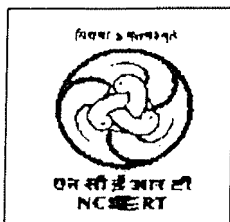


**TRAINING PROGRAMME ON DIAGNOSIS
AND REMEDIATION OF DIFFICULTIES IN
LEARNING MATHEMATICS FOR TEACHERS
OF A.P. RESIDENTIAL SCHOOLS SOCIETY**

Report

Dr B S P Raju
Programme Coordinator



REGIONAL INSTITUTE OF EDUCATION, MYSORE – 6
[National Council of Educational Research and Training]

DECEMBER 2004

PREFACE

The PAC programme titled "Training Programme on Diagnosis and Remediation of Difficulties in Learning Mathematics for teachers of Andhra Pradesh Residential Schools Society at Secondary Level" was taken up at the request of the A.P. Government

It is evident from the syllabi of preservice courses that much less emphasis was given on the aspects of diagnosis and remediation of students' learning difficulties in Mathematics. Also since there were many changes in the curriculum at secondary level in recent years and this requires the development of the skills and competencies of a professional quality among teachers of mathematics and so this programme is designed to equip the teachers to develop teaching strategies which cope with various learning difficulties of students.

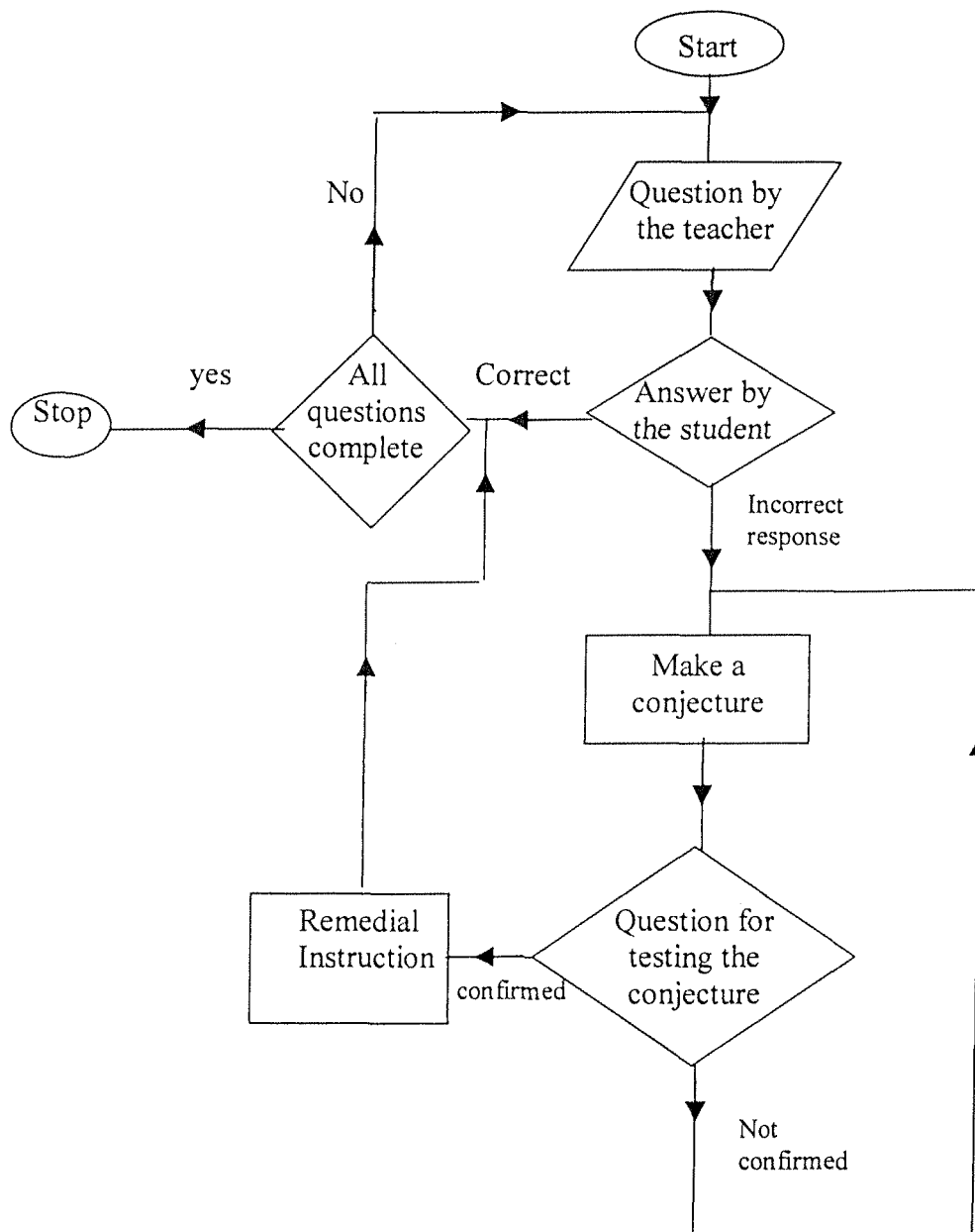
Main specific objectives of the programme are

- a) to diagnose the difficulties of students in Mathematics
- b) to identify suitable teaching strategies for the identified learning difficulties.

The programme was held here at RIE, Mysore from 6.12.04 to 13.12.04. In all, 18 teachers of mathematics from A.P. Residential Schools Society participated in the programme. They have been given training in analyzing a concept, the ways of teaching a concept, the different strategies of teaching a concept that leads to the difficulties in learning the concepts, conjecturing the cause of errors, the testing of

the conjectures and to prepare remedial teaching to remove the difficulty. (See the following flow chart).

Flow Chart for Diagnosis and Remediation for Difficulties in Learning Mathematics



CONTENTS

	Page No.
1. Introduction	1
2. Approach Paper	4
3. Time Table	9
4. Teaching of Mathematical Concepts	10
5. Sample Answer Sheet of the Pretest	19
6. Sample Content analysis by the Participant	31
7. Diagnosis and Remediation of difficulties in learning Concepts – K Dorasami	34
8. Sample Answer sheet of the Post-test on Content Categories on the Lesson – Circle	36
9. Sample Material developed by the Participants after the inputs are given.	37
10. A copy of research paper “An Inspiration to Learn and Teach Mathematics” by Dipendra N Bhattacharya	41
11. Instruction Comprehension Test (Anonymous)	44
12. List of Participants	45

Introduction

Recently many national reports like NCFR has expressed much concern for the professional development of teachers of mathematics. As a result, there were many inservice programmes for mathematics teachers that includes (i) content enrichment, (ii) identification and experience with “what if” situations, (iii) problem solving; (iv) solving problems in multiple ways; (v) facilitating discussions and questioning, (vi) diagnosis and remediation; (vii) manipulatives as one way to explore mathematical content and (viii) proofs in mathematics; at all levels being conducted by NCERT, RIE, SCERT, etc. so that the teachers at different locations can meet with each other to share concerns, confer about individual students, and plan mathematics and inter-disciplinary programmes, clarifying their misconceptions in content and pedagogy and plan their instruction.

As a part of improving the school mathematics instruction through professional development of the teachers at the request of the A.P. Government, this “Training Programme on Diagnosis and Remediation of Difficulties in Learning Mathematics for Teachers of A.P. Residential Schools Society” is being taken up here at RIE, Mysore for secondary school maths teachers of A.P. School Society from 6.12.2004 to 13.12.2004.

The reasons for selecting this particular theme is evident from the syllabi and preservice courses for teachers that much less emphasis was given on the aspects of diagnosis and remediation of students’ learning difficulties in mathematics. In the absence of a training programme, the

process of identification of learning difficulties is not easy and that of remediation is even more difficult.

