
i) Draw a circle with radius r.



ii) Cut the circle into large number of equal sectors as follows :



iii) Arrange the sectors as shown below.



If the number of sectors is very large, the resultant figure will be approximately a rectangle.

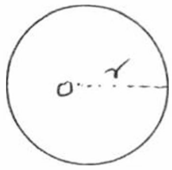
Length of the rectangle = Perimeter of the circle / 2

$$= \frac{2\pi r}{2}$$

$$= \pi r$$

The breadth will be r.

iv) Area of the circle = Area of the rectangle



Area of the rectangle = $l \times b$

$= \pi r \times r$

$= \pi r^2$ sq. units

= Area of the circle

Activity – 27

Topic : Area of the circle.

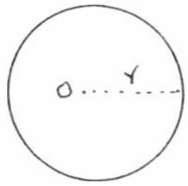
Objective : To find the area of the circle by using the area of the small sector.

Pre-knowledge : Knowledge of the terms associated with a circle and triangle.

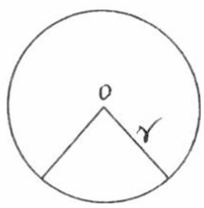
Material Needed : Chart paper, pencil, scissors.

Construction :

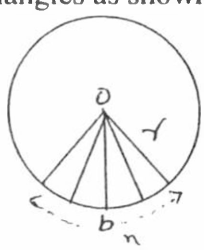
i) Take a circle of radius 'r'.

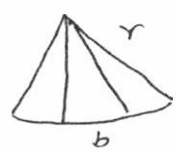


ii) Draw a sector of the circle.



iii) Divide the sector into 'n' thin triangles as shown in the figure.





Area of each triangle = ½ . b . r

The total area of the sector A = n × (½ . b . r)

= ½ r l (l = nb) length of the arc.

= ½ . r × (arc length of the sector)

The area of the circle= ½ r . circumference of circle

$= \frac{1}{2} \times r \times 2\pi r$

$= \pi r^2$

Activity – 28

Topic : Geometry

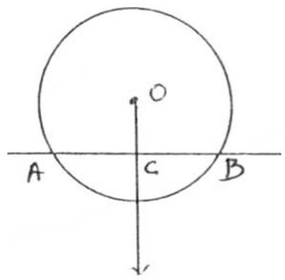
Objective : Every chord of a circle is bisected by the perpendicular from the centre.

Pre-knowledge : Circle, chord of a circle.

Material Needed : Drawing board and a horizontal scale.

Construction :

Consider a circle drawn on a vertically mounted drawing board. A horizontal scale is fixed.



A string hangs from the centre of the circle carrying a bob (weight) at the lower end, passing through C. C is the midpoint of AB and OC is the perpendicular bisector of the chord AB. This can be checked for various position of AB.
Note : If a circular protractor is fixed at O, we can observe that OC bisects $A\hat{O}B$.

Conclusion :
The perpendicular from the centre of a circle to any of its chord is
a) the perpendicular bisector to the chord
b) the bisector of the angle subtended by the chord at the centre.

Activity – 29

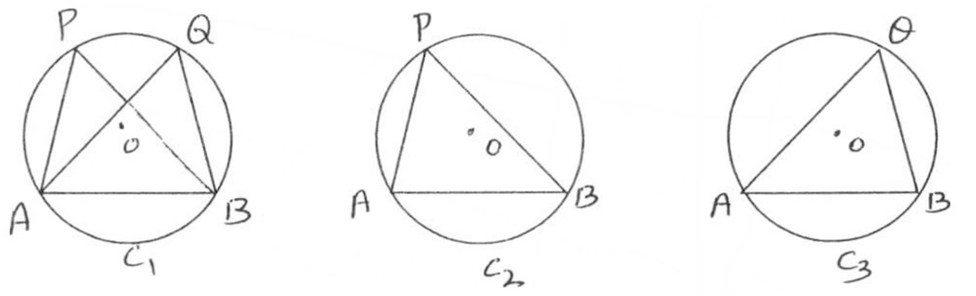
Topic : Geometry

Objective : A given chord in a circle subtended the same angle at every point on its circumference.

Pre-knowledge : Circle, chord of a circle.

Material Needed : Three transparent circular paper sheets

Construction :



Take three transparent circular discs C_1, C_2, C_3 of the same size.

On C_1, draw a chord AB and construct two angles $A\hat{P}B$ and $A\hat{Q}B$, taking points P and Q on the circle.

On C_2, show the chord AB and $A\hat{P}B$.

On C_3, show the chord AB and $A\hat{Q}B$.

Mount C_1, C_2, C_3 on the same pivot at the centre. By rotating, bring C_1, C_2, C_3 to the same position on C_1.

Rotate C_3 around O so that Q is brought to the position of D.

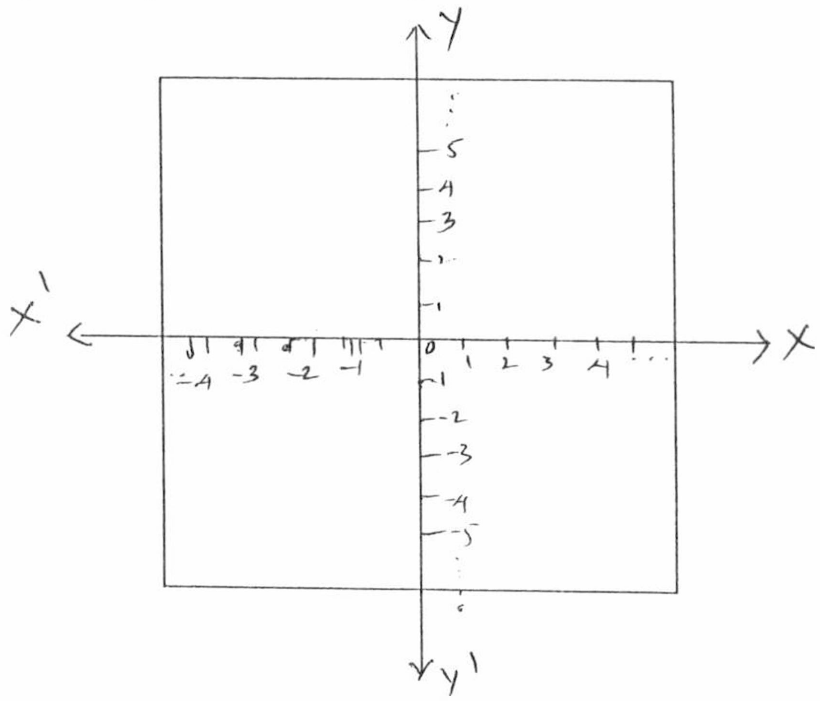
We find $A\hat{P}B = A\hat{Q}B$.

COORDINATE GEOMETRY

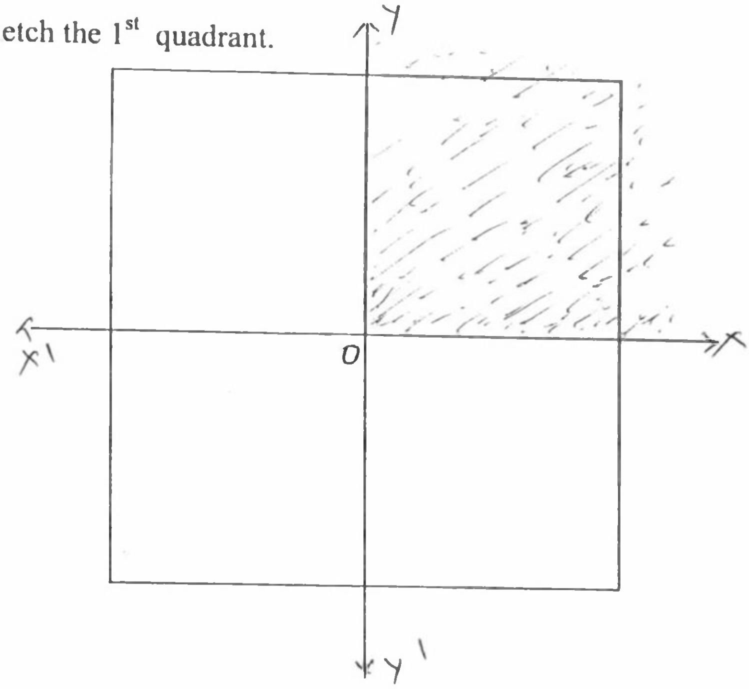
Activity – 30

Topic	:	**Co-ordinate Geometry**
Objective	:	To understand four quadrants and co-ordinates of a point.
Pre-knowledge	:	x-axis and y-axis in the coordinate plane. Meeting point of x-axis and y-axis is origin. Positive numbers and negative numbers.
Material Needed	:	Chart, colour pencils and scale.

i) Represent a coordinate plane as follows.

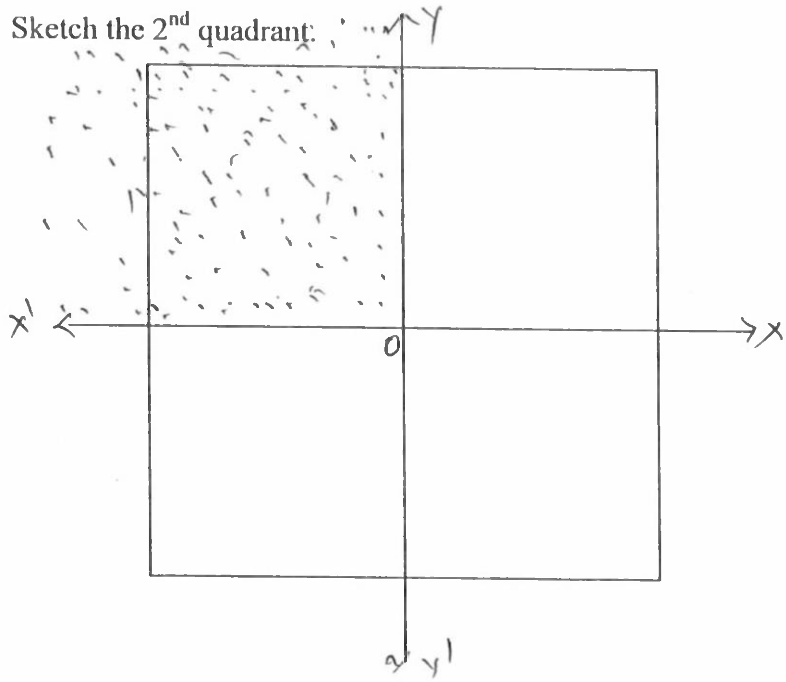


ii) Sketch the 1^{st} quadrant.