In order to identify learning difficulties of students in Mathematics, the teachers should know what they are doing when they teach, why they are doing what they do and how consciously and deliberately they are doing the activities to increase students' learning. To know about these the teachers (the participants) are asked to answer a set of 10 questions; each having approximately 10 each; the first one about the identifying the content categories, the second one about the correctness of definitions; the third one about the hierarchy of the concepts, fourth one about the restating the definitions in terms of its much simpler terms (technically speaking, using the concepts in such superordinate concepts in the hierarchic level), the fifth one about the necessary and sufficient conditions i.e. to identify the sufficiency of the essential attributes; the sixth is about creating the counter examples, the seventh is to identify the moves in teaching the activities, the eighth one about evaluation, the ninth one about learning difficulties and the last one about the instructional objectives.

Since the teachers were not expected, this type of teaching learning process, as evident by the responses from the above test (a response sheet is enclosed for reference); the programme coordinator has discussed along with Prof K Dorasami about the content categories, strategies of teaching and learning concepts and generalizations and teaching and learning generalizations.

The deliberations were held in the regional language Telugu to make the learning more meaningful.

In order to keep the sustenance of interest among the participants, a copy of the research paper by Dipendra N.Bhattacharya on “An Inspiration to Learn and Teach Mathematics” and also a new way of finding the logarithm of a number of any base by B.S.P.Raju has been discussed and also an instruction Comprehension Test is also given and found that almost all the participants except one has failed.

Approach Paper

Students differ in intellectual ability to abstract, generalize, reason and remember. Because of these varying abilities, some students learn readily and usually understand what they are taught, while others are not.

The trouble in understanding the concepts, principles may be due to several factors like physiological, social, emotional, intellectual and pedagogical. Here we limit ourselves to the difficulties of the students in learning mathematics that are due to pedagogical factors.

For the students who have trouble in understanding the concepts, principles, etc. the teacher can diagnose the trouble and provide remedial instruction.

In this training programme, we are going to answer the following questions?

1. What are the difficulties students typically manifest in learning mathematics?
2. How a teacher can improve in diagnosis of the difficulties of the students that manifest in learning?
3. How remedial teaching can be done to remove the difficulties of the students in learning mathematics?

Steps in Diagnosis

1. **Discover which students have difficulty :**
 - a) When students cannot answer certain questions

- b) When students cannot apply concepts and principles they have been taught
- c) Make the same mistake repeatedly.

Exercise 1

A 12 year old boy in grade 6, solved the following problems of converting fraction to decimals as given below :

1. $\frac{2}{10} = 1.2$

2. $\frac{5}{10} = 1.5$

3. $\frac{27}{15} = 4.2$

4. $\frac{4}{6} = 1.0$

5. $\frac{429}{100} = 5.29$

6. $\frac{3}{2} = .5$

7. $\frac{2}{3} = .5$

Steps :

- 1. Add the numerator and denominator.
- 2. Insert a decimal point after the 1st digit if there are two or more digits obtained in the step 1. Otherwise insert the decimal point to the left of the first digit.

Note: It is a faulty procedure.

Exercise 2

Adding Decimals

$$.3 + .4 = .07$$

Steps :

1. Add the numbers.
2. Count the number of digits to the right of decimal point in '.3'.
Count the number of digits to the right of decimal point '.4' , add the result ($1+1=2$).
3. Locate the decimal point in the output of step 1 so that there are two digits to the right of the decimal point.

Note: The procedures used are important ones, with legitimate roles to play. But the procedure used in step 3 should not have been called into action in this situation.

2. **What kind of errors a student or a group of students are making ?**
 - a) May be able to state part of a definition but not the complete definition.
 - b) May be able to repeat a statement of a principle given in the textbook but not be able to state it in their own words or give instances of it.
 - c) May not be able to abstract a pattern from a set of instances and hence not able to discover a generalization the teacher is trying to teach them.
 - d) May be able to apply a principle when the teacher or textbook tells them that it is relevant but not be able to decide which principles are relevant when faced with a problem to solve.

3. Conjecturing the cause for errors

Errors are observable while cause of errors (why the error is made) is not. But it can be inferred.

Steps in finding the causes for errors :

- a) Recalls how he taught the particular concept, principle or skill.

Ex: *For subtraction of rational numbers, the principle taught was :*

To subtract b from a , add the additive inverse of b to a .

- b) There was plenty of practice with a variety of exercises.
- c) The particular student appeared attentive while the operation was explained.
- d) he has done the home work.

Possible Conjectures

- a) The student does not know what an additive inverse is.
- b) The student does not have the concept of additive inverse of a rational number.
- c) The student does not know how to add two rational numbers.

4. Testing the conjectures

Use the conjecture as hypothesis and make some predictions by means of it. If the predictions are confirmed by the subsequent data, then the confidence in the hypothesis is enhanced.

Example:

For the conjecture

“the student does not know what an additive inverse is” in step3;

the testing can be done as follows :

- a) Ask the student to state the additive inverse of several rational numbers.

- b) Ask the student to identify the additive inverse of a given rational number, in a multiple choice test item.

If the students answer for both the above questions wrong, then most probably your conjecture is correct.

Remedial Teaching

Once the conjecture has been determined, the teacher can decide what kind of remedial teaching to employ.

If the above conjecture is confirmed in step 4; then remedial teaching can be as follows:

Make students understand the concept of additive inverse, give several examples and also enable them to subtract a rational number from another.

**TRAINING PROGRAMME ON DIAGNOSIS AND REMEDIATION OF DIFFICULTIES IN LEARNING MATHEMATICS
FOR TEACHERS OF A.P. RESIDENTIAL SCHOOLS SOCIETY
(6 – 13 December 2004)**

Days	9.30 – 11.10	11.10-11.25	11.25-1.05	1.05-2.05	2.05-3.35	3.35-3.50	3.50-5.20
6.12.2004 Monday	Registration and Inauguration	Pre-test (B S P Raju)		Teaching Content Categories (B S P Raju)		Content Categories (B S P Raju)	
7.12.2004 Tuesday	Strategies of Teaching and Learning of Concepts (B S P Raju)	Strategies of Teaching and Learning Concepts (K Dorasami)		Presentation and Discussion on Teaching and Learning of Concepts – Group Work (B S P Raju)		Presentation and Discussion on Teaching and Learning of Concepts – Group Work (B S P Raju)	
8.12.2004 Wednesday	Diagnosis and Remediation (B S P Raju)	Identification of Difficulties in Learning Concepts (B S P Raju)		Remediation in Learning Concepts – Group work (B S P Raju & K Dorasami)		Remediation in Learning Concepts – Group Work (B S P Raju & K Dorasami)	
9.12.2004 Thursday	Simulated Lesson on Diagnosis and Remediation in Learning Concepts (B S P Raju)	Simulated Lesson on Diagnosis and Remediation in Learning Concepts (B S P Raju)		Strategies of Teaching and Learning of Generalizations (B S P Raju)		Strategies of Teaching and Learning of Generalizations (B S P Raju)	
10.12.2004 Friday	Identification of Difficulties in Learning Generalisation (B S P Raju)	Identification of Difficulties in learning generalization (B S P Raju)		Presentation and Discussion on Remediation in Learning Generalisation – Group Work (B S P Raju)		Presentation and Discussion on Remediation in Learning Generalisation – Group Work (B S P Raju)	
11.12.2004 Saturday	Simulated Demonstration Lesson on Diagnosis and Remediation in learning generalization (B S P Raju)	Simulated Demonstration Lesson on Diagnosis and Remediation in learning generalization (B S P Raju)		Identification of Difficulties in Solving verbal Problem (B S P Raju)		Identification of Difficulties in Solving verbal Problem (B S P Raju)	
12.12.2004 Sunday	Remediation in solving verbal problems (B S P Raju)	Remediation in solving verbal problems (B S P Raju)		Presentation and Discussion on Remediation of solving verbal problems (B S P Raju)		Presentation and Discussion on Remediation of solving verbal problems (B S P Raju)	
13.12.2004 Monday	Presentation and Discussion (B S P Raju)	Finalisation and Consolidation of the material developed (B S P Raju) – In Telugu		Finalisation and consolidation of the material developed (B S P Raju) – in Telugu		Valedictory	

Teaching Mathematical Concepts

The study of mathematics deals with certain objects such as Natural numbers, Circles, Triangles, functions and proof.