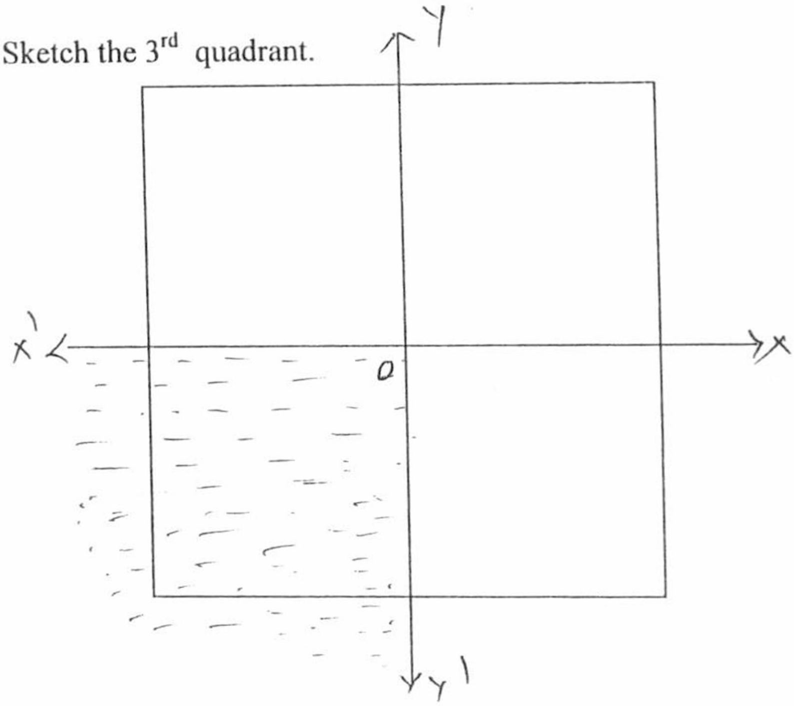
Here we see that $x > 0$ and $y > 0$.

iii) Sketch the 2^{nd} quadrant.



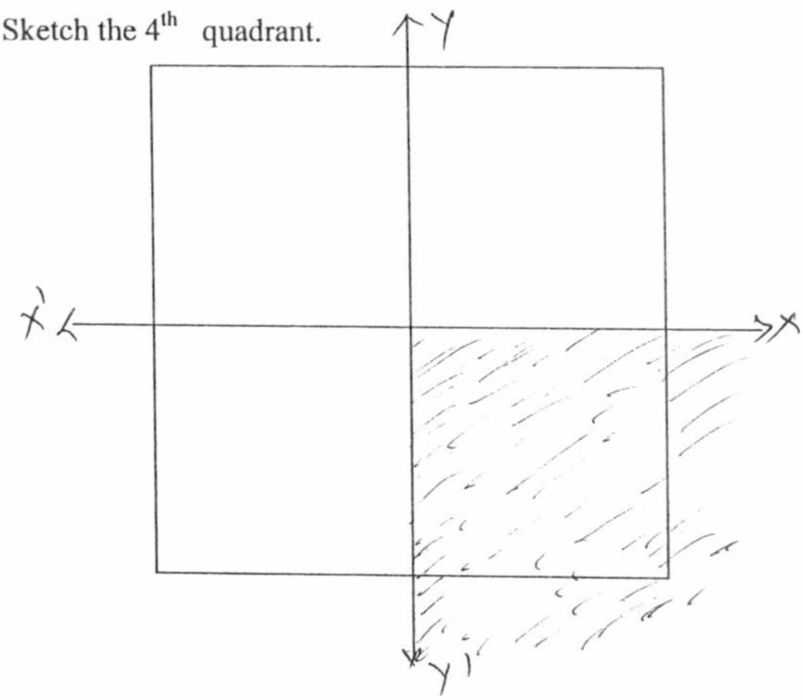
Here we see that $x < 0$ and $y > 0$.

iv) Sketch the 3rd quadrant.

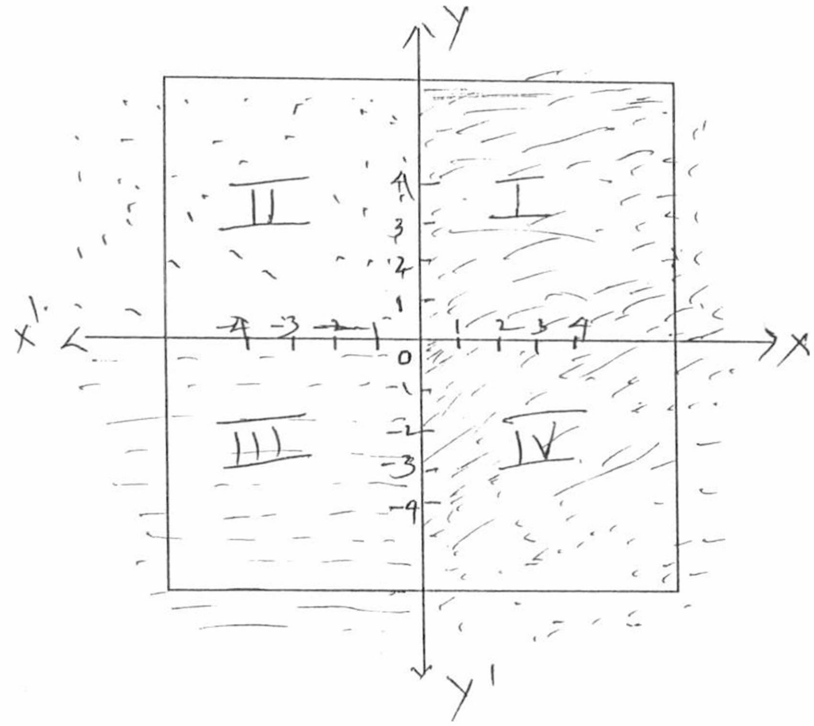


Here we see that $x < 0$ and $y < 0$.

v) Sketch the 4th quadrant.



Here we see that $x > 0$ and $y < 0$.



Summary : Any point in the coordinate plane either belongs to a quadrant or lies on axis.

Exercise : 1. The point (2,3) lies in the first quadrant. Find the position of the points (-2,3), (2,-3) and (-2, -3).

1. Mark the points (-4, 0), (0, -1), (2,0), (0,3) on the graph sheet.

Activity – 31

Topic : **Co-ordinate Geometry**

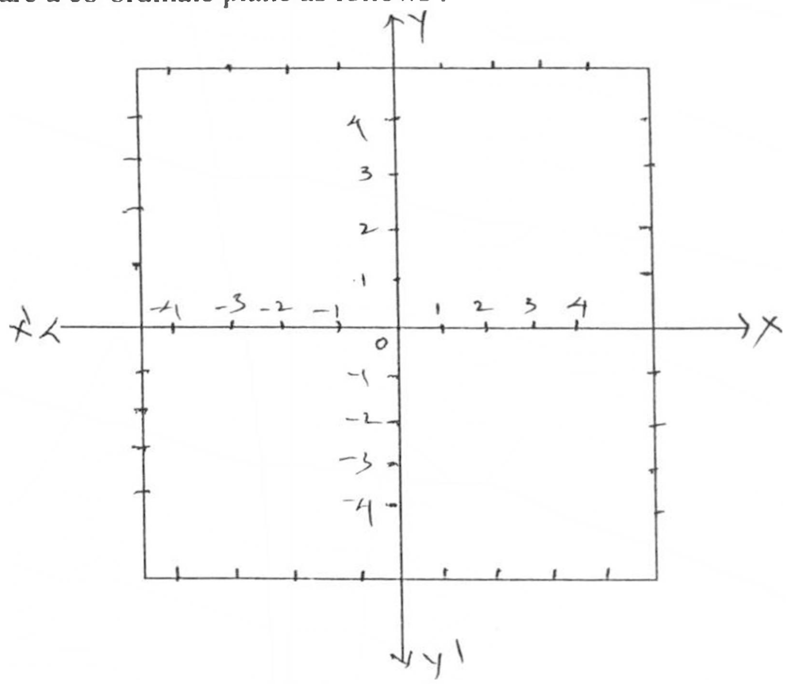
Objective : To locate the given points in the co-ordinate plane.

Pre-knowledge : Knowledge of co-ordinate plane.

Material Needed : Rubber band, beads, hard board, nails.

Activities :

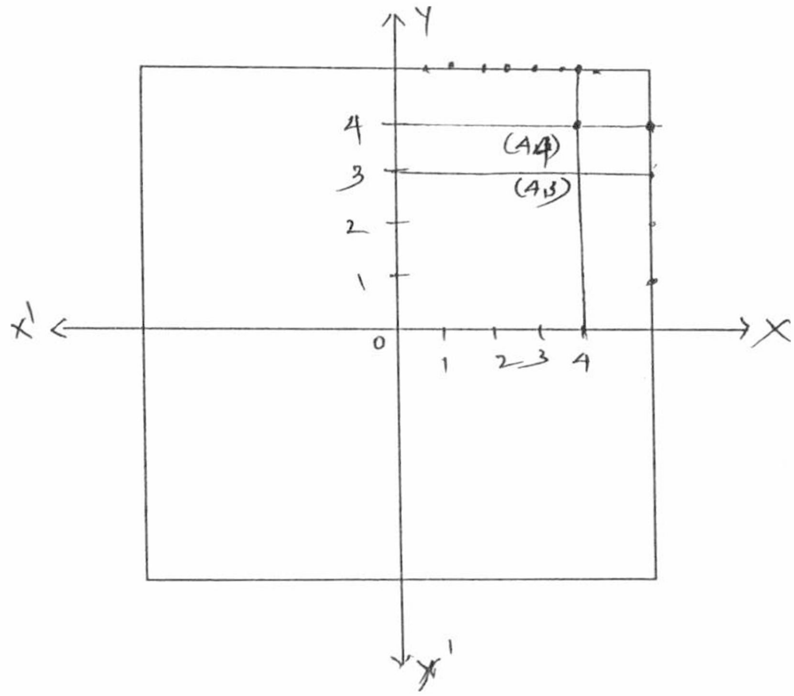
i) Prepare a co-ordinate plane as follows :



Fix nail in each number and opposite to the corresponding number. (As drawn in the figure). Note : is nail.

ii) For example, locate the point (4,3). Now, tie a rubber band with a bead from number 4 in x-axis to the corresponding opposite nail and tie another rubber band with a bead from number 3 in y-axis to the corresponding opposite nail. The rubber band should intersect each other.

Do the above process in the co-ordinate plane as follows:



The intersecting point of the rubber bands is known as (4,3). This activity has to be repeated till the students become familiar to locate the points in all four quadrants.

Activity – 32

Topic : Linear Equations

Objective : To demonstrate an easy method of drawing the graph of a linear equation.

Pre-knowledge : The equation $ax + by + c = 0$ represents a straight line.