In learning about these mathematical objects, we are concerned with what these objects are. For example

1. What an angle is how to call whether or not something is a rectangle, what is the definition of a parallelogram ?
2. What are the relations among mathematical objects ?

When we learn what an object is, we are learning a concept of that object.

When we teach students what an object is, and how to identify it, we are teaching a concept of that object.

Concepts are the most basic learnable objects and the first things learned by young children.

By means of concepts, other concepts and other kinds of subject matter are learned.

A concept is the meaning of a term used to designate the concept.

According to Hunt, Marin and Stone (1966), "A concept is a decision rule which, when applied to the description of an object, specifies whether or not a name can be applied. Thus a student who knows the definition of a circle as the locus of points in a plane from a

given point in the plane has a rule that can be used to tell whether any given object is to be called a circle.

Moves in Teaching a Concept

Some concepts are taught, for others the term designating the concepts are used.

For example, a teacher who has deliberately taught a concept of a finite set might not teach a concept of an infinite set but would simply use the term.

1. Defining

Because most concepts in mathematics are precise, definitional moves can be used.

Definition is an elegant move since it employs minimum language. But the very elegance may be a block to learning.

Definitions are often written in the form (1) is a (2) such that (3).

The first space is filled by the term being defined, the second space is filled by a term denoting a superset in which the set of objects denoted by the term defined are included and third space is filled by one or more conditions that differentiate the set of objects denoted by the term defined from all the other subsets of the superset.

2. Stating a sufficient condition or sufficient condition move

It is the form in which a characteristic or a property of an object is stated that identifies it as a sufficient condition.

A rhombus is an equilateral parallelogram. Being an equilateral parallelogram is sufficient for being rhombus.

The sufficient condition is more clear in the statements :

“If a quadrilateral is an equilateral parallelogram, it is a rhombus”.

Other forms are:

“If a parallelogram is a square, it is a rhombus”.

A triangle is a right angled triangle provided that it has one right angle.

The logic of the move of sufficient condition enables a student to find examples of objects denoted by a concept, assuming such an example exists.

3. Giving one or more examples

Examples are objects denoted by the concept i.e. members of the set determined by the concept.

Examples clarify concepts because they are definite, specific and if well chosen familiar.

Teachers frequently elicit examples of concepts from students to decide whether the students have acquired the concepts.

Examples cannot be given for every concept. For example, even prime number greater than 2, greatest integer, and for self-contradictory concepts like square circle, six-sided pentagon.

4. Giving an example Accompanied by a Reason why it is an example

Accompanying an example with a reason that it is an example is an effective move because the reason is a sufficient condition.

This move is helpful to slow learners, because the logical connection is made explicit by supplying a reason.

5. Comparing and Contrasting Objects denoted by the Concept

By comparing objects of the concept being taught with objects with which students are familiar, a bond of association can be established between familiar and less familiar.

In teaching a concept of parallelogram, the teacher may compare it with non-parallelogram (trapezium).

Comparison points out similarities. But since objects compared are not identical, a contrast identifies some of the differences, if not all.

If a teacher has taught a concept of equal set and then teaches a concept of equivalent set, the next step may be to contrast these two concepts in order that the students do not miss the distinction between them.

6. Giving a Counter Example

A counter example is an example that disproves a false definition of a concept.

Two kinds of counter examples are possible for an incorrect definition.

1. Give a member (an example) of the set determined by the term defined that is not a member of the set determined by the defining expression.
2. Give a member (an example) of the set determined by the defining expression that is not a member of the set determined by the term defined.

Though this kind of move is effective in sustaining thinking and ultimately facilitates comprehension of the desired concept, students may feel that the teacher was badgering and embarrassing them. Teachers have to exercise good judgement when deciding how frequently to use counter example moves.

7. Stating a necessary Condition

If two sides are parallel, a quadrilateral is a parallelogram. This statement indicates the absence of a necessary condition for a quadrilateral to be a parallelogram.

One form of the definition of a parallelogram to satisfy the necessary condition is,

If both pairs of opposite sides are parallel, a quadrilateral is a parallelogram.

Another form in which a necessary condition is stated uses *only if*.

Ex. A quadrilateral is a parallelogram *only if* both pairs of sides are parallel.

A necessary condition move enables a student to identify examples of objects *not* denoted by a concept.

8. Stating a necessary and sufficient condition

This move is used, if condition by which objects can be denoted by a concept is both necessary and sufficient condition. One form for this is the explicit use of the terms necessary and sufficient, as

It is both *necessary* and *sufficient* that a parallelogram be equilateral for it to be a rhombus. Another form is the use of *if and only if*. Thus the statement is equivalent to,

A parallelogram is a rhombus *if and only if* it is equilateral.

The definition in terms of necessary and sufficient condition proceeds by subsuming the set of objects to be defined from all other subsets of the superset. Thus, a definition of a rhombus might be

A parallelogram having pair of adjacent, congruent sides is a rhombus.

The definition implies that there are two conditions necessary for an object to be a rhombus.

(1) being a parallelogram and (2) having a pair of adjacent congruent sides. The combination of these two necessary condition is sufficient. But for some students, the necessary and sufficient conditions in the above statement may not be clear. For them, the teacher can make use of *if and only if* form.

A sufficient condition move enables a student to identify examples and a necessary condition move enables students to identify non-examples of a concept. A combination of these enables students to discriminate both examples and non-examples of a concept.

An object not in the set determined by a concept is a non-example of the concept.

9. Giving non-examples

Like the move of giving examples, giving non-examples helps to clarify a concept. Definition of a concept followed by examples and non-examples of the concept is a common move for a teacher.

10. Giving a non-example accompanied by a reason why it is a non-example

This move is similar to that of giving an example together with a reason that is an example. The reason that accompanies the non-example is the failure to satisfy a necessary condition.

Its logic is that of conditional reasoning,

“If a quadrilateral is not a parallelogram, it is not a rhombus. This quadrilateral is not a parallelogram. Therefore it is not a rhombus”.

Strategies of Teaching a Concept

A *strategy* is defined as a temporal sequence of moves.

So, theoretically, there are thousands of strategies for teaching a concept, of which some are logically impossible.

Examples of some Strategies of teaching a concept

1. Definition ----- Examples ----- Example with a reason
Non-Example with a reason
2. Example ----- Non-example-----Comparison and Contrast -----
Characteristic -----Definition -----Example with a reason----
Non-example with a reason.

In such strategies, the definition identifies the necessary and sufficient conditions, examples clarifies them and reasons reinforce necessary and sufficient conditions.

Use of Concepts

1. Knowledge of a concept helps in classifying given objects into examples and non-examples of the concept.
Since we can classify, we can discriminate. For example, a student who has concept of rhombus can pick out rhombus from other quadrilaterals.
2. Knowledge of concepts helps in communication

Communication breaks when people do not have the knowledge of certain concepts.

A definition of a term tells you both how to use the term and also how to avoid using it.

Ex: A rhombus is an equilateral parallelogram.

This definition tells that a rhombus means, “an equilateral parallelogram” and if the students do not have the concept of an equilateral parallelogram, the teacher can think of the definition.

An equilateral parallelogram is a four-sided figure whose sides are line segments having the same length.

3. Concepts helps in Generalisation
4. Concepts help in discovery of new knowledge

Each statement given below describes a content category in Mathematics. All that you have to do is to identify the concepts, generalizations, facts and rules in the list. For example, if a statement describes a concept write 'concept' in the blank provided/against the statement.