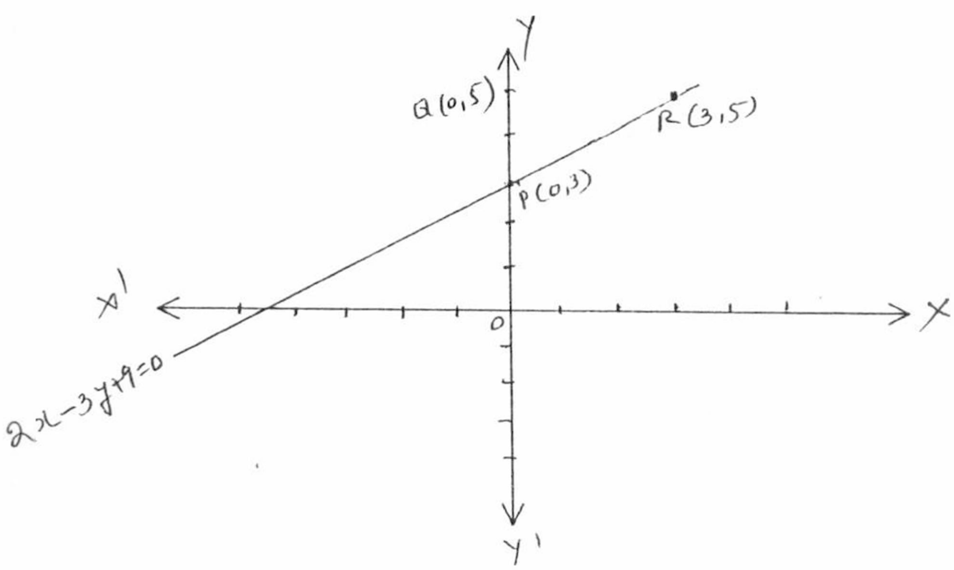
Material Needed : 1. Graph sheets, 2. Geometry Box.

Activities :

- Let $2x - 3y + 9 = 0$ be the given equation.
- Reduce it to the form $y = mx + c$.

  $$y = \frac{2}{3}x + 3$$

- Comparing with $y = mx + c$ we get $m = \frac{2}{3}$ and $c = 3$.
- On a graph sheet draw the co-ordinate axes to intersect at O.
- Since $y = 3$ when $x = 0$ in the given equation, mark a point P(0,3).
- From P move 2 units on y axis to Q (0,5). (Why?) and from Q move 3 units horizontally in the positive direction to reach the point R(3,5) as the slope $m = \frac{2}{3}$. Join PR and extend.



Follow up activity : 1. Let A = (-3,1) and B = (3,3). Does the points A and B lie on the line $2x - 3y + 9 = 0$? Explain.

2. Draw the graph of

i) $2x + 3y + 9 = 0$

ii) $2x - 3y - 9 = 0$

iii) $2x + 3y - 9 = 0$

Activity – 33

Topic : Linear equations

Objective : To verify the conditions for consistency of a system of linear equations in two variables graphically.

Pre-knowledge : Graphing linear equations in two variables.

Material Needed : 1. Graph sheets, 2. Geometry box.

Activities :

Case (i) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

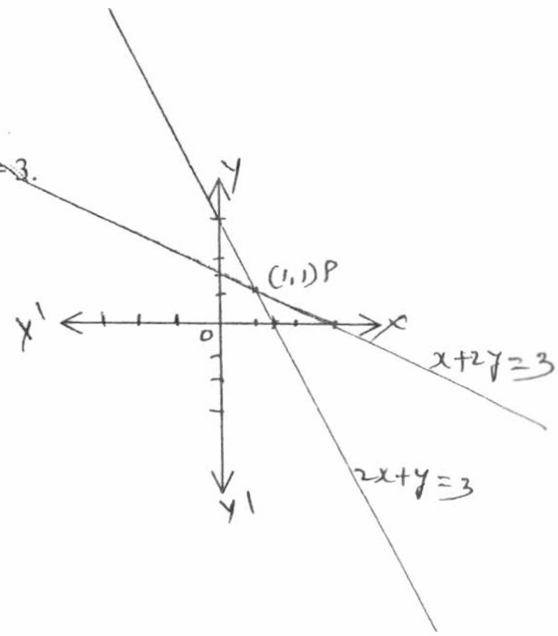
- Consider the equations x + 2y = 3 and 2x + y = 3.

Here $a_1 = 1, b_1 = 2, c_1 = -3$ and
$a_2 = 2,\ b_2 = 1, c_2 = -3$.

Now, $\frac{a_1}{a_2} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{2}{1}$

i.e. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

- For each of the two equations, draw the graph on a same graph paper.



Case (ii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

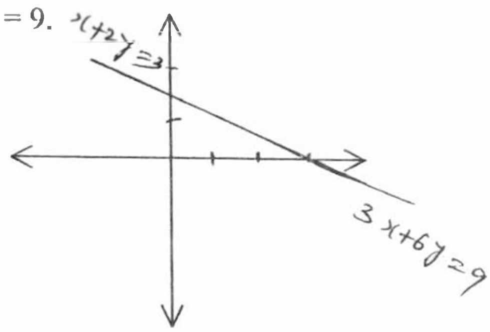
- Consider the equations x + 2y = 3 and 3x + 6y = 9.

Here $a_1 = 1,\ b_1 = 2, c_1 = -3$ and
$a_2 = 3,\ b_2 = 6,\ c_2 = -9$

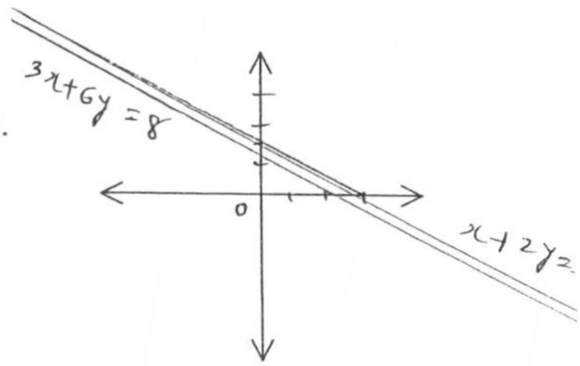
Now $\frac{a_1}{a_2} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{1}{3}$

i.e. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

- For each of the two equations draw the graph on the same graph paper.