- | | |
|------------------------|---|
| <u>Concept</u> | 1. The empty set is a subset of every set. |
| <u>Fact</u> | 2. The set of rational numbers is an abelian group under multiplication. |
| <u>Fact</u> | 3. Twin Primes are primes which differ by 2. |
| <u>Generalisation</u> | 4. An Isosceles triangle is a triangle having two congruent sides. |
| <u>Fact</u> | 5. A trapezium is a quadrilateral with a pair of opposite sides parallel. |
| <u>Concept</u> | 6. The natural number 1 is the unit for decimal system. |
| <u>Fact & Rule</u> | 7. Angles inscribed in the same area of a circle are congruent. |
| <u>Concept</u> | 8. Null matrix is a matrix in which every element is zero. |
| <u>Concept</u> | 9. A function is a relation in which no two distinct ordered pairs have the same first co-ordinate. |
| <u>Generalisation</u> | 10. Median of a triangle is a line segment whose end points are a vertex and the midpoint of the opposite side of the triangle. |

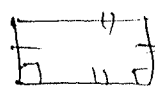
Note : I am facing language problem
 due to we are working
 Telugu medium school.
 and so -

Q.2 The definitions given below may have one or more of the following limitations to be a good definition.

- Being circular i.e., using one or more terms which need to be defined.
- Having unnecessary information
- Being too general/vague/not comprehensive using the above criteria, judge the goodness of the following definitions. If a definition possesses all the characteristics of a good definition, write 'good definition'. If a definition is not good, mention the limitation to be a good definition and rewrite the definition so that the rewritten definition is good.

1. A square is a rectangle with equal sides and equal angles

A rectangle with equal sides and angle with 90° .



2. Transversal is a line which intersects two or more lines in distinct points

3. Singular matrix is a matrix

- Good Definition.

4. Diameter of a circle is a chord passing through the centre

Good Definition.

5. A composite number is a number other than one which is not prime

Good Definition.

6. Incircle is a circle inside a triangle

A circle which is touching the triangle.



7. Identity matrix is a square matrix in which the leading diagonal elements are one and all other elements zero

Good Definition

8. Parallel lines are lines which are parallel

Parallel lines are lines which the length is between two lines.

9. Cyclic quadrilateral is a quadrilateral inside a circle

vertices are on circle.



10. Finite set is a set which is finite

Limited number containing the set.
Call finite set.

1. A square matrix is a Matrix having Equal no of Rows and column

1. A square matrix is a Matrix having Equal Rows and column

2. A chord is a line segment whose Two end points are on the circle

3. Circumcircle is a Circle which passes through the vertices of the triangle.

4. Coplanar lines are Some which Belongs to same
of lines

5. Union of two sets is a Set containing all which elements are belongs to two sets. $\therefore A = \{1, 2\}$
 $A \cup B = \{1, 2, 3\}$

6. Complement of a set is a Set having some elements containing a group.
 Ex: Prime no. Set = $\{2, 3, 5, 7, \dots\}$

7. Least common multiple of two natural numbers is l.c.m
which

Ex. 2 mul. - $\{2, 4, 6, 8, 10, \dots\}$ Above 2. C.M. is = 4
 $4 = \{4, 8, 12, \dots\}$

8. A prime number is a natural number which is not divisible
except 1 and the same number.

9. Column matrix is a Matrix having 1 column and several Rows. $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

10. Mode of a distribution is a ~~mode~~ ^{number} which
 a number ^{which} more times in given data.

En: 10, 11, 12, 13, 14, 12, 15, 70, 81, 82, 13
15, 15 =

If the two no. are same, the mode is two number, 15 (3 times)

Q.4 Each statement given below is a definition of a concept (with the concept name shown in capital letters). Rewrite the definition by replacing the term underlined in the statement.

1. RECTANGLE is a parallelogram in which each angle is right angle.
 Parallelogram as a Rectangle of its each angle as right angle.
2. ORTHOCENTRE of a triangle is a the point of concurrency of the attitudes of the triangle.
 Altitudes of the triangle is a the point of concurrency of orthocentre triangle.
3. DISJOINT SETS are sets whose intersection is a null set.
 A null set is disjoint sets are sets whose intersection.
4. SECANT is a line on the plane of a circle containing a chord of the circle.
 Chord of the circle which is circle containing a secant - in a line on the plane of circle.
5. ACUTE-ANGLED TRIANGLE is a triangle having each angle acute.
 Acute angles having a triangle is called a Acute Triangle.
6. SUPPLEMENTARY ANGLES are angles whose sum is a straight angle.
 Straight angle is called sum of supplementary angles.
7. PERPENDICULAR LINES are coplanar lines which intersect each other at right angles.
 Coplanar lines which intersect each other at right angles called perpendicular lines.
8. RHOMBUS IS an equilateral parallelogram.
 Equilateral parallelogram is a Rhombus.
9. MEAN DEVIATION of a set scores is mean of the deviations of the scores from their mean.
 Mean is the which is of the deviations of the scores from their mean deviation.
10. REGULAR POLYGON is a polygon which is equiangular and equilateral.
 Equiangular and Equilateral polygon is called Regular Polygon.

Q.5 Each statement given below describes condition(s) for a mathematical object to be an example of a concept. Specify whether the condition(s) is/are 'necessary' or 'sufficient' or 'necessary and sufficient'. Write 'undecided', if you cannot identify the type of the condition.

necessary and sufficient 1) If a number is divisible by two, it is an even number.

~~necessary~~
Undecided 2) If two lines on a plane do not intersect, they are not perpendicular.

Sufficient 3) A geometric figure is a circle 'if and only if' it is the locus of points in a plane which are equidistant from a given point.

necessary and sufficient 4) A line is a tangent to a circle only if it touches the circle at one and only one point.

Sufficient 5) A function is a linear function provided that its graph is a straight line.

necessary 6) If the ratio of each pair of consecutive terms is a constant, a sequence is geometric.

necessary 7) If both pairs of opposite sides are not parallel, a quadrilateral is not a parallelogram.

Sufficient and necessary 8) Two complex numbers are equal provided that their real parts are equal and their imaginary parts are equal.

necessary 9) A triangle is an isosceles triangle only if it has two congruent sides.

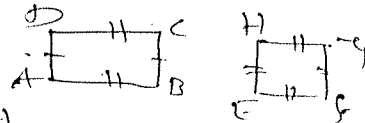
Sufficient 10) A matrix is an identity matrix only if it is a square matrix.

Q.6 Given a counter example to show that each of the definitions given below is incorrect

1. Equal matrices are matrices having equal number of rows and columns

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad A \neq B$$

2. Rectangle is a quadrilateral having congruent sides and angles



$$ABCD \neq EFGH$$

3. Diameter of a circle is a line segment passing through the centre of the circle

Def: Diameter of a circle is a chord passing through the centre of the circle. True.

4. Perpendicular lines are lines which intersect at right angles

Lines which intersect at right angle are perpendicular.

5. A rational number is a number of the form P/q where P and $q \in \mathbb{N}$

Ex: $-2/5$ is rational number.

A rational number is number of form P/q where $P, q \in \mathbb{Z}$

6. Singular matrix is a matrix whose transpose is equal to the given matrix $A \neq 0$

7. Rhombus is a parallelogram having the opposite pairs of sides equal



Parallelogram having the opposite pairs of sides equal called Rhombus.

8. Intersection of two sets is a set containing the elements in both the sets

Intersection of two sets is containing common the elements in both sets. Ex: $A = \{1, 2, 3\}$
 $B = \{2, 3, 4\}$

9. Scalene triangle is triangle in which each angle is acute. $A \cap B = \{2, 3\}$

Scalene triangle is triangle in which each angle is acute not necessary. Ex: $\angle A = 100^\circ, \angle B = 50^\circ, \angle C = 30^\circ$

10. Binomial is an open phrase of two terms of which one is a constant

Q.7 Given below are the names of some of the moves activities of teaching a concept in mathematics.

1. Defining
2. Stating a necessary condition
3. Comparing and contrasting
4. Stating a sufficient condition
5. Stating a necessary and sufficient condition
6. Giving a counter example

Using these moves, identify the type of each of the moves in a strategy of teaching the concept 'Isosceles Trapezium'. The moves to be identified are numbered and write the number in blank against a move to indicate the type of the move. You can use a name more than once in the identification.

Strategy

T : In the last class we discussed about a special kind of a geometric figure. We started with a closed four-sided plane figure what do you call such figures ?

.....1..... S_1 ?

S_1 : Quadrilaterals

T : Good. We also discussed about some quadrilaterals called trapeziums. What makes a quadrilateral a trapezium ?

.....2..... S_2 can you ?

S_2 : If a pair of opposite sides are parallel, it is a trapezium.

T : Right. If a pair of opposite sides of a quadrilateral are parallel, it is a trapezium.

Just as we found some special quadrilaterals that we called trapeziums, we are going to learn about some special trapeziums that we shall call Isosceles Trapezium.

A set of drawings of trapeziums is drawn on the chalk-board. Of these some are isosceles and others are not and are labeled accordingly. Measurements of nonparallel sides are indicated in all the drawings.

2.T: From these two categories, find out the characteristics that the isosceles trapeziums have in common that does not exist in other trapeziums ?2..... S_3

S_3 : A pair of sides are equal in length.

T : S_3 said that an isosceles trapezium has two equal sides. Can anyone draw a trapezium with two sides equal which is not an isosceles trapezium ?4..... S_4 draw the figure ?

3. S_4 : Draws a trapezium that is not isosceles.

T : Very good. This is not an isosceles trapezium. So there must be some other specification to describe an isosceles trapezium. What is it ? Observe carefully the congruent sides (S_5 volunteers to respond). Yes, S_5 ?

4. S_5 : In an isosceles trapezium the nonparallel sides are congruent.

T : S_3 you agree with S_5 ? (S_3 nods his head indicating his agreement with S_5 ? Now, write the definition of isosceles trapezium in your note books
state the definition? S_6 ?

5. S_6 : Isosceles Trapezium is a trapezium with the nonparallel sides equal.

6 T : Very good. We have learnt that trapezium is isosceles provided that it has the non-parallel sides equal.

Q.8. In each item given below, judge whether the test item measures the objective prefixed to it. Indicate your judgement by 'yes' if the item measures the objective and 'no' if it does not. Write 'undecided', if you cannot make judgement about the measurability of an objective.

1. Objective: Students will be able to define a symmetric matrix.

Test item: What is a symmetric matrix ? *Yes,*

2. Objective: Students will be able to state the condition for a parallelogram to be a rectangle.

Test item: Which of the following characteristic of a rectangle will not be found in all parallelograms ? *Undecided,*

1. Opposite sides parallel
2. Right angles
3. Opposite sides equal
4. Diagonals bisect each other

3. Objective: Student will be able to state the necessary condition(s) for a parallelogram to be square.

Test item: A parallelogram is a square only if it is equilateral. True/False. *Undecided,*

4. Objective: Students will identify correctly the primes among the given natural numbers.

Test item: Circle all the numbers below which are primes 4, 7, 9, 13, 15, 28, 31. *Yes.*

5. Objective: Students will be able to state the definition of a rectangle.

Test item: Which of the following is the best description of the meaning of a rectangle? *Yes.*

- a. A rectangle is a four-sided geometric figure with opposite sides parallel and equal.
- b. A rectangle is a four-sided geometric figure with opposite sides parallel and all sides equal.
- c. A rectangle is a four-sided geometric figure with opposite sides parallel and equal and the interior angles congruent.
- d. A rectangle is a four-sided geometric figure with equal sides and angles.

- Q.9. Following are some of the difficulties students may be confronted with, in learning/using mathematical manifested by the students' response to teachers' question in each of the items. Indicate your response to teach item by writing the number representing the kind of difficulty in the blank provided against each item.

Kinds of Learning Difficulties

1. Not knowing the term designating the concept (concept name)
2. Inability to state the meaning of the term designating a concept.
3. Inability to remember the condition(s) necessary for a mathematical object to be an example of the concept.
4. Misclassifying an example as a non-example for a concept and vice-versa.
5. Inability to deduce useful information from a concept.

Teaching - Learning Situations

1.T: When do you say that two sets equal? You (points to a student)?

S₁ : When the sets have same number of elements. (1)

T: Yesterday we talked about a special kind of geometric figure. We started with a closed four-sided figure. What do we call these? S₂?

S₂ : Sorry, I don't remember.

4.T: Yes, I is the point of concurrency of angle bisectors of a triangle. What is it called?

S: No response. 2

5.T: Which of the following numbers are composite? S₃?

6, 9, 13, 17, 25

S₃ : 13 and 17 (1)

6.T: What is a power set?

S: Sorry, I don't know. (1)

7.T : Right. ABCDEF is regular hexagon. So what is the measure each angle of this hexagon?

S : I know that the angles are congruent, but I can't tell the measure of an angle. (3)

8.T : Why do you say that $5/2$ is a proper fraction?

S : Because the numerator is greater than the denominator. (1)

9.T : What are concentric circles?

S : Concentric circles are a kind circles. (3)

10.T : If A & B are equal sets, then ABC and BCA. Is this statement true?

S : I have no idea. — (2)

Q. 10. Write the specific instructional objectives of teaching concept, function.

1. Reading writing.
2. Knowledge : Recall ; Recognize
3. Understanding : Concept, principle.
how to solve Problems
4. Application : Using principles.
Drawing skills. Simplification
Constructing.

① Name of the Definition:

Complimentary Angles: (ସମ୍ପର୍କିତ କෝණ);

① Def: ଦୁଇ କିମ୍ବା ଅଧିକ 90° କୋଣ ଯଦି ସମ୍ପର୍କିତ କିମ୍ବା କୋଣ.

② Sufficient condition: (i) ଦୁଇ କିମ୍ବା ଅଧିକ 90° କୋଣ ଯଦି ସମ୍ପର୍କିତ.

(ii) ସମ୍ପର୍କିତ କୋଣ ଦୁଇ କିମ୍ବା ଅଧିକ $= 90^\circ$.

③ Example: ଦୁଇ କିମ୍ବା
 ~~$45^\circ, 45^\circ$~~ ; ~~45°~~ .
 $50^\circ, 40^\circ$; $30^\circ, 60^\circ$; $88, 10^\circ$

(4) Example with Reason: $50^\circ + 40^\circ = 90^\circ$. ଦୁଇ କିମ୍ବା
 $30^\circ + 60^\circ = 90^\circ$. "

(5) Non Example: $40^\circ, 60^\circ$. ଦୁଇ କିମ୍ବା
 90° ନୁହେଁ.

(6) Non Example with reason: $40 + 60 = 100$. " ନୁହେଁ.

(7) Necessary condition: ଦୁଇ କିମ୍ବା ଅଧିକ $= 90^\circ$
 $\angle A + \angle B = 90^\circ$.

(8) Necessary condition: ସମ୍ପର୍କିତ: $45^\circ, 45^\circ$; $45^\circ + 45^\circ = 90^\circ$

(9) Contrast compare. Contrast: Concept condition: ଦୁଇ କିମ୍ବା ଅଧିକ $= 90^\circ$

(10) Counter Example: $40 + 60 = 100^\circ$ ନୁହେଁ କିମ୍ବା ସମ୍ପର୍କିତ ନୁହେଁ.