Case (iii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.



* Consider the equations x + 2y = 3 and 3x + 6y = 8.

Here $a_1 = 1$, $b_2 = 2$, $c_1 = -3$ and
$a_2 = 3$, $b_2 = 6$, $_2 = -8$.

Now $\frac{a_1}{a_2} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{3}{8}$

i.e. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

- For each of the two equations draw the graph on a same graph paper.

Observation :

1. In case (i) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the graph of the two equations intersect in one point.
2. In case (ii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the graph of the two equations is the same line.
3. In case (iii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the graph of the two equations are parallel to one another.

Conclusion : The system of linear equations in two variables of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is

i) consistent with unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ [The graph is a pair of intersecting lines].

ii) Consistent with *infinite* solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ [The graph is a pair of overlapping lines].

iii) *inconsistent* if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ [The graph is a pair of parallel lines].

Follow up activity : Check the consistency of the following system of equations.

i) $x + 3y = 4$
$2x + 6y = 8$
$3x + 9y = 12$

ii) $2x + y = 6$
$x - y = 0$
$x + y = 4$

iii) $3x + y = 8$
$2x - y = 2$
$x + y = 0$

Activity – 34

Topic : Trigonometry

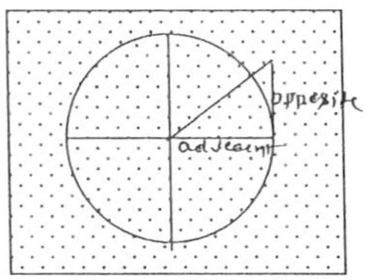
Objective : To find trigonometric ratios of any angle – sine, cosine, tangent.

Pre-knowledge : Concept of Centre, circumference and radius of a circle, right angled triangle.

Material Needed : Circle trig Geoboard and some rubber bands. (In this Circle Trig geoboard, the circumference is divided into 360 parts so as to represent each part as one degree. Horizontal and vertical scales are marked on the board).

Construction :

Step 1 : Anchor a rubber band at the centre of the board and then pull it to just edge of the geoboard so that it passes over the desired angle (for present only upto 45^o) as shown in the following figure.



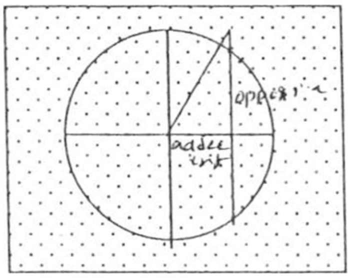
Step 2 : Read the value where rubber band crosses the right side tangent line.

Step 3 : Read the radius of the circle and the angle made by the rubber band at the centre with the help of the markings on the circumference.

Step 4 : Find the ratio of the opposite and adjacent sides of the triangle shown. This ratio, $\frac{opposite}{adjacent}$ is the tangent (tan) value of the angle measured at Step 3.

Step 5 : Change the angles at 30^o, 45^o and calculate the values of tan 30^o. tan 45^o.

Step 6 : Similarly form the triangles corresponding to the angles between 45^o and 90^o as shown below.

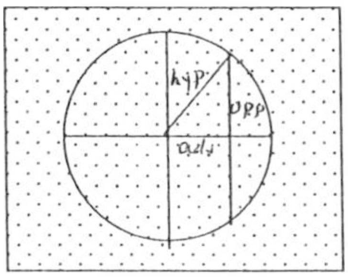


Step 7 : Read the angle, adjacent and opposite sides.

Step 8 : Calculate ratio $\frac{opposite}{adjacent}$ to get the tangent of the angle under consideration.

Step 9 : Observe that tan 45^O = 1 and tan 90^o is undefined as the adjacent is zero in this case. Similarly tan 0^o = 0. Also observe that tangent of an angle increases as the angle increases from 0^o and 90^o.

Step 10 : Form a triangle corresponding to a particular angle as shown in the figure.



Step 11 : Read the angle, opposite, adjacent and hypotenuse of the triangle. Then calculate $\frac{opposite}{hypotenuse}$. This gives the value of sine of the angle measured. Similarly calculate the cosine of the angle by the value of $\frac{adjacent}{hypotenuse}$. Here hypotenuse is equal to the radius of the circle.

Step 12 : Using the above procedure, calculate sine and cosine of 0^o, 15^o, 30^o, 45^o, 60^o, 75^o and 90^o.

Activity – 35

Topic : Trigonometry

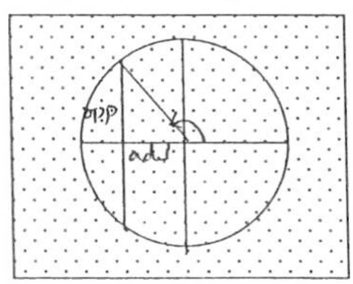
Objective : To calculate sine, cosine and tangent of angles between 90^0 and 360^0.

Pre-knowledge : Definitions of sine, cosine and tangent of an angle, coordinate axes, legs and hypotenuse of a right angled triangle.

Material Needed : Circle trig geoboard and rubber bands.

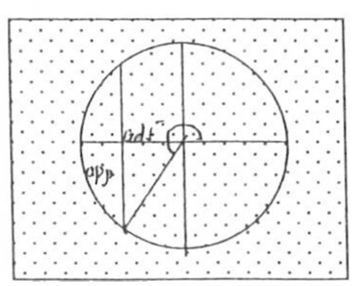
Construction :

Step 1 : In geoboard form a triangle as shown in the following figure corresponding to any angle between 90^0 and 180^0 and use the definitions as in the earlier activities to find the values of sine, cosine and tangent. Consider the signs of the values of adjacent and opposite as of coordinate axes in that quadrant.