$40^\circ, 50^\circ$	$30^\circ, 50^\circ$
$30^\circ, 60^\circ$	$40^\circ, 60^\circ$
$20^\circ, 70^\circ$	$50^\circ, 70^\circ$

(2) କିମ୍ବା ଅଧିକ 90° କୋଣ
କିମ୍ବା ସମ୍ପର୍କିତ ନୁହେଁ.
 $50 + 40 = 90$

(10) Counter Example: 80° ನಿಗಲ ಮೊತ್ತ 90°
ಇಕಂಪಿ ಲೇ ಪ್ರಮಾಣ
ಇರಲಿ.

Ex: 50°, 60°.

$$50 + 60 = 110$$

$$110 \neq 90$$

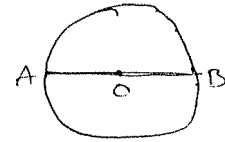
∴ ಲೇ ಪ್ರಮಾಣ.

u ——— m

① Name of the concept: ವೃತ್ತದ ವೃತ್ತದ (Diameter)

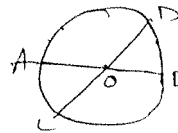
② Def:- 'ವೃತ್ತದ ಕೇಂದ್ರವು ಹಾದು ಹೋಗುವ
ವೃತ್ತದ ವೃತ್ತದ ಬಿಂದು.'
'Diameter is cord passing through
the centre.'

(3) - Essential attributes: ವೃತ್ತದ ಕೇಂದ್ರವು ಹಾದು ಹೋಗುವ.

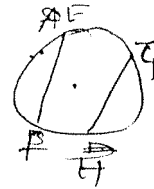


(4) Non Essential attributes: Length of the
Diameter.

(5) Exa:- \overline{AB} , \overline{CD}



(6) Non Exa:- \overline{AB} , \overline{CD}
 \overline{EF} , \overline{GH}



(7) Conceptual Hierarchy:

(a) Super Ordinate concept: Secant-line.

(b) Subordinate " " Radius, ವೃತ್ತದ.

(c) Co-ordinate " " Chords.

Diagnosis and Remediation of Difficulties in Learning Concepts

1. Inability to recall (remember) the concept name

Ex: Not being able to call a line segment whose end points are two non-adjacent vertices a polygon as a diagonal.

Remediation: Remind the concept name and use exemplification moves. For example, tell that the name of such a line segment is DIAGONAL and ask students to give the name of line segments whose end points are two non-adjacent vertices the polygon or ask students to draw one or more diagonals of a polygon.

2. Inability to define a concept

Ex: A student may be unable to say what the term SINGULAR MATRIX means. That is, not being able to give a definition of the singular matrix.

Remediation: Use definition move (The definition can also be elicited from other students). Check to see that the student understands the language being used. Using exemplification moves can do this.

3. Inability to give or recognize an example of a concept

Ex: Not being able to recognize 25 as a perfect square.

A cause of the difficulty could be not knowing/remembering a sufficient condition.

Remediation: Use sufficient condition move (this may be done by asking students when a number is a perfect square). Use exemplification moves to reinforce the concept.

4. Inability to remember one or more conditions necessary for an object to be an example of a concept

Ex: For example, a student may not remember that, if a parallelogram has right angles but has no congruent sides, it is not a square. As a result the student may think some parallelograms are squares that are not.

Remediation: Remind the student of the necessary conditions and use exemplification moves.

5. Inability to remember a condition sufficient for an object to be an example of a concept.

Ex: For example, a student may not remember that, if a rhombus is equiangular, it is a square.

Remediation: Remind the student of the sufficient condition.

6. Misclassifying a non-example as an example of a concept and vice-versa.

Ex: A student cites similar triangles that are congruent triangles as examples for congruent triangles. A student who cites a non-example, as an example probably does not know that a condition, which is necessary, is not sufficient. The necessary condition restricts the set of objects denoted by the concept. Not knowing that a necessary condition, which is not sufficient, makes the student to include in the set, objects that are not examples of the concept.

Ex: A student does not consider congruent triangles as examples of similar triangles. The cause for this kind of misclassification of an example as a non-example does not know that a condition, which is necessary is also sufficient. The sufficient condition admits objects to the set denoted by the concept; not knowing the sufficient condition makes the student exclude objects that should be in the set.

Remediation: Identify first the kind of misclassification manifested by the student's errors. If a student does not know a necessary or sufficient condition, use the necessary or sufficient condition move and exemplification moves.

7. Inability to deduce useful information from a concept

Ex: A student may not prove that the diagonals of a rhombus are perpendicular, if he/she cannot prove that a pair of triangles are congruent. The student cannot prove that the triangles are congruent because he/she cannot deduce from the hypothesis (that the parallelogram is a rhombus) that a pair of adjacent sides are congruent.

Remediation: It is harder to offer any methodological suggestions of a general nature to help a teacher help students who have this kind of difficulty. However, the teacher can focus on the concept and ask probing questions to direct the student's thinking.

8/12/04

Circle :

- (1) $\text{Circumference} = C = 2\pi r$ — Generalisation
- (2) $\pi = \frac{22}{7}$ — Fact
- (3) $\text{Area of Circle} = A = \pi r^2$ — Generalisation
- (4) Segment of Circle — Concept
- (5) $\text{Radius} = r$ — Concept
- (6) Centre Point of circle — Concept
- (7) $\text{Line segment} = \text{chord}$ — Concept
- (8) $\text{Distance} = d$ — Concept
- (9) $\text{Diameter} = d$ — Concept
- (10) $\text{Radius} = r = \frac{d}{2}$ — Generalisation
($r = \text{Radius}$, $d = \text{Diameter}$)
- (11) $\text{Area of Half Circle} = \frac{\pi r^2}{2}$ — Generalisation
- (12) Length of Half circle — $\frac{36}{7} r$ — Generalisation
($\text{Circumference} = 2\pi r$) (Concept)
- (13) Tangent —
- (14) Secant — 36 —

Q. no.	Question	Answer given by student	Correct Answer	Possible conjecture	Try for Conjecture	Remedial instructions
1.	6^2 ఎలా?	12	36	Inability to remember a condition sufficient for an object to be an example of a concept.	3^2 ఎలా? 4^2 ఎలా?	$6^2 = 6 \times 6 = 36$ $3^2 = 3 \times 3 = 9$ $4^2 = 4 \times 4 = 16$
2.	'm' గానే వివరించుకు?	'm' ను భావించండి.	'm' ను భావించండి.	Inability to define a concept.	am అని 'a' ను వివరించు?	am అని 'a' ను భావించు, m ను భావించు. రెండూ.
3.	$8x^3 + 6x^2 + 5x + 4$ బహుపది పేరుకా?	8	3	Inability to remember one or more conditions necessary for an object to be an example of a concept.	$2x^3 + x^2 + x + 1$ బహుపది పేరుకా? $x^3 + x^2 + x + 1$ బహుపది పేరుకా?	బహుపది అని పదాల పేర్లు కాక, అక్షరాల పేర్లు కూడా ఉంటాయి. బహుపది పదాలతో కూడినది.
4.	XC ఎలా?	110	90	Misclassifying a non-example as an example of a concept and vice versa.	CX ఎలా?	ఒక రెండో పదపద్యం ఇవ్వండి, దాని రెండో పదం సరిగ్గా రావాలి, ఒక రెండో పదం ఇవ్వండి, రెండో పదం సరిగ్గా రావాలి.
5.	$45^\circ, 45^\circ, 90^\circ$ వ రేఖామిత్రాలు?	లంబకోణం త్రిభుజం.	లంబకోణం సమదీర్ఘ త్రిభుజం.	Inability to remember a condition sufficient for an object to be an example of a concept.	$50^\circ, 50^\circ, 100^\circ$ వ రేఖామిత్రాలు?	ఒక త్రిభుజం అని ఏదైనా రెండు కోణాలు సమానమైతే, ఆ త్రిభుజం లంబకోణం త్రిభుజం.