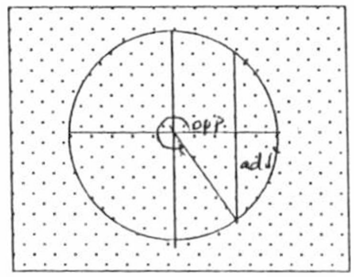
Step 2 : Find sine, cosine and tangent values of 120^o, 135^o, 150^o, 180^o.

Step 3 : Corresponding to any angle between 180^0 and 270^o, form the triangle as shown below. You can find the values of sine, cosine and tangent, etc. of the given value as above.



Step 4 : Find sine, cosine and tangent values of 210^o, 225^o, 240^o, 270^o.

Step 5: To find the values of sine, cosine and tangent of any angle between 270° and 360° consider the triangle as shown below.



Step 6 : Use the definitions and find the required ratios as in the above activities. Don't forget to consider signs of the measurement of 'opposite' and 'adjacent'.

Step 7 : Find sine, cosine and tangent values of 300°, 315°, 330°, 360°.

Activity – 36

Topic : Trigonometry

Objective : To compute the values of cosecant, secant, cotangent of any given angle.

Pre-knowledge : Definitions of cosecant, secant and cotangent as ratios and as respectively the reciprocals of sine, cosine and tangent of the given angle.

Material Needed : Circle Trig geoboard and rubber bands.

Construction :

Step 1 : Use similar steps of the previous activities and compute the values. Also try to find the values by computing the reciprocals of sin, cos and tan values of the required angle respectively.

Step 2 : If the radius of the circle is one then that circle under consideration is called a unit circle and the measures of adjacent and opposite sides are the x and y coordinates of the respective points. In this case, sine, cosine and tangent values can be calculated with the help of these x and y coordinates.

Activity – 37

Topic : Trigonometry

Objective : a) To demonstrate the basic trigonometric identities like $\tan = \frac{\sin}{\cos}$

b) To find sines, cosines and tangents of complementary angles, supplementary angles and opposite angles.

Pre-knowledge : Knowledge of sin and cos identities.

Material Needed : Circle Trig geoboard and rubber bands.

Construction :

Step 1 : Use the individual values of sine, cosine, tangent, etc. obtained earlier to verify different identities like $\cot = \frac{\cos}{\sin}$, etc.

Step 2 : Using the values of sin, cos and tan verify that
$\sin(90 - \theta) = \cos(\theta)$, $\cos(90 - \theta) = \sin(\theta)$, $\tan(90 - \theta) = \cot(\theta)$
$\sin(180 - \theta) = \sin(\theta)$, $\cos(180 - \theta) = -\cos(\theta)$, $\tan(180 - \theta) = -\tan(\theta)$
$\sin(-\theta) = -\sin(\theta)$, $\cos(-\theta) = \cos(\theta)$, $\tan(-\theta) = -\tan(\theta)$.

Activity – 38

Topic : Trigonometry

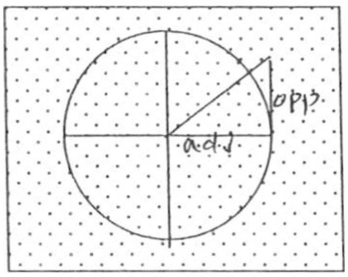
Objective : To find the angles corresponding to given trigonometric ratios (Inverse functions)

Pre-knowledge : Concept of inverse trigonometric functions and trigonometric ratios.

Material Needed : Circle trig geoboard ;and rubber bands.

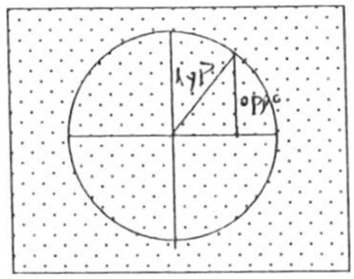
Construction :

Step 1 : Form the triangle with the given value of $\frac{opposite}{adjacent}$ as shown in the following figure. To form the triangle, we have to calculate the opposite side with the help of the given ratio and radius of the circle and by adjusting the rubber band so as to equate the opposite side to the calculated value.



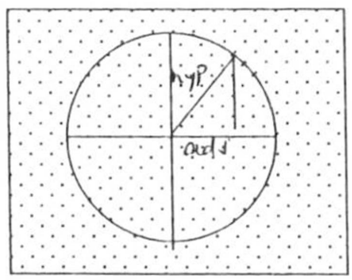
Step 2 : Now measure the angle at the centre. This angle is equivalent to $\tan^{-1}$ (given ratio), that is arctan.

Step 3 : As in step 2, form the triangle as shown in the following figure corresponding to the given ratio, $\frac{opposite}{hypotenuse}$ as sine of an angle. Here we to find arcsin. To form the triangle, it is required to calculate opposite side with the help of radius and the given ratio. Adjust to form the triangle having opposite side is equal to the above calculated value.



Step 4 : Read the angle at the centre. This gives the arcsin corresponding to the given ratio.

Step 5 : As in the above steps, form a triangle corresponding to the given ratio $\frac{adjacent}{hypotenuse}$ to find arcos as shown in the following figure.



Step 6 : Read the angle at the centre. This gives the arcos of the given ratio.

Step 7: Using the above steps, find arcsin arcos and arctan of 0, ½ , 1.