Topic: అభిప్రాయ

VII class.

Teacher's } : ఇది అభిప్రాయం అని నిర్ణయించుకోవడం, దానిని నిరసించడం

Students } : అవును. (wrong)

Correct } : అవును.

Possible } : Inability to deduce useful information from a
conjecture } : concept.

Test for } : 1. అభిప్రాయం నిజమేనా? అవును.
Conjecture } : 2. అభిప్రాయం నిజం కాదునా? అవును.

Remedial } : 1. అభిప్రాయం అంటే ఏమిటి?
Instructions } : 2. అభిప్రాయం అంటే ఏమిటి?
3. అభిప్రాయం నిజమేనా? అవును.
4. అభిప్రాయం నిజం కాదునా? అవును.
5. ఇది అభిప్రాయం, దానిని నిరసించడం
అవును.

Question

సమాధి పాట

తెనకా పాట

Answer given by the student

తెనకా పాట సమాధి పాట నా పాట నా పాట నా పాట

Correct Answer

సమాధి పాట నా పాట నా పాట నా పాట నా పాట

Possible Conjectures

1) సమాధి పాట నా పాట నా పాట నా పాట నా పాట

Text for Conjectures

1) 4a, 5a పాట నా పాట నా పాట నా పాట నా పాట

Reminded Introduction

1) సమాధి పాట నా పాట నా పాట నా పాట నా పాట

2) జ్ఞాన

1, iii ప్రతిపక్ష సమాధి పాట

సమాధి పాట" అని 4) త

ప్రతిపక్ష సమాధి పాట" సమాధి పాట

కానీ" అని ప్రతిపక్ష సమాధి పాట పాట

పాట" సమాధి పాట సమాధి పాట

పాట నా పాట నా పాట సమాధి పాట

పాట" అని ప్రతిపక్ష సమాధి పాట

ప్రతిపక్ష సమాధి పాట

(ii) సమాధి పాట సమాధి పాట

సమాధి పాట సమాధి పాట

సమాధి పాట సమాధి పాట

సమాధి పాట సమాధి పాట

P. Rave Ram

(P. Rave Ram)

T. R. M. M.

A. P. R. School, Vemuravolu

Questions

వికవది యొక్క పరిమాణం
రెనానా ఎటు?

Answer given
by the student-

వికవది వికవది యొక్క
పరిమాణం ఎంతమంది
రెనానా యొక్క
పరిమాణం ఎంతమంది

Correct
Answer

వికవది వికవది యొక్క
పరిమాణం ఎంతమంది
రెనానా యొక్క
పరిమాణం ఎంతమంది

Possible
Conjectures

(1) వికవది యొక్క పరిమాణం ఎంతమంది?
(2) వికవది యొక్క పరిమాణం ఎంతమంది?

Text for
Conjectures

(1) వికవది యొక్క పరిమాణం ఎంతమంది?
(2) వికవది యొక్క పరిమాణం ఎంతమంది?

Remind! Instructions

(1) వికవది యొక్క పరిమాణం ఎంతమంది?
(2) వికవది యొక్క పరిమాణం ఎంతమంది?

(1) వికవది యొక్క పరిమాణం ఎంతమంది?

(2) వికవది యొక్క పరిమాణం ఎంతమంది?

(1) వికవది యొక్క పరిమాణం ఎంతమంది?

(2) వికవది యొక్క పరిమాణం ఎంతమంది?

AN INSPIRATION TO LEARN AND TEACH MATHEMATICS

by
Dipendra N. Bhattacharya
Clarion University of Pennsylvania

This article is based on a talk "A Journey to Truth, Goodness and Beauty in Mathematics (How to Motivate the Unmotivated)", given at the 66th Annual NCTM Meeting in Chicago.

I will begin by stating what must be understood above all: The learning and teaching of mathematics is a journey, not a destination; and, motivation is the key to success.

A long time ago, my father told me that anything worthy of study must have three qualities. They are Satyam, Sivam, and Sundaram which in English means Truth, Goodness and Beauty.

Let us examine mathematics, and determine if it renders any of these qualities...Is math based on truth? I am sure, that your students will not argue with that. Mathematics has survived five thousand years of human history; surely, to do so it must be based on truth.

Does math contain any goodness? Is math good for us? Well, I am a brown man making a living in a foreign country because of it -- it has been good to me!

So, we have determined that math is based on truth, and it is good...what about beauty? Is math beautiful? At this point, most students will say, "Please give us a break! We will agree with the first two, but there is no way you can convince us that math is beautiful!" Well, with the assistance of a few examples, I will show you the beauty in mathematics...

EXAMPLE #1 One day when I was teaching high school in Canada, I was asked to substitute for the Chemistry teacher. I do not know much about Chemistry, but I thought I might be able to teach them something anyway. I asked them what the topic of the day was, and they replied "Catalytic Agents". I was fairly sure that a catalytic agent is a chemical element or compound which takes part in a chemical process, but remains unaltered after the reaction has taken place. I checked with one of their books, and then told them I was

going to tell them a story to illustrate the term. A long time ago, a very wealthy king left a will stating that horses shall be divided as such:

$1/2$ shall go to his first son
 $1/3$ shall go to his second son
 $1/9$ shall go to his third son

When he died, his three sons found 17 horses in the stable. The sons, not being very bright, concluded that the only fair way to do as the will wished was to cut the seventeenth horse up with a saw! Just as they were about to begin sawing, an old (and very wise) minister happened by and was shocked by their intended actions. "What ever are you doing?," he exclaimed, shocked by their stupidity. When the three dim-witted princes explained that their only alternative was to cut up the poor horse, the minister made a very wise suggestion..."since everything I own is truly the property of my master, add my horse to the seventeen. Please allow me to help you divide them, and thus fulfil my master's last wish."

The brothers agreed to the wise man's suggestions, and the horses were divided in this manner...

$17 + 1 = 18$, so...
 $1/2$ of $18 = 9$, which were given to the oldest son
 $1/3$ of $18 = 6$, which he gave to the second son
 $1/9$ of $18 = 2$, which the third son took

Total = 17

So, the last son took his 2 horses, and the old minister rode off on the same horse he started out on, proving that a horse can make a very good catalytic agent!

EXAMPLE #2 Ronald Regan was born in 1911, and he is 77 years old. He took office as the President of the United States of America in 1980, and has been in office for 8 years. If a person were to add up these numbers ($1911 + 77 + 1980 + 8$), they would add up to 3,976.

Mikhail Gorbachev was born in 1931, and he is 57 years old. He became the Boss of the Soviet Union in 1985, and has been in charge for 3 years. If a person were to add these numbers $(1931 + 57 + 1985 + 3)$, the answer will again be 3,976!

Is this some kind of eerie coincidence?

Are you mystified, or did you figure out that this strange coincidence will even work for you?

The reason, of course, is that anyone's year of birth and present age will add up to 1988! Similarly, the other two dates will also always add up to 1988! And, $1988 + 1988 = 3,976$!

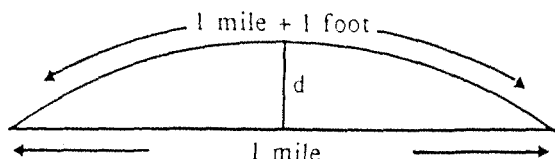
This is an example of a fixation, which is one of the biggest difficulties a person faces when trying to learn mathematics. The following examples are of fixations. You will like them...satisfaction guaranteed!

EXAMPLE #3 There are gaps between railway ties, so that they look like this:



This is done, so that they do not bend when the tracks expand due to the rise in temperature. Suppose we do not leave these gaps. Consider the following situation:

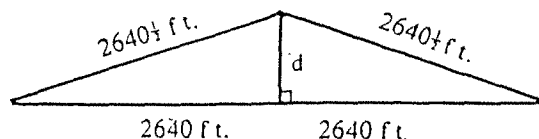
The line is one mile long, and has been expanded one foot, so that the new length is 1 mile + 1 foot. As a result of this expansion, the tracks buckle in the center, causing a bulge. Consider:



How high is bulge d = ?

Please do not calculate, just guess. Most people guessed that d is less than 4 inches. Nobody guessed more than 10 inches. Then we used the Pythagorean Theorem to find d .

1 mile = 5280 feet



$$d^2 + (2640)^2 = (2640\frac{1}{2})^2$$

$$d = \sqrt{(2640\frac{1}{2})^2 - (2640)^2} > 51 \text{ feet}$$

Being a fixation, no one guessed that d could be that big.

EXAMPLE #4 A class has 99 girls and 1 boy, how many girls must leave the class, so that the percentage of girls becomes 98%?

The answer is 50, because $99 - 50 = 49$, and 49 girls out of 50 is 98%.

EXAMPLE #5 A pole is 30 feet high. A monkey climbs the pole 10 feet during the day (6 a.m. to 6 p.m.), working nonstop and comes down 8 feet during the night. How many days will it take the monkey to reach the top of the pole?

Did you notice that the answer is not 15, but 11 (or $10 \frac{1}{2}$) days? Note that after 10 days, the monkey will have climbed 20 feet. So, what happens after day 10?

EXAMPLE #6 A trucker drives 20 mph for the first half of his trip from New York to Philadelphia. At what speed should he come back so that the average speed (total distance/total time), for the entire trip is 40 mph? Most people guess 60 mph. However, he will never make it! Can you figure out why?

EXAMPLE #7 Find the pattern (*):

$$\begin{aligned} 3 * 5 &= 4 \\ 6 * 2 &= 4 \\ 9 * 3 &= 0 \end{aligned}$$

And, what is...

$$\begin{aligned} 6 * 6 &= ? \\ 5 * 5 &= ? \\ 1 * 8 &= ? \end{aligned}$$

The sum of 3, 5 and 4 is 12, and so are the others.

EXAMPLE #8 "How much will one cost?"

"Twenty cents," replied the clerk in the hardware store.

"And how much will twelve cost?"

"Forty cents."

"O.K., I'll take eight hundred and twelve."

"That will be sixty cents."

What was the customer buying? House numbers! (20 cents a piece.)

Now that you have somewhat of an idea about fixations, please allow me to demonstrate it to you:

I am going to fixate you. Please try to hang tight, and not get fixated. Well, first let me tell you a little story. There is a corner store ... a Seven Eleven. One evening at 7:30 p.m., a mute person came into the store. When the clerk asked him what he wanted, the mute pretended to puff on an imaginary cigarette. The clerk immediately understood, and handed him a pack of smokes. Next, a blind man came into the store wanting to buy a pair of scissors. The clerk asked him what he wanted to buy. What do you think he did to convey his wishes to the clerk? If you got fixated, you will assume that he made a cutting motion with his fingers, which he did not. He simply asked for them!

Remember, in order to be successful in mathematics, one has to be alert and awake at all times. Don't get fixated!

The session was ended with the following story...

There once was a very frustrated math teacher who had just explained the rules for adding fractions at least a dozen times. In spite of this, one student still was not catching on. The teacher became so upset by this that she yelled, "You are so stupid. You are like the blackness of coal that will not disappear even after a hundred washings!"

At this time, the school principal happened to be passing by and heard the teacher's hasty comment. He walked in to the class, and looking directly into the teacher's eyes said, "Ah, but the blackness of that same coal disappears when fire ignites it!" In other words, if teachers are good, care about their students, and know how to motivate them, just as the blackness of coal disappears when fire enters it, so does confusion.

I hope that from my talk you will be able to see the Satyam, Sivam, and Sundaram that can be found in mathematics. It is my wish that from this you may find your own way to motivate the unmotivated, to help them see the truth, goodness, and beauty in math -- to help them want to find it for themselves.

CALL FOR ARTICLES

The theme for the September, 1989 issue will be
RELATIONS, PATTERNS & FUNCTIONS.

Authors are requested to submit articles
(both short & long, practical and theoretical)
for all ages and ability levels.

Instruction Comprehension Test

This test has to be completed in 2 minutes. Read all the questions before starting the test. Mark all the answers on the question paper itself. Work swiftly and accurately. GOOD LUCK!

1. Write down all the odd numbers from 0 to 10

2. When you divide 12 by 3 you get _____
3. Write down your name backwards _____
4. Put your hand on your head and solve $2/3 + 5 - 4.5 + 7.9$

5. If you reach this point shout "I'm the fastest" _____
6. Hold your pencil in your left hand and write "Hello"

7. Turn 180 degrees in your seat and face the person behind you.
8. If at this point you think that you will be the first to complete this test shout "I will win".
9. Write down three features that you think makes your face look good.
10. Shake the hand with the person next to you.
11. Now that you have read all the questions, answer only the first three.
12. Think kind thoughts of the person who set this test.

LIST OF PARTICIPANTS

- | | |
|---|---|
| <p>1. K C S Raju
A P Residential School
Kalvabugga (village)
Orvakal (Mandal)
Kurnool 518 011, AP</p> | <p>10. P Dattu Rao
TGT in Mathematics
A P Residential School (G)
Pochampad
Nizamabad Dist, AP</p> |
| <p>2. N Rambabu
TGT in Mathematics
A P Residential School
Bhupathipalam
East Godavari Dist, AP</p> | <p>11. K Rama Krishna Reddy
TGT in Mathematics
A P Residential School
Lepakshi
Anantapur Dist 515 331, AP</p> |
| <p>3. D Sathyanarayana Reddy
TGT in Mathematics
A P Residential School
Mukkavori Palli
Kadapa Dist</p> | <p>12. B Narayana Murthy
TGT in Mathematics
A P Residential School (G)
Thatipudi, S.Kota (Mandal)
Vizianagaram Post 535 160, AP</p> |
| <p>4. Parasuramapura Siva Prasad
TGT in Mathematics
A P Residential School
Vomaravalli
Srikakulam Dist, AP</p> | <p>13. C Srinivasa Murthy
TGT in Mathematics
A P Residential School
Nimmakuru
Krishna Dist 521 157, AP</p> |
| <p>5. T Raghavulu
TGT in Mathematics
A P Residential School (G)
Thatipalli Mandal
Malyal Dist, Karimnagar, AP</p> | <p>14. Mancharla Moses
TGT in Mathematics
A P Residential School
Nagireddygudem
Warangal Dist, AP</p> |
| <p>6. V Anjaneyulu Goud
TGT in Mathematics
A P Residential School
Nagakurnool
Mahabubnagar 509 209, AP</p> | <p>15. K Rajaiah
TGT in Mathematics
A P Residential School (G)
Sofinagar, Nirmal
Adilabad Dist 504 106, AP</p> |
| <p>7. V Madan Mohan Rao
TGT in Mathematics
Arekal, APR School (BC) (G)
Kurnool Dist, AP</p> | <p>16. M Srinivasa Rao
TGT in Mathematics, A P
Residential School B C Boys,
Pennahobilam, Uravakonda
Anantapur Dist 515 812, AP</p> |
| <p>8. P Kotaiah
TGT in Mathematics
A P Residential School
Gandipalem, Nellore Dist, AP</p> | <p>17. M S S V V Prasad
TGT in Mathematics
A P Residential School
Musunuru 521 207, AP</p> |
| <p>9. S Sathyanarayana
TGT in Mathematics
A P Residential School
Girls Vangara
Karimnagar Dist, AP</p> | <p>18. Narasimha Reddy
TGT in Mathematics
A P Residential School
Nagarjuna Sagar
Nalgonda Dist 508 202, AP</p> |