Step 8 : To find arccot, arc cosec and arc sec use arctan, arcsin and arc cos values. For example, we know $\cot^{-1} \frac{1}{2} = \tan^{-1} 2$.

Activity – 39

Topic : Geometry

Objective : To demonstrate by paper folding and cutting the formula for the

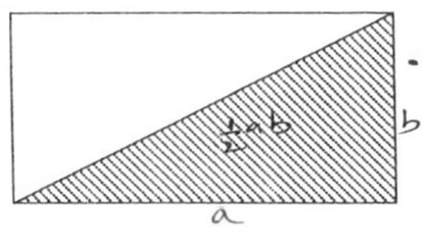
a) area of a right angled triangle
b) area of any quadrilaterals with same height and bases are equal
c) area of any triangle
d) area of parallelogram
e) area of quadrilateral

Pre-knowledge : Knowledge of triangles and quadrilaterals

Material Needed : Paper cuts of triangles and quadrilaterals

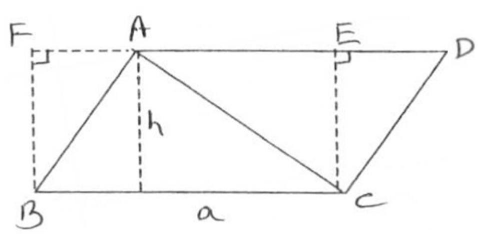
Construction :

a) Cut out a rectangle. Cut it into two identical right angled triangles, by cutting along a diagonal.



The area of the right angled triangles with sides a and b is equal to ½ the area of the rectangle.
$\therefore A = \frac{1}{2}ab$

b) Cut a parallelogram ABCD such that half of it is a triangle ABC.

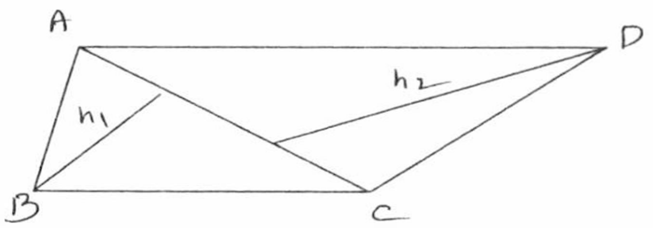


Cut out the right angled triangle CDE and join it to the position BAF.
Then the area of the parallelogram ABCD is equal to the area of the rectangle BCEF which is equal to ah.

c) The area of the triangle ABC = ½ ah.

d) Area of the parallelogram = ah = base height.

e) Given a quadrilateral ABCD with AC = d and perpendicular from B and D to AC are h_1 and h_2.



Area of ABCD = ΔABC + ΔACD

= ½ AC h_1 + ½ AC × d_2

= ½ AC (h_1 + h_2)

= ½ d (h_1 + d_2)

Activities on Paper folding

1. **Create a line :** Take a sheet of paper. Fold the paper once. The fold got is a straight line.
2. **Create a point using two Intersecting Lines :** Fold the paper twice so that one fold cuts the other fold. The intersection of the two folds (two lines) is a point (the point of intersection of the lines).
3. **Show that one and only one line passes through two given distinct points.** Mark two points on a sheet of paper. Fold it so that, it contains both the marked points. A single fold (only one line) is got.
4. **Create a perpendicular to a line:** Fold the sheet. One line (l) is got. Refold the sheet so that the fold passes through a point on l and the path of the line are brought to coincide. The second fold got is the line (l^1) perpendicular to the given line l.
5. **Create a perpendicular to a given line through a point (a) on the given line, (b) outside the given line.**
   a) Fold the sheet to get the given line l. Mark a point P on l. Fold the paper through P perpendicular to l. The line l^l so got passes through P and is perpendicular to l.
   b) Mark P outside the line l (obtained by folding the sheet). Fold the sheet through P perpendicular to l. The line l^l so got passes through P and is perpendicular to l.
6. **Fold a pair of parallel lines and create a parallelogram.**
   Fold the sheet along a line l_l on a rectangular sheet of paper. Fold the sheet again so that the fold got $l_l{}^l$ is parallel to the earlier fold l_l. Now l_l is parallel to $l_l{}^l$.
   Similarly create two parallel lines l_2 and $l_2{}^l$ where l_l and l_2 intersect on the sheet. The two pairs of lines l_l, $l_l{}^l$, l_2, $l_2{}^l$ form a parallelogram.
7. **Create (a) the perpendicular bisector of a line segment, (b) the angle bisector of a given angle.**

a) Mark a line *l* and mark points A and B on *l*. Fold the sheet so that A and B are brought to coincidence. The fold so got is perpendicular bisector of AB.

b) Create an angle with vertex at O. Fold so that the crease passes through 'O' and the arms of the angle are brought to coincidence. The fold got is the angle bisector.

8. **Getting the (a) centroid, (b) orthocentre, (c) incentre and (d) the circumcentre of a triangle. Mark a triangle ABC on a sheet of paper.**

a) Mark the midpoints of the sides BC, CA and AB as D, E, F respectively. Fold the sheet thrice passing through A, D, D, E and E, F. The folds AD, BE and CF which are the medians pass through the centroid G of the triangle.

b) Fold the sheet thrice so that the folds passes through the vertices and perpendicular to the opposite sides. These folds are the altitudes of the triangle passing through the orthocentre of the triangle.

c) Fold thrice to get the angle bisector of the triangle which passes through the incentre of the triangle.

d) Fold thrice to get the perpendicular bisectors of the sides. These pass through the circumcentre of the triangle